Robust Stability in Matching Markets*

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Abstract

In a matching problem between students and schools, a mechanism is said to be robustly stable if it is stable, strategy-proof, and immune to a combined manipulation, where a student first misreports her preferences and then blocks the matching that is produced by the mechanism. We find that even when school priorities are publicly known and only students can behave strategically, there is a priority structure for which no robustly stable mechanism exists. Our main result shows that there exists a robustly stable mechanism if and only if the priority structure of schools is acyclic (Ergin, 2002), and in that case, the student-optimal stable mechanism is the unique robustly stable mechanism.

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1 Introduction

Matching theory has influenced the design of labor markets and student assignment systems. Stability plays a central role in the theory: A matching is stable if there is no individual agent who prefers being unmatched to being assigned to her allocation in the matching, and there is no pair of

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agents who prefer being assigned to each other to being assigned to their respective allocations in the matching. In real-world applications, empirical studies have shown that stable mechanisms often succeed whereas unstable ones often fail.

In recent years, the incentive properties of stable mechanisms have attracted much attention. Roth (1982) shows that any stable mechanism is manipulable. However, if preferences of one side of the market are common knowledge, as in school choice (Abdulkadiroğlu and Sönmez, 2003) where school priorities are exogenously given by law, the student-optimal stable mechanism is both strategy-proof and stable (Dubins and Freedman, 1981; Roth, 1982). Indeed, the student-optimal stable mechanism has been adopted in practical assignment problems, such as student assignment in New York City and Boston and the National Resident Matching Program.

However, most existing analysis has overlooked other types of manipulation, as stability and strategy-proofness have been studied separately. If agents are capable of misreporting their preferences during the centralized matching process and also rematching (blocking) after the matching is announced, then they may be able to use the combination of these manipulations to their advantage. Chakraborty, Citanna, and Ostrovsky (2009) consider the combination of these manipulations and propose a strong stability concept requiring robustness against these manipulations in a matching problem with interdependent values.[1] Adapting their concept to the standard matching model without interdependent values, we say that a mechanism is robustly stable if no student is made strictly better off by a combined manipulation of misreporting preferences and rematching.

Although this departure from the standard concepts may seem small, it has very different implications on the design of matching mechanisms. First, we demonstrate that, even when school priorities are exogenously given and only students can behave strategically, there is no robustly stable mechanism in general.

Given the above impossibility result, a natural question is what conditions allow for a robustly stable mechanism. Our main result characterizes the existence of a robustly stable mechanism in terms of the priority structure of schools. More specifically, we show that there is a robustly stable mechanism in a market if and only if the priority structure of schools in that market is acyclic (Ergin, 2002). Moreover, if there is a robustly stable mechanism, then it coincides with the student-optimal stable mechanism.

The analysis of this paper suggests that one cannot expect complete elimination of manipulations even when only students can act strategically. If the

[1]The relation to their paper will be discussed subsequently.

social planner can influence the priority structure, as in the case of student placement in public schools, the theory suggests that acyclicity is likely to make the system immune to manipulations. However, acyclicity is a very demanding condition, and so this paper suggests that robust stability is hard to guarantee, even when the social planner can influence the priority structure to some extent.

Are combined manipulations important in real world? While a comprehensive analysis is beyond the scope of this paper, a suggestive example can be found in the school choice problem (Abdulkadiroğlu and Sönmez, 2003). For instance, in New York City, many students participate in an appeals process to be assigned to a school they like better than their prescribed assignment (Abdulkadiroğlu, Pathak, and Roth, 2005, 2009). About 300 appeals out of about 5,000 were from students who received their stated first choices. This may suggest that students can engage in rematching in the school choice setting. A more detailed discussion is given in the Conclusion.

Section 2 presents the model and the results. The relation to the literature will be discussed after the main result of the paper. Section 3 concludes.

2 Model and Results

A matching problem is tuple $(S, C, P, \succ, q)$. S and C are finite and disjoint sets of students and schools. For each student $s \in S$, P_s is a strict preference relation over C and being unmatched (denoted by $\emptyset$). We write cR_sc' (where $c, c' \in C \cup \{\emptyset\}$) if either cP_sc' or $c = c'$. For each school $c \in C$, $\succ_c$ is a **priority**, which is a strict, complete and transitive binary relation over S.[2] We write $\succ = (\succ_c)_{c \in C}$. For each $c \in C$, q_c is the **quota** of c. A **matching** is a vector $\mu = (\mu_s)_{s \in S}$ assigning a seat at school $\mu_s \in C$ or $\emptyset$ to each student s, with seats in each school c assigned to at most q_c students. We write $\mu_c = \{s \in S | \mu_s = c\}$ for the set of students who are assigned seats at school c. The set of a student's possible preferences is denoted by $\mathcal{P}$.

We say that matching μ is **blocked** by $(s, c) \in S \times C$ if $cP_s\mu_s$ and either (1) $|\mu_c| < q_c$ or (2) $|\mu_c| = q_c$ and $s \succ_c s'$ for some $s' \in \mu_c$. A matching μ is **individually rational** if $\mu_s R_s \emptyset$ for every $s \in S$ and $|\mu_c| \leq q_c$ for every $c \in C$. A matching μ is **stable** if it is individually rational and is not blocked.

We refer to a tuple $(S, C, \succ, q)$ as a **market** and consider a situation where only student preferences are private information while the market $(S, C, \succ, q)$ is given. A **mechanism** is a function φ from $P^{|S|}$ to the set of all matchings. Mechanism φ is **stable** if $\varphi(P)$ is a stable matching for every $P \in \mathcal{P}^{|S|}$.

[2]As we are primarily interested in the school choice problem, we assume that every student is acceptable to every school.

Mechanism φ is **strategy-proof** if $\varphi_s(P) R_s \varphi_s(P'_s, P_{-s})$ for every $P \in \mathcal{P}^{|S|}$, $s \in S$ and $P'_s \in \mathcal{P}$. Note that we only allow students to report preferences; school priorities are publicly known. This assumption simplifies the analysis and helps illuminate the consequences of the stability concept of this paper. Publicly known school priorities arise naturally in the school choice setting: As Abdulkadiroğlu and Sönmez (2003) point out, school priorities are exogenously given by law in many school districts. Similarly, Chakraborty, Citanna, and Ostrovsky (2009) consider two-sided matching between students and colleges in which preferences of students are publicly known. They motivate their assumption by noting that (i) information about colleges is mostly public in practice, and (ii) because of extant impossibility results, such an assumption is *necessary* to obtain positive results. These points hold in our setting as well.

Definition 1. A mechanism φ is **robustly stable** if the following conditions are satisfied.

(1) φ is stable,

(2) φ is strategy-proof, and

(3) there exist no $s \in S$, $c \in C$, $P \in \mathcal{P}^{|S|}$ and $P'_s \in \mathcal{P}$ such that (i) $c P_s \varphi_s(P)$ and (ii) $s \succ_c s'$ for some $s' \in \varphi_c(P'_s, P_{-s})$ or $|\varphi_c(P'_s, P_{-s})| < q_c$.

In words, a mechanism is robustly stable if it is stable, strategy-proof and also immune to a combined manipulation, where a student first misrepresents his or her preferences and then blocks the matching that is produced by the centralized mechanism. Condition (3) is the additional requirement over the combination of stability and strategy-proofness, and it plays a central role in our analysis. To the best of our knowledge, Chakraborty, Citanna, and Ostrovsky (2009) are the first to consider this combined manipulation in two-sided matching.[3] They consider a Bayesian game of matching with interdependent values in which a player can both misreport in the matching process and rematch afterwards. They say that a mechanism is stable if there is a perfect Bayesian equilibrium in which all players report their signals truthfully and all players accept their assigned partners on the equilibrium path. Although the direct comparison is somewhat subtle because of modeling differences, the robust stability concept defined here is conceptually close to and is motivated by the stability concept employed by Chakraborty, Citanna, and Ostrovsky (2009).

[3]In a different context of principal-agent problem, Myerson (1982) considers a similar notion of combined manipulations.

Given P, the student-proposing **deferred acceptance algorithm** produces a stable matching $\varphi^S(P)$ (Gale and Shapley, 1962). The **student-optimal stable mechanism** is a mechanism φ^S that produces $\varphi^S(P)$ for every $P \in \mathcal{P}^{|S|}$. It is well known that φ^S is stable (Gale and Shapley, 1962) and strategy-proof (Dubins and Freedman, 1981; Roth, 1982). Moreover, Alcalde and Barberà (1994) show that φ^S is the unique stable and strategy-proof mechanism. The following example demonstrates, however, that φ^S is not immune to the combination of these two kinds of manipulations even though it is immune to each of them separately.

Example 1. Consider a problem with $S = \{1, 2, 3\}, C = \{a, b\}$, and

$$\begin{aligned} &P_1 : b, a, \emptyset, \\ &P_2 : a, \emptyset, \\ &P_3 : a, b, \emptyset, \\ &\succ_a : 1, 2, 3, && q_a = 1, \\ &\succ_b : 3, 1, 2, && q_b = 1. \end{aligned}$$

Under the true preferences $P = (P_1, P_2, P_3)$, the student-optimal stable mechanism φ^S produces $\varphi^S(P) = (\varphi^S_1(P), \varphi^S_2(P), \varphi^S_3(P)) = (a, \emptyset, b)$.

Now consider a false preference P_2' of student 2, $P_2' : \emptyset$. Then, under $P' = (P_2', P_{-2})$, φ^S produces $\varphi^S(P') = (b, \emptyset, a)$. Since $aP_2\emptyset = \varphi^S_2(P)$ and $2 \succ_a 3 \in \varphi^S_a(P')$, φ^S is not robustly stable: More specifically, student 2 has incentives to first report P_2' and then block $\varphi^S(P')$, violating condition (3) of the definition of robust stability.

Mechanism φ^S is the only mechanism that is stable and strategy-proof (Alcalde and Barberà, 1994). Thus Example 1 implies that given a priority structure, there does not necessarily exist a robustly stable mechanism:

Theorem 1. *There exists a priority structure for which there is no robustly stable mechanism.*

The next question to ask is whether we can say a mechanism is robustly stable in a *specific market*. In other words, we investigate conditions on a pair $(\succ, q)$, called a **priority structure**, under which the mechanism is robustly stable. The following concept will prove useful.

Definition 2 (Ergin (2002))**.** Let $(\succ, q)$ be a priority structure. A **cycle** is $a, b \in C$, $i, j, k \in S$ such that

- $i \succ_a j \succ_a k$ and $k \succ_b i$, and

- There exist disjoint sets of students $S_a, S_b \subset S \setminus \{i, j, k\}$ such that $|S_a| = q_a - 1$, $|S_b| = q_b - 1$, $s \succ_a j$ for every $s \in S_a$ and $s \succ_b i$ for every $s \in S_b$.

A priority structure $(\succ, q)$ is **acyclic** if there exists no cycle.

With the above notion, we can now present our main result, which is a characterization of markets for which a robustly stable mechanism exists.

Theorem 2. *For market $(S, C, \succ, q)$, φ^S is robustly stable if and only if the priority structure $(\succ, q)$ is acyclic.*

Proof. See Appendix. □

Given that φ^S is the unique stable and strategy-proof mechanism (Alcalde and Barberà, 1994), this theorem implies that, given the market, there exists a robustly stable mechanism if and only if the priority structure is acyclic.

To obtain intuition for Theorem 2, it is useful to review Example 1. If student 2 declares all schools unacceptable in the student-proposing deferred acceptance algorithm, then students 1 and 2 apply to schools b and a respectively and both are admitted. On the contrary, if 2 reports that a is her first choice, then that will displace 3 from a. Then 3 applies to his second choice b, displacing 1 from her first choice b, resulting in her applying to her second choice b. Then b rejects 2 and the algorithm terminates. By refraining from applying to a, student 2 can change matching of other students without changing her own matching ($\emptyset$ in both cases). This enables her to engage in a combined manipulation if she finds a to be acceptable: First misreport preferences so that other students are matched differently than under truthtelling, and then rematch with a more preferred school a after the matching is prescribed. The property that students cannot influence matchings of others without changing, called nonbossiness, turns out to play a key role more generally. Ergin (2002) shows that φ^S is nonbossy if and only if the priority structure is acyclic, and the proof of Theorem 2 is based on his result.

Theorems 1 and 2 suggest that manipulations may be unavoidable even when only students can act strategically. If the social planner can influence the priority structure, as in the case of student placement in public schools, the theory suggests that acyclicity would make the system immune to manipulations.[4] This point of view is shared by a number of studies, from related but different aspects. Ergin (2002) shows that the student-optimal

[4] Alternatively, the social planner could regulate the rematching process so that a student cannot be matched to a more preferred school even if she has high priority.

stable mechanism is group strategy-proof if and only if the priority structure is acyclic. Haeringer and Klijn (2007) show that, in the school choice setting, the set of Nash equilibrium outcomes under the student-optimal stable mechanism (possibly with constraints on the length of rank order lists) coincides with the set of stable matchings if and only if the priority structure is acyclic. Kesten (2008) shows that the student-optimal stable mechanism is immune to capacity manipulation (Sönmez, 1997) if and only if the priority structure of schools is acyclic. Following the current paper, Afacan (2010) recently introduces the concept of *group* robust stability and investigates priority structures guaranteeing the condition. Kesten (2006) introduces a slightly stronger acyclicity concept and shows that the top trading cycles mechanism coincides with the student-optimal stable mechanism if and only if the priority structure satisfies his version of acyclicity. The concept of acyclicity has been generalized to coarse priorities and acceptant and substitutable priorities (as defined by Kojima and Manea (2009)) by Ehlers and Erdil (2009) and Kumano (2009), respectively.

An important related paper is Chakraborty, Citanna, and Ostrovsky (2009). They consider a matching market with interdependent values and introduce a stability concept with the possibility of combined manipulations. In that environment, they establish impossibility theorems which assert that there is no stable mechanism in their sense. Meanwhile they also note that their impossibility theorems can be obtained even with a weaker notion of stability, namely the combination of traditional stability and strategy-proofness as required separately. The current study complements their study by showing that there does not necessarily exist a robustly stable mechanism even if there is no interdependent value component, and then characterizing the condition necessary and sufficient for the existence of a robustly stable mechanism. Note that our characterization result critically depends on the assumption of private values. With interdependent values, Chakraborty, Citanna, and Ostrovsky (2009) show the impossibility of stable mechanisms even when the priority structure is acyclic, so our private values assumption is important in Theorem 2.

The definition of robust stability requires that the mechanism be immune to combined manipulations *even if a student knows everything about the environment and reported preferences of other students.* Clearly, perfect information is a strong assumption in many applications. However, it turns out that combined manipulations are easy to carry out without *any knowledge* other than the student's own preferences. Specifically, consider the following strategy of a student: (1) Declare all schools to be unacceptable to the mechanism, and (2) then rematch with her most preferred school available once the matching is prescribed by the mechanism.[5]

Proposition 1. *In φ^S, any student who uses the above strategy is matched to a school that she weakly prefers to the school matched under truthtelling.*

Proof. See Appendix. □

A related question is whether combined manipulations are expected in large markets.[6] Proposition 1 implies that incentives for combined manipulations remain in large markets (although the magnitude may as well become small). This is because a student can safely misreport preferences and rematch with the same school as under truthtelling even in the worse case.

3 Conclusion

This paper introduced a new stability concept, robust stability. Theorem 1 demonstrates that, given a priority structure, there does not necessarily exist a robustly stable mechanism. This result suggests that one cannot eliminate manipulations completely even when agents on only one side of the market have private information. On the other hand, Theorem 2 characterizes the market structures that enable robustly stable mechanisms to exist. If the social planner can design the priority structure, as in the case of student placement to public schools, the theory suggests that acyclicity is likely to make the system robust to manipulations. However acyclicity is a very demanding condition, so one possible way to read this paper is to say robust stability is not only impossible for arbitrary markets (Theorem 1), but also is hard to guarantee by judiciously specifying a priority structure (Theorem 2).

The extant literature has also found acyclic priority structures to be key in producing desirable properties in matching markets. Papers cited in this paper are only a few examples. This paper identifies one more sense in which such a structure proves critical for the design of matching markets. We envision that investigating further implications of priority structures may be a fruitful direction of future research.

Before concluding the paper, we comment on a conceptual issue. The model assumes that school priorities are publicly known. Publicly known

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[5] I am grateful to anonymous referees for encouraging me to consider this issue and for suggesting Proposition 1.

[6] In the two-sided matching setting, Roth and Peranson (1999), Immorlica and Mahdian (2005), and Kojima and Pathak (2008) show that manipulation incentives become small under φ^S as the market size grows.

school priorities arise naturally in the school choice setting: As Abdulkadiroğlu and Sönmez (2003) point out, school priorities are exogenously given by law in many school districts. In such a case, however, one might argue that schools are not strategic players and hence do not participate in rematching, so combined manipulations are unimportant and instead stability and strategy-proofness are sufficient. Even in school choice, however, robust stability may be important. For instance, consider the appeals process. In student placement to high schools in New York City, many students participate in an appeals process to be assigned to a school they like better than their prescribed assignment (Abdulkadiroğlu, Pathak, and Roth, 2005, 2009). For the academic year 2003-2004, the first year when the student-optimal stable mechanism was implemented there, more than 5,000 students appealed their assignments, and about 300 appeals were from students who received their stated first choices.[7] The Department of Education granted about half of the appeals. This suggests that students may be able to engage in rematching even in the school choice setting.[8]

Needless to say, the above interpretation is only suggestive. First, students are often required to offer a reason in order to appeal, for instance a new address. Second, it is not clear whether the same school priorities as those used in the initial allocation process are respected during the appeals process. Also, it is difficult to see whether students engage in combined manipulations in actual school choice problems (the appeals may be due to different reasons such as changes in student preferences). Even so, the analysis of this paper raises the possibility that combined manipulations may happen in matching markets, and suggests that the market organizer take into account such possibilities when designing a mechanism.

Another possible application is to labor markets where preferences of one side of the market are publicly known. In this context, the assumption that preferences of one side of the market are publicly known may be too strong. However, it may be a reasonable first approximation in some cases. For instance, firms may have sufficiently established reputations so that workers' preferences over firms can be estimated from them with reasonable precisions. A similar application is college admission. The assumption that student preferences are known may be a reasonable approximation of actual college admission because information about colleges is mostly public in practice. A more thorough analysis of these issues is beyond the scope of this paper and is left for future research.

[7] Interestingly, successful manipulations that appear in our analysis involve students rematching after they receive their stated first choices.

[8] I am grateful to Al Roth for suggesting this example.

Appendix: Proof of Theorem 2

We say that mechanism φ is **nonbossy** if $\varphi_s(P'_s, P_{-s}) = \varphi_s(P)$ implies $\varphi(P'_s, P_{-s}) = \varphi(P)$. The following result proves useful.

Result 1 (Ergin (2002))**.** *Mechanism φ^S is nonbossy for market $(S, C, \succ, q)$ if and only if $(\succ, q)$ is acyclic.*[9]

Proof of the "only if" direction We show the claim by contraposition. Suppose that the priority structure is not acyclic. Then, by definition, there exist $a, b \in C$, $i, j, k \in S$ such that

- $i \succ_a j \succ_a k$ and $k \succ_b i$, and
- There exist disjoint sets of students $S_a, S_b \subset S \setminus \{i, j, k\}$ such that $|S_a| = q_a - 1$, $|S_b| = q_b - 1$, $s \succ_a j$ for every $s \in S_a$ and $s \succ_b i$ for every $s \in S_b$.

Consider the following preferences of students:

$$\begin{aligned}
P_i &: b, a, \emptyset, \\
P_j &: a, \emptyset, \\
P_k &: a, b, \emptyset, \\
P_s &: a, \emptyset, \text{ for every } s \in S_a, \\
P_s &: b, \emptyset, \text{ for every } s \in S_b, \\
P_s &: \emptyset, \text{ for every } s \in S \setminus [\{i, j, k\} \cup S_a \cup S_b].
\end{aligned}$$

It is easy to see that $\varphi^S_j(P) = \emptyset$. Now consider a false preference of student j, $P'_j : \emptyset$. We write $P' = (P'_j, P_{-j})$. Then $\varphi^S_k(P') = a$. Since $a P_j \emptyset = \varphi^S_j(P)$ and $j \succ_a k \in \varphi^S_a(P')$, φ^S is not robustly stable.

Remark. Intuitively, the proof of the "only if" direction is similar to Example 1. It proceeds by essentially "embedding" a small market as in Example 1 into any given market with a cyclic priority structure by appropriately specifying student preferences.

[9] Part of Theorem 1 of Ergin (2002) states that φ^S is group strategy-proof if and only if the priority structure is acyclic. Result 1 follows from the following two well-known facts: (i) φ^S is strategy-proof for any priority structure, and (ii) a mechanism is group strategy-proof if and only if it is both strategy-proof and nonbossy.

Proof of the "if" direction We show the claim by contradiction. To this end, suppose that $(\succ, q)$ is acyclic but φ^S is not robustly stable. Since φ^S is stable and strategy-proof, this assumption implies that

Condition (A): there exist $s \in S$, $c \in C$, $P \in \mathcal{P}^{|S|}$ and $P'_s \in \mathcal{P}$ such that (i) $cP_s\varphi^S_s(P)$ and (ii) $s \succ_c s'$ for some $s' \in \varphi^S_c(P'_s, P_{-s})$ or $|\varphi^S_c(P'_s, P_{-s})| < q_c$.

Letting $P' = (P'_s, P_{-s})$, we consider the following cases.

(1) Suppose $\varphi^S_s(P') = \emptyset$. Let $P''_s : c, \emptyset$ and $P'' = (P''_s, P_{-s})$.

 (a) Suppose $\varphi^S_s(P'') = \emptyset$. Then, by definition of P''_s we have

$$c\, P''_s\, \varphi^S_s(P''). \tag{1}$$

 Moreover, since $(\succ, q)$ is acyclic by assumption and hence φ^S is nonbossy by Result 1, we have $\varphi^S(P'') = \varphi^S(P')$. This property and Condition (A) imply that either $s \succ_c s'$ for some $s' \in \varphi^S_c(P') = \varphi^S_c(P'')$ or $|\varphi^S_c(P'')| = |\varphi^S_c(P')| < q_c$. This and relation (1) mean that $\varphi^S(P'')$ is unstable under P'', contradicting the assumption that φ^S is a stable mechanism.

 (b) Suppose $\varphi^S_s(P'') = c$. Then this is a contradiction to strategy-proofness of φ^S, since $\varphi^S_s(P'') = cP_s\varphi^S_s(P)$.

(2) Suppose $\varphi^S_s(P') \neq \emptyset$. Let

$$P''_s : \emptyset,$$

 and $P'' = (P''_s, P_{-s})$. By the well-known comparative statics by Kelso and Crawford (1982) and Gale and Sotomayor (1985), $|\varphi^S_c(P')| \geq |\varphi^S_c(P'')|$ and, if $|\varphi^S_c(P')| = |\varphi^S_c(P'')| = q_c$, then there exists $s'' \in \varphi^S_c(P'')$ such that $s' \succeq_c s''$ for all $s' \in \varphi^S_c(P')$. Thus Condition (A) is satisfied with respect to s, c and P''_s (instead of P'_s) and, since $\varphi^S_s(P'') = \emptyset$, the analysis reduces to Case (1) above.

Appendix: Proof of Proposition 1

Let P be the true preference profile of students and consider an arbitrary student $s \in S$. First note that the result of the deferred acceptance algorithm is independent of the order that the applications are processed (McVitie and Wilson, 1970), resulting in $\varphi^S(P)$. Let $c = \varphi^S_s(P)$. Now consider a reporting P'_s of student s that declares all schools to be unacceptable. Then the matching $\varphi^S(P'_s, P_{-s})$ produced under the deferred acceptance algorithm under (P'_s, P_{-s}), coincides with a matching in which all applications are processed by the deferred acceptance algorithm with respect to P except that no applications by student s are processed. By the result of McVitie and Wilson (1970) cited above, if we let student s apply according to her true preferences P_s from then on, then the resulting matching is $\varphi^S(P)$ and thus matches s to $c = \varphi^S_s(P)$. This is only possible if s is accepted by c at some step of the deferred acceptance algorithm since $\varphi^S(P'_s, P_{-s})$ was produced, so s can also match with c in the rematching stage if s has declared all schools unacceptable at the deferred acceptance stage.

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