Center for Turbulence Research
Proceedings of the Summer Program 2004

MHD turbulence

The Summer Program 2004 included a substantial MHD session. It consisted of 6 projects, involving 14 contributors. Furthermore, the solar turbulence group also includes MHD as one of its basic constituents with 4 projects involving 8 contributors. This means that the fundamental coupling between the Navier-Stokes equations and the Maxwell-Ohm equations (themselves often written under the form of the induction equation) has become a significant and quite challenging domain of application for the modeling of turbulent fields.

Among the basic features of MHD turbulence, the most important seems to be that, beside viscous dissipation, a second diffusive mechanism is present. It is related to the Joule dissipation and its efficiency is controlled by the value of the magnetic diffusivity η. Therefore, beside the usual Reynolds number Re, an analogous non-dimensional number, named the *magnetic Reynolds number* and denoted Rm, where the magnetic diffusivity is substituted to the viscosity ν, plays an important role. Clearly, Re is always much larger than unity in turbulence. This is not true of Rm and depending on its value one gets very different classes of turbulent flows. At the laboratory scale, whatever the actual fluid used in the experiments (liquid metal or ionized gas), Rm is always much smaller than unity. This implies that the applied magnetic field B is weakly perturbed ($b \ll B$), although the magnetic field perturbation b ($b \approx RmB$ if $Rm \ll 1$) is large enough to be measured. This asymptotic limit may be characterized by two key effects: i) b obeys a simple linear equation and behaves like a passive quantity, inertia remaining the unique source of non-linear effects, ii) the Alfven waves, one of the major MHD characteristics, degenerate into a simple diffusion because their damping time is significantly shorter than their transit time. As soon as Rm is in the range $1-50$, the former simplicity is not anymore valid and one must consider the full induction equation instead of its linear approximation. This implies that the magnetic field, although still transported by the fluid, does not remain passive (such a transport of a non-passive vector field is still a fully open challenge in turbulence). Furthermore, the Alfven waves start to be relevant when the applied magnetic field is large enough to yield a Lundquist number Al/η larger than unity, or as soon as the wave transit time l/A is significantly shorter than their damping time l^2/η. Here, $A = B/\sqrt{\mu\rho}$ denotes the Alfven waves celerity and l a characteristic lengthscale of the flow. This relevance of Alfven waves may start to be effective even when $Rm \ll 1$.

The domain of very large Rm is quite fascinating, since the Navier-Stokes equation and the induction equation are both highly non-linear. In that regime, the Alfven waves are of prime importance. And, among the guiding ideas, the analogy between the induction equation and the equation of vorticity in ordinary hydrodynamics suggests that the magnetic field is frozen in the fluid and behaves exactly like fluid particles; this is a consequence of the Alfven theorem, an MHD equivalent of the Kelvin theorem in ordinary hydrodynamics. One of the noticeable aspects of this class of phenomena is the fact that they cannot be observed in well-controlled laboratory experiments. But they are relevant for the Earth core (except at small scales) and for all astrophysical phenomena, in stars as well as in the interstellar medium. As a consequence, the numerical modeling of that class of phenomena is essential since there is no hope to understand them directly through controlled experiments.

Among the electrically conducting fluids present on Earth at small and moderate scales, one encounters (beside liquid metals) ionized gases, also referred to as plasmas. They are used in different sorts of engineering techniques and thus deserve some particular attention. Very often, they are far from thermodynamic equilibrium and as a consequence, they cannot be modeled as fluids, even multi-phase fluids. This is very much the case for fusion plasmas ($T \approx 10^7 - 10^8 K$) confined by a strong magnetic field and these were not included in the group of projects for the Summer Program 2004. In the Summer Program 2004, the only problem involving plasmas which is investigated is the particular case of Hall thrusters used to propel satellites by producing forces of the order of a few milli-Newtons. It is addressed with the purpose of understanding and modeling the generally observed high transport of electric charges. The key idea is that the high level of observed fluctuations could explain the increase of the apparent electric conductivity.

In spite of the apparent diversity of the MHD phenomena considered during the Summer Program 2004, there is a remarkable unity in the questions being investigated. Among them, the understanding of the anisotropy, due to the privileged direction imposed by the presence of an applied magnetic field, is at the center of the majority of projects. At low Rm, this anisotropy, which seems to be scale independent, yields quasi-two-dimensional structures aligned in the B-direction (however, it is worth noticing that even though the morphology of the turbulent structures is B-dependent, the fluid flow organization within any Q2D structure remains not directly influenced by B). At large Rm, the results obtained by Cho and Lazarian show that the anisotropy is more pronounced in the small scales. As soon as walls perpendicular to the magnetic field are present, as is the case in most of the experiments, some extra effects arise: the velocity component parallel to the applied magnetic field is rapidly suppressed, whereas the perpendicular ones may survive during a much longer time related to the Joule dissipation which takes place within the thin Hartmann layers. The modeling of this influence of the Hartmann walls, which is a necessary step before the development of models capable of computing actual channel flows, is then a significant originality of the Summer Program 2004.

To conclude this brief summary of the most important questions addressed by the participants to the MHD part of the Summer Program 2004, it is important to stress that this international effort seems to be quite timely. Indeed, numerical models should become important tools in the coming years, either to compute industrial MHD flows (e.g. in metals processing or in the design of the blanket of the future nuclear fusion reactor), or to model geophysical and astrophysical flows. In relation to other projects hosted during the Summer Programs 2004, it is worth noting that also in MHD, LES seems to have important advantages over any other modeling techniques, either because the spectral range is so large that DNS is far from being achievable, or because of the presence of an inverse energy cascade when the turbulence becomes Q2D as in liquid metals.

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