Exploration of fundamental matters of acoustic instabilities in combustion chambers

By O. M. Umurhan

1. Motivations and objectives

The excitation of acoustic oscillations in combustion chambers is well-known as an unwelcome phenomenon that impedes efficient burning and promotes structural stresses that can lead to engine and combustor failure. Understanding, categorizing, and ultimately controlling thermoacoustic excitation is an important goal and is of current interest to designers and theoreticians alike. To this day, however, accurate predictions for the onset of acoustic-combustor instabilities are difficult to make, and models created to account for these difficulties (Crocco's time-lag prescription) make use of parameters that are specific to the combustor in question and whose mathematical origins are usually shrouded in mystery.

It is our aim to study this problem and shed a little light upon it by means of numerical tools (LES of combustion) and theoretical means (asymptotic analysis) in order to clarify and predict the conditions leading to the ubiquitously observed acoustic-combustor instabilities. The focus of this review will be on the latter efforts while the former approach will be discussed in the section on ongoing and future work.

In particular, we wish to lay out the general groundwork for current and future stability analysis to be used to predict acoustic amplification in real combustors *and* to develop some fundamental physical notions behind the source and nature of wave instabilities in heat releasing environments. In this latter respect there are two fundamental physical questions initially of interest:

- The first of these is: if a combustor is designed in which the Rayleigh criterion is used as the barometer or gauge to control thermoacoustic instability, does this necessarily imply that all possibly self-excited oscillations are suppressed? In other words, if the combustor does not violate the criterion, does that mean all self-excited sound will be suppressed in the combustion? The answer to this first question appears to be "no" unless isentropic conditions can be maintained at the inlet boundary of the system.
- Second: how sensitively is the Rayleigh criterion itself effected by the introduction of *non-isentropic boundary conditions* at the combustor inlet? The current answer to this is that the stability profiles are profoundly altered due to introduction of an *entropy or drift-mode* into the acoustic dynamics.

In the following we lay out a brief background about nonadiabatic acoustic instabilities and the conditions in combustors in which to consider these instabilities. We will then follow this by investigating these effects and issues in an idealized one-dimensional scenario for a combustor.

1.1 Background

Sound in a chamber where burning occurs may be self-excitated under two conditions. The first of these is the classic Rayleigh mechanism, which states that an acoustic mode will be rendered unstable when the phase of the unsteady burning of the chamber coincides with the sound's pressure fluctuation (Rayleigh, 1896). Typically, this condition is quantified by evaluating the *Rayleigh criterion* (Rayleigh, 1896) for infinitesimal fluctuations,

$$\frac{C_P - C_V}{C_P} \int_V \int_t^{t+T} \frac{Q'P'}{P_0} d^3x dt' > 0$$

where the integral is taken over the domain volume V and acoustic mode period T. P_0 and P' are the pressure steady state and fluctuation while Q' is the heat release fluctuation. Once Q' and its spatial distribution is determined along with the knowledge of the domain geometry, one may evaluate whether or not a chamber sound mode will be stable to infinitesimal perturbations.

The other mechanism involves the interaction of a so-called *entropy mode* (also referred to as a *drift-mode*) with the flow field (Dowling, 1995, Umurhan, 1999a). Dowling considers this an important yet commonly ignored feature for acoustics in a flow field since the entropy mode can non-negligibly influence a chamber's acoustic mode frequency (Dowling, 1995) and illicit instability (Umurhan, 1999a) through coupling at a chamber's inlet and outlet boundaries and at the flame position. A necessary integral condition for the stability of linearized acoustics in a one-dimensional flow field is expressed by,

$$\frac{\gamma}{2} \int_V \int_t^{t+T} \rho' u' D u_0^2 dx > 0$$

where ρ' is the fluctuating density which carries the entropy mode signature (Bloxsidge *et al.* 1988, Dowling 1995, Umurhan 1999a).

We will consider longitudinal one-dimensional disturbances in a combustion chamber shaped as a simple cylindrical tube of length L and an input velocity u at some speed less than the sound speed but not necessarily infinitesimally small. (See Fig. 1.) We wish to isolate the effects of the entropy mode and demonstrate how it renders oscillations unstable. To this end, we will pay attention to (but not limited to) results under circumstances where there are no variations of the heat production term in the evolution equations describing acoustic propagation.

2. Model problem

2.1 One-dimensional linearized equations of motion

We begin the following analysis of the nondimensionalized Euler equations supplemented with an energy equation and an equation of state for thermodynamic

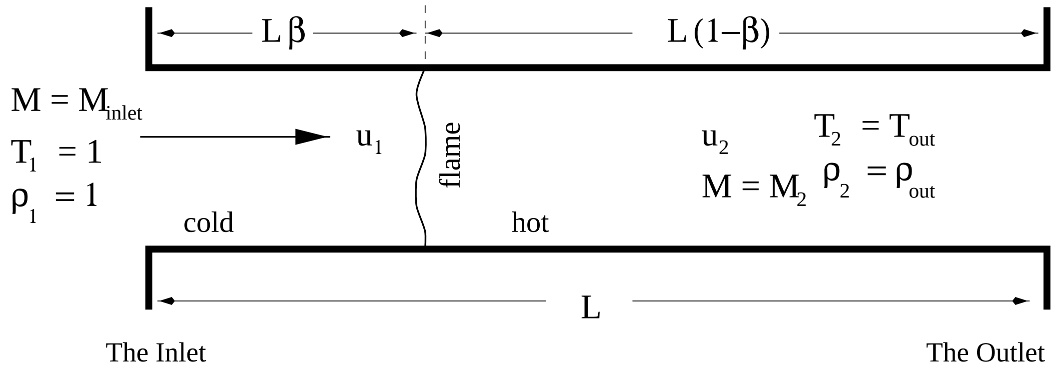

FIGURE 1. Schematic of model problem. In a general sense, the burning takes place over the entire domain of tube. In the special case where the burning is distributed on a plane, we call it a *flame* whose position is given by $0 < \beta < 1$.

pressure. After perturbing those equations around a steady state, we find that the equations describing acoustics disturbances are

$$\partial_t \rho' + D\Omega' = 0 \tag{1}$$
$$\gamma\rho_0 \partial_t u' + \gamma M D u' + \gamma \Omega' D u_0 = - DP' \tag{2}$$
$$\partial_t P' + u_0 DP' + u' DP_0 + \gamma P' D u_0 = - \gamma P_0 D u' + (\gamma - 1) Q' \tag{3}$$
$$P' = \rho_0 T' + T_0 \rho' \tag{4}$$

where derivatives with respect to the spatial coordinate x are given by D. P', ρ', u', T' denote pressure, density, velocity, and temperature perturbations respectively while Q' denotes unsteady heating. The term Ω' refers to the perturbed mass flux and is defined by $\rho_0 u' + u_0 \rho'$. The quantities u_0, ρ_0, P_0, and T_0 are defined below.

A few words about what was used to obtain the nondimensional equations are presently in order. The dimensional density was scaled by a characteristic value at the inlet, $\rho_{\rm in}$, while the dimensional temperature was similarly scaled by $T_{\rm in}$. Lengths were scaled by the tube size, L, while velocities were scaled by the sound speed, c_a, at the inlet, or $(\gamma \mathcal{R} T_{\rm in})^{1/2}$; here $\mathcal{R}$ is the gas constant and is where the other constant, $\gamma = C_P / C_V$, is the ratio of specific heats of the ideal gas in consideration. Time scales are characterized by the fundamental acoustic period scale of the tube given by L/c_a. The usual energy conservation equation is rewritten in the form of an evolution operator on the pressure with the aid of the equation for mass conservation and the ideal gas law.

The inlet velocities in our nondimensional scalings will be described by an inlet Mach number,

$$M_{\rm in} = M = \frac{u_{\rm in}}{\sqrt{\gamma \mathcal{R} T_{\rm in}}} \ ,$$

where $u_{\rm in}$ is the dimensional inlet velocity. M will be one of our main tuning parameters. The "0" subscripted terms refer to steady state quantities which satisfy

$$\rho_0 u_0 = M \tag{5}$$

$$P_0 + \gamma M u_0 = 1 + \gamma M^2 \tag{6}$$
$$u_0 D P_0 + \gamma P_0 D u_0 = (\gamma - 1) Q_0 . \tag{7}$$

Since Eqs. (1, 2, 3) form a third order system in spatial derivatives, we require imposition of three boundary conditions. To this end we use the rather simplified requirements that pressure fluctuations be fixed at both the inlet ($x = 0$) and outlet ($x = 1$). The third condition will be that the *mass-flux is constant at the inlet*, which implies that the fluctuating mass-flux, Ω', is zero at $x = 0$. We note that enforcing the mass flux condition introduces non-isentropicity into the acoustic evolution. An alternate third condition would be to require no entropy fluctuations at the inlet. This *isentropic* condition removes the driftmode effect observed by Bloxsidge (1988), Dowling (1995), and Umurhan (1999a) in studies of acoustic-flame couplings. In the subsequent analysis, we will usually adopt the former condition.

2.2 Integral relationship

An integral relationship may be simply derived for the total perturbation acoustic energy of the set (1)-(5). If we multiply Eq. (2) by u' and, similarly, multiply Eq. (3) by $P'/\gamma P_0$, add the two resulting equations together followed by an integration over the domain length we find,

$$\begin{aligned}
&\frac{\partial}{\partial t}\int_0^1 \left(\gamma \rho_0 u'^{\,2} + \frac{1}{\gamma P_0} P'^{\,2}\right) dx = \\
&-\int_0^1 \frac{DP_0}{\gamma P_0} u' P' dx - \frac{1}{2}\int_0^1 \left(\frac{2\gamma - 1}{\gamma} - M^2\right)\frac{Du_0}{P_0} P'^{\,2} dx \\
&-\frac{1}{2}\gamma M \int_0^1 Du'^{\,2} dx - \int_0^1 \gamma \Omega' u' D u_0 dx + (\gamma - 1)\int_0^1 \frac{1}{\gamma P_0} Q' P' dx
\end{aligned} \tag{8}$$

where we have used the boundary conditions specified in the previous subsection.

We note that aside from boundary terms (which are certainly relevant) that would arise from non-homogeneous boundary conditions, the terms containing the integral relations in (8) are general.

The meaning of Eq. (8) when the RHS vanishes is that the acoustic energy within the domain neither grows nor decays. Naturally, the elements on the RHS of Eq. (8) represent the input/drain of energy throughout the system. The classical Rayleigh criterion routinely used in literature is recovered when there is no steady flow throughout the domain because those terms containing gradients of the steady state pressures and velocity fields, DP_0, Du_0, and the Mach number M, vanish in that limit.

2.3 Forms of Q_0 and Q' and corresponding analysis strategies

Classic thermoacoustic effects relating to the Rayleigh criterion rely on the fluctuations in the heat release (in Q'). The process leading to instabilities associated with the criterion are well known, and procedures in which to parameterize those effects to carry out complex eigenmode analysis are frequently used (Crocco, Bloxsidge, and others).

In order to isolate the influence of the entropy mode upon the stability of the chamber, we pay special attention to (but are not limited to) disturbances in which there are *no heat release fluctuations.*

Case I*:* $Q_0(x) = \epsilon q_n(x) \quad , \quad Q' = 0$

This is the simplest form of the heating law that can be expressed. The heating is distributed over the domain but with an overall amplitude which is small, a scaling which we can tune arbitrarily with the parameter ϵ. The advantage of this prescription is that it allows for analytical calculations of the growth rates simply by expanding all solutions in powers of ϵ.

We adopt several forms for the heat release; in particular, we adopt the following forms

$$q_n(x) = \begin{cases} 1, & \text{if } n = 0; \\ 2x, & \text{if } n = 1; \\ 3x^2, & \text{if } n = 2; \\ 6x(1-x), & \text{if } n = 3; \\ \frac{3(1-x^2)}{2}, & \text{if } n = 4. \end{cases}$$

The normalization constants in each expression above are chosen so that the total integrated heat release in the chamber is equal to 1. Evidently, the shape of the heat release over the course of the tube length dictate the character of the stability profile.

By introducing the heating function as a small order ϵ quantity, we must correspondingly expand both steady states and perturbation solutions in similar powers of ϵ. In more concrete terms, if ϕ is some fluid quantity (density, velocity, etc.) and if ϕ_0, ϕ' represents this quantity's steady state and perturbations, then we formally expand all these solutions as,

$$\phi_0 = \phi_{00} + \epsilon\phi_{01} + \mathcal{O}\left(\epsilon^2\right)$$
$$\phi'_1 = \phi'_0 + \epsilon\phi'_1 + \mathcal{O}\left(\epsilon^2\right),$$

whereupon we would reinsert these expansions into the acoustic disturbance set Eqs. (1-4) and carry out a formal perturbation analysis.

Case II*:* $Q_0 = \hat{Q}\delta(x-\beta) \quad , \quad Q' = \Theta(T', u')\delta(x-\beta)$

In this formulation, the burning region is compacted onto a a plane, appearing as a discontinuity. Consequently, the basic states are uniform on either side of the discontinuity and are connected to each other through the usual Rankine-Juegout jump conditions for deflagration fronts (Williams, 1978).

$$\Big[\rho u\Big]_{\beta^-}^{\beta^+} = 0 \tag{9a}$$

$$\Big[\gamma\rho u^2 + P\Big]_{\beta^-}^{\beta^+} = 0 \tag{9b}$$

$$\rho u\left[\frac{1}{\gamma-1}T + \frac{1}{2}u^2\right]_{\beta^-}^{\beta^+} = (\gamma-1)\hat{Q} \tag{9c}$$

where β^+, β^- signify approaching the discontinuity $x = \beta$ from either above or below. These jump conditions are general and are appropriate for both steady states and perturbations alike provided all the functional forms are well behaved and integrable.

Note that for the steady state configuration, given an inlet pressure and temperature while also given the heat release at the discontinuity, there is a critical inlet Mach number (M_{cr}) for which the outflowing velocity is supersonic given by the solution to

$$\hat{Q} - \frac{\gamma(M_{cr}^2 - 1)^2}{2M_{cr}(\gamma^2 - 1)} = 0. \tag{10}$$

Perturbed disturbances are calculated for each region (burnt and unburnt) with its corresponding uniform steady states. The disturbances are subsequently connected to each other by satisfying the perturbed form of the jump conditions in Eqs. (9a-c). The details and subtleties of this procedure can be found in Umurhan (1999b).

The purpose of this case is to illustrate in very general terms the competition between classical thermoacoustic processes (fluctuating heat release, the Rayleigh mechanism) and the entropy wave mode of destabilization. In particular we represent the heat release fluctuation by

$$Q' = \Big[\nu T' + \mu u'\Big]\delta(x - \beta) \tag{11}$$

where ν and μ are to be generally taken as tunable constants. This form of the burning fluctuation was derived in Umurhan (1999b) as a simple model for spray combustion and burning. In this model the spray droplets are introduced at the inlet and convected along with the background fluid oxidizer towards the flame position, the latter of which was kept fixed for simplicity. During the flameward convection phase, fuel droplets experience evaporation variations on account of two processes: (1) enhanced evaporation due to fluctuations of the ambient temperature and (2) enhanced evaporation resulting from the fluctuations seen by the fuel droplet of the difference between the fuel droplet speeds and the speed of the ambient oxidizing fluid. In this model, all variations of ambient temperatures and fluid flow are attributed to the passage of acoustic disturbances. To complete the model, the total amount of burning/heat-release at the fixed flame position is set to be linearly proportional to the total amount of evaporated fuel reaching the flame. Thus, we can see that the total amount of evaporated (premixed) fuel reaching the flame will be a function of the history of the fuel droplet beginning from initial injection up to the point of flame crossing. Generally, the constants ν and μ will reflect this aggregate history.

For this presentation we leave out the complexities involved with the form of ν and μ (see Umurhan 1999b for details) and simply treat these parameters as freely tunable constants.

2.4 Normal mode stability results

One may embark on several paths to investigate the stability of acoustic disturbances. We choose the standard normal mode method because it is the most transparent of calculations. We adopt the following usual Fourier form for general perturbations ϕ',

$$\phi'(x,t) = \hat{\phi}(x)e^{i\omega t}$$

where ω is both the complex frequency and the eigenvalue for the boundary value problem to be solved. A final note: unless otherwise indicated, we use the no mass-flux fluctuation boundary condition at the inlet.

2.4.1 <u>*Case I*</u>

In Fig. 2 we present the results of the stability analysis for several forms of the domain heat distribution. We find that as the Mach number is lowered, wiggles as a function of Mach number appear in the nondimensional growth rate. The amplitude of the growth rate appears to grow with an increased inlet Mach number. We also find the low Mach number wiggles become more pronounced the closer to the outlet the heat release is. We see that for all $q_n(x)$ there is a minimum Mach number beyond which the standing acoustic mode in the chamber is unstable. Finally, we observe that if the heat release is concentrated closer to the inlet, instability sets in at lower Mach numbers than otherwise. When the fixed mass-flux boundary condition at the inlet is relaxed and, instead, isentropic conditions are adopted, we find that the acoustic modes neither grow nor decay, which is in concordance with already known results of Munjal & Prasad (1986), Peat (1988), and Kumar & Sujith (1997).

2.4.2 <u>*Case II*</u>

One noteworthy consequence of this delta function procedure has to do with the nature of the normal modes themselves. In general each fluid quantity (irrespective of which side of the flame one is on) is characterized by three waves: $\mathrm{Exp}(ik^{(+)}x)$, $\mathrm{Exp}(ik^{(+)}x)$ and $\mathrm{Exp}(ik^{(0)}x)$. The wavenumbers corresponding to $k^{(+)}$ and $k^{(-)}$ represent the usual right and left propagating Doppler shifted acoustic waves. The $k^{(0)}$ wavenumber represents the drift/entropy mode introduced through the presence of the mean flow.

The analysis rather transparently shows that the pressure and velocity fluctuations are characterized by a linear combination of the left and right propagating acoustic waves *but not the drift/entropy mode.* However, the analysis also demonstrates that the density and temperature fluctuations are represented by linear combinations of the left and right propagating acoustic modes *as well* as the the drift/entropy mode if the inlet boundaries are non-isentropic. Given isentropic inlet boundaries, the drift/entropy mode is also absent from the temperature and density fluctuations.

There are a number of results here worth looking at and the first that we present is the asymptotic limit where the steady state nondimensionalized heat release Q_S is small compared to the parameters μ and ν. This limit is much like the expansion

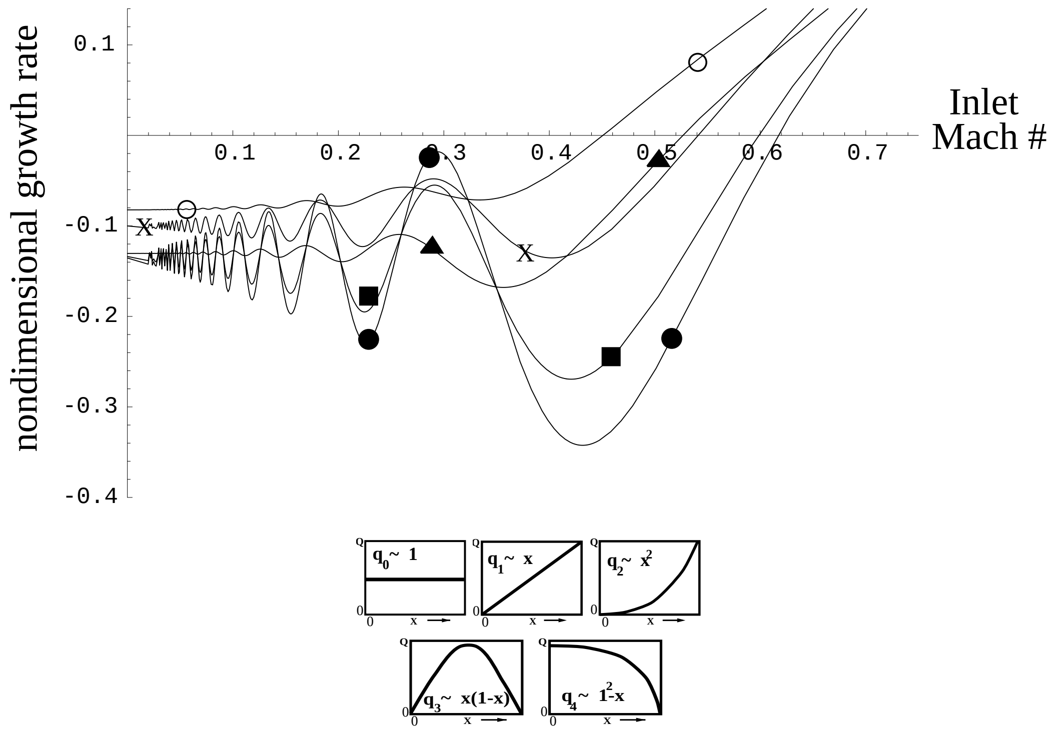

FIGURE 2. The nondimensional growth rates vs. inlet Mach numbers for the fundamental mode of the tube for burning rates given by $Q_0 \sim \epsilon q_n(x)$ along with no fluctuating burning terms. The stability profile exhibits low Mach number oscillations. All modes become unstable for sufficiently large inlet Mach numbers. —×— : $q_0 \sim 1$. —■— : $q_2 \sim x$. —●— : $q_2 \sim x^2$. —○— : $q_3 \sim 1 - x^2$. —▲— : $q_4 \sim x(1-x)$.

employed in Case I except that we take the fluctuating heat release to *be very sensitive* to the ambient hydrodynamic quantities at the flame position. An approximate first order correction for the fundamental mode can be obtained through a series of Taylor expansions,

$$\omega(\text{correction}) \sim i\Big[(1+M)e^{-2i\pi\beta} + (M-1)\Big]\times \Big[M\mu(1+e^{-2i\pi\beta}) - (\gamma-1)M\nu(e^{-2i\pi\beta}-1) - 2\psi\nu e^{\frac{-i\pi\beta(1-M)}{M}}\Big] \tag{12}$$

The parameter ψ denotes whether we adopt isentropic ($\psi = 0$) or fixed mass-flux ($\psi = 1$) conditions at the inlet. Figures 3-4 exhibit typical solutions. We observe the following features:

- That isentropic conditions result in fairly uniform stability boundaries.
- The instability here is purely thermoacoustic, but the pressure-temperature phasing may be the reverse of what usual intuition would tell otherwise. Consider the case where ν is positive and μ is zero. Consequently, in terms of the form of Q'

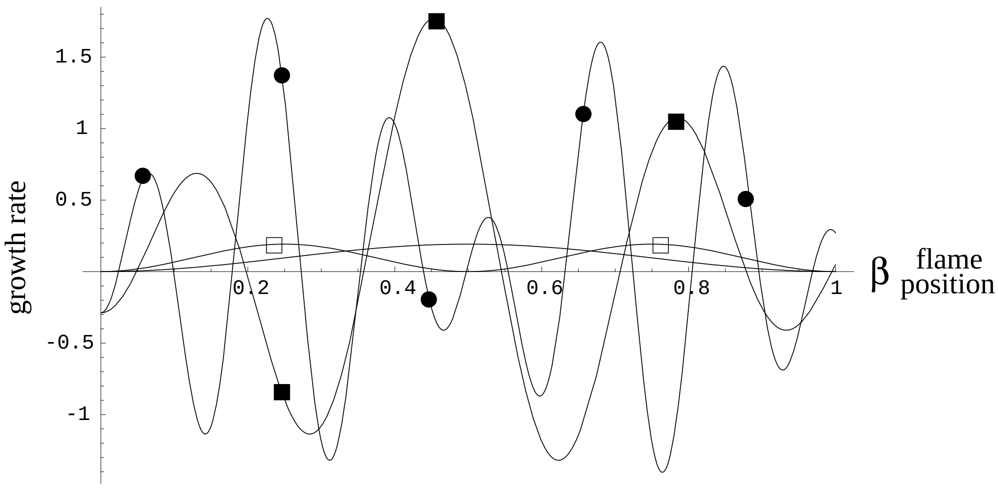

FIGURE 3. Mode comparison for $M = 0.18$, $\mu = 0$, and $\nu = 0.20$ for different inlet boundary conditions. Notice how extreme the the growth rates are for the fixed mass-flux condition as opposed to the isentropic inlet. Fundamental mode: fixed mass-flux, —●— ; insentropic, ——— . First overtone: fixed mass-flux, —■— ; isentropic, —□— .

used, positive temperature fluctuations correspond to positive fluctuating heat release. By inspecting the generalized Rayleigh criterion in Section 2.2, we see that the necessary condition for acoustic instability is guaranteed as long as the the temperature and pressure fluctuations are in phase at the flame position. However, the subtlety of the results of Figs. 3 and 4 can be recognized by noting the different phasing relationships between disturbance types. If the acoustic disturbances are isentropic, then indeed the necessary condition for instability as predicted by the Rayleigh criterion is satisfied because temperature and pressure fluctuations are everywhere in phase for isentropic inlet conditions. If the disturbances are not isentropic, then the introduction of the drift/entropy mode into the temperature perturbation profile yields a phase relationship between the pressure and temperature fluctuation that is a function of position within the combustor. Naturally, there are situations where the pressure and temperature can be as extreme as π radians out of phase at the position of the flame.

- Heat release fluctuations that are dependent upon the velocity seen at the flame position are *insensitive* to the entropy mode influence. This is because the velocity fluctuations' eigenfunctions do not carry any entropy mode signature, unlike the density and temperature fluctuations as were commented on above.

- Like in the case of a moving flame (Umurhan 1999a) the stability boundaries become highly oscillatory with respect to the flame position β as the Mach number gets smaller. The implications of this are profound: the slower the inlet flow, the *more sensitive to flame position* the acoustic stability becomes. Any flame that moves about even slightly (in the extremely small Mach number limit) might find itself drifting into acoustically unstable places in the combustor.

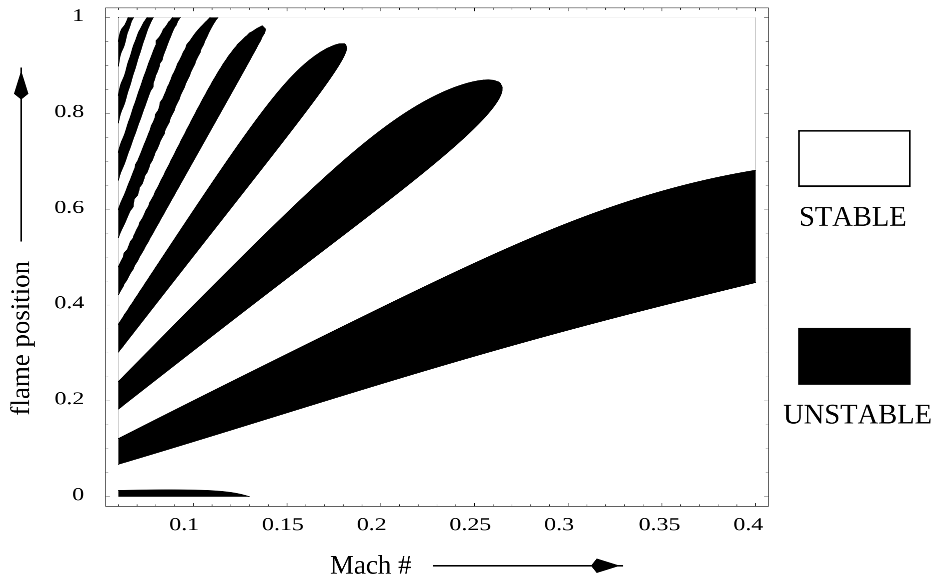

FIGURE 4. Stability boundaries of the fundamental mode with fixed mass-flux inlet *bc*. Presented here is the limiting case in which $Q_0 \to 0$. The parameters are $\nu = 0.52$, $\mu = 0.0$.

2.5 Weakly nonlinear analysis of Case I.

In the limiting situation of Case I, the frequencies of all harmonics of the tube's standing acoustic waves are commensurate. This means that a weakly nonlinear analysis must terminate at quadratic powers of perturbation amplitude. Therefore, we adopt a two-time multiple scale analysis (Bender & Orszag, 1978) in which the time derivatives are written as

$$\partial_t \to \partial_t' + \epsilon \partial_T$$

where T represents a long time scale. The steady states and perturbations are expanded as

$$\phi \to \phi_0(x) + \epsilon \phi'(x,t)$$

with

$$\phi_0(x) = \phi_{00}(x) + \epsilon \phi_{01}(x) + \epsilon^2 \phi_{02}(x) + \mathcal{O}\left(\epsilon^3\right)$$
$$\phi_1'(x,t) = \phi_0'(x,t) + \epsilon \phi_1'(x,t) + \epsilon^2 \phi_2'(x,t) + \mathcal{O}\left(\epsilon^3\right)$$

We note that the slow-time (ϵT time scale) nonlinearity is introduced into the amplitudes of the first order *linear* solutions,

$$\phi_0'(x,t) = A(T) f(x) e^{i\omega t} + c.c.$$

where $f(x)$ is the eigenfunction of the linear operator at that given order of ϵ and where *c.c* denotes complex conjugate. Thus, at lowest order, f(x) appearing above is the same linear acoustic mode that we analyzed in Case I. *Since linear boundary value problems involving eigenvalues introduce an arbitrary constant in the form of an amplitude, we choose this amplitude to reflect the long-time behavior of a standing acoustic wave's "envelope".*

Returning to the full (nonlinear) Euler equations with a heat source and adopting $Q_0 = \epsilon q_n$, we perform a multiple scale analysis which yields, rather generally, a countably infinite set of ordinary nonlinear differential equations for each acoustic harmonic m.

$$\partial_T A_m = \beta_m A_m + \sum_{k=1}^{\infty} \gamma_{mk} A_{m-k} |A_k|$$

In general, γ_{mk} are complex. For demonstration here, we present a reduced set of these amplitude ode's truncated at the second harmonic for the marginal case $M \approx 0.52$ (see Fig. 2) along with q_n for $n = 4$.

$$\begin{aligned} \partial_t A &= \alpha_1 A + \gamma_{21} B|A| + \gamma_{32} C|B| \\ \partial_t B &= \alpha_2 B + \gamma_{11} A^2 + \gamma_{31} C|A| \\ \partial_t C &= \alpha_3 C + \gamma_{12} AB \end{aligned}$$

Figure 5 is a preliminary numerical analysis and sample solution of this set. We see rather clearly that a steady amplitude is not attainable in this case; the amplitudes for a given fundamental mode appear chaotic and intermittent.

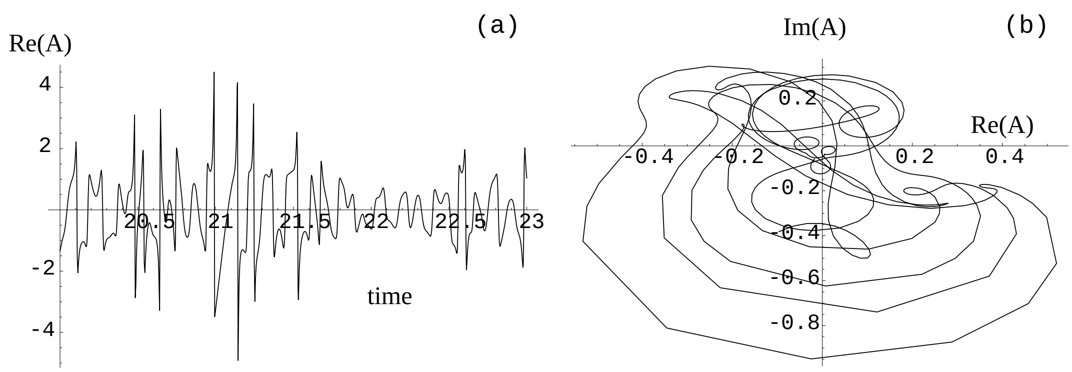

FIGURE 5. Nonlinear evolution of the truncated weakly nonlinear solutions for Case I. Chaos and intermittency (phase chaos) appear readily. M = 0.52. (a) is the amplitude and (b) is the phase diagram of the fundamental mode.

3. Immediate directions and current conclusions

The purpose of this presentation was to demonstrate the procedure one must employ to test/evaluate the stability against general wave motions inside a combustor. We have laid out the elements that must be addressed and taken into account, and we have shown ways to handle and interpret their effects. In particular, we have shown that ignoring the effects of entropy waves upon the overall wave dynamics (including wave responses with a flame) can significantly alter the predicted frequencies and growth rates of chamber acoustic modes.

Yet, we have utilized a highly idealized model for the heat release distribution and its dynamical response to fluctuating flow variables. It is of practical import to apply the stability analysis approach discussed in this work to real turbulent combustors in which acoustic instabilities need to be understood and controlled. Specifically, the form of heat release we used here was prescribed analytically either from a perspective of mathematical convenience (Case I) or as being derivative from a crude and idealized model of some burning process (spray combustion - Case II).

Real turbulent combustion defies such simplistic ease, and we must, consequently, rely on semiempirical burning laws for these situations. To this end, work is currently being conducted in order to computationally derive heat release functionals for combustors supporting diffusion flames. The philosophy behind the approach is exactly that spearheaded by Poinsot and collaborators (see for example, Veynante & Poinsot, 1997, and Schönfeld & Poinsot, this volume) for similar investigations of low Mach number numerical simulations of premixed flames. The approach involves observing the response of premixed flames inside a ducted combustor to periodic pulsations of the duct's inlet velocities and temperatures. The periodic modulations are meant to mimic the passage of a (relatively) long-wavelength acoustic wave through the combustor chamber. Their approach was motivated by real experiments of turbulent premixed combustors where enhanced vortex creation and shedding followed by subsequent enhanced fuel burning *result* from the passage of a strong amplitude acoustic mode; in turn, the enhanced burning feeds the very same acoustic wave and the process is maintained (Poinsot, *et al.* 1987).

This sort of analysis has not been systematically applied to simulations of diffusion flames where the burning response ought to be much different in character than that for premixed flames (Dowling, 1995, Kosaly, *private discussions*). We are conducting low Mach number LES simulations of 3D jet diffusion flames using the code developed by Charles Pierce and collaborators. We have been initially focusing upon modeling one-step reactions characterized by infinitely fast burning rates. This latter feature implies that the time scale of real burning is limited by the time and efficiency by which the turbulence mixes fuel and oxidizer. This mixing (and, hence, total heat release) will be enhanced by the generation of vortices which can occur if the inlet conditions are modulated (for instance, a pulsed inlet velocity field).

The end goal is to derive radially averaged axial empirical burning functions for such diffusion flame combustors for which we can apply the sort of stability analysis developed in this work. See strategy diagram in Fig. 6.

Through the analysis we have presented here, we have gained a number of valuable insights into previously overlooked mechanisms and culprits responsible for acoustic generation in heat releasing tubes. Nonisentropic inlet boundary conditions can effectively alter the phase relationship between pressure and temperature fluctuations (see the Rayleigh criterion) which can lead to wave instability. Whereas for isentropic inlet boundaries the pressure and temperature fluctuations are in constant phase with each other up to the position of the flame, nonisentropic conditions can generally render the their phasing to be a function of position within the combustor.

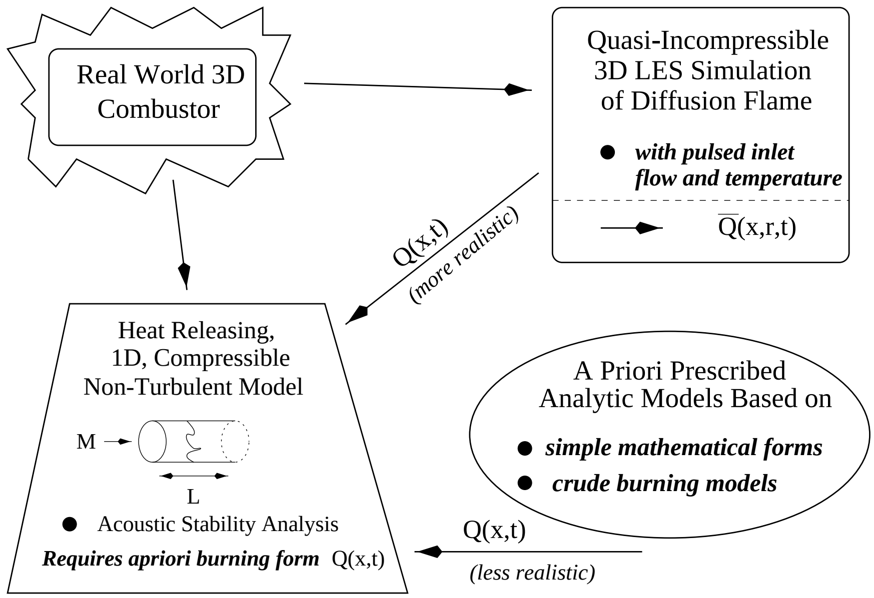

FIGURE 6. Strategy diagram.

On a speculative note: it appears that this phasing mechanism may be responsible for an interesting effect observed in spray combustion experiments performed by C. Edwards and collaborators. Usual spray combustors such as the tube design developed in C. Edwards' lab exhibit chamber instability when fuel is injected into the combustor uniformly. However, C. Edwards points out that the implementation of a combustor design in which spray fuel is injected into the combustor from different positions and randomized in time *prevents acoustic generation within the combustor*. It seems reasonable to suppose that the source of the instability is in part related to the drift mode effect discussed in this review **and** that the reason why the fuel injection randomization procedure works to control generating acoustics in the design discussed by C. Edwards is because the phase induced instability is effectively averaged out over a sufficient period of time. There is not enough time to build up a coherent acoustic mode because the chamber itself is slipping in and out of acoustically unstable configurations through fuel mass-flux modulations at the inlet.

8. Acknowledgments

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