An approach to systems modeling for real-time control of jet flows

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1. Introduction

The past 25 years have seen many examples of open-loop control of jet flows with significant practical importance. During the same time, there have been tremendous developments in the area of closed-loop feedback control strategies for linear and nonlinear systems. It has been seen in many applications that coordination of control application with state measurements in the closed-loop setting is essential for optimum system performance. Thus, the possibility of feedback control of flow systems such as turbulent jets should be carefully examined. However, such control problems also pose technical difficulties as turbulent flow systems are multi-scale and difficult to compute with a high degree of fidelity. Thus, the present work explores the development of low-order system models for use in the feedback control framework for the jet control problem. The present work is part of a collaboration involving Georgia Tech, Stanford University, UCLA, UCSD, and The Boeing Company under the support of the Air Force Office of Scientific Research.

To design feedback control algorithms, it is very useful to have a simple model that accurately captures the relevant physics of the phenomena under consideration. Such a model, which should have sufficient simplicity to enable real-time state estimation, is developed in the present work using linear stability theory to model the initial development of the instabilities leading to the turbulent breakdown of a jet. Direct numerical simulations of turbulent jets carried out by Freund *et al.* (1998) will be used as validation for these models; when performed properly, such simulations can capture the relevant flow physics "exactly", albeit at a very large computational expense.

System identification techniques may also be used (instead of linear stability theory) to develop input-output system models of jets for use in the feedback control setting, as explored by Ikeda (1998). Such models may be constructed without any reference to the equations which govern or approximate the flow physics. The present work is an intermediate-level approach that uses inviscid linear stability theory to approximate the jet system. A key feature of the present work is a piecewise quadratic approximation of the mean flow that permits rapid solution of the equations. The authors acknowledge the careful work of Pal (1998) in deriving the stability equations used.

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2. Large scale structures in inflectionally-dominated flows

It is well established that the inviscid inflectional instability characterized by Kelvin and Helmholtz dominates the large-scale development of inflectionally-dominated flows even into non-linear turbulent regimes. An excellent review discussing many important aspects of this problem is given by Ho and Huerre (1984). Supporting articles covering other aspects of the problem (including the nature of nonlinear interaction and wall effects) are described in Cain and Thompson (1986) and Cain, Roos, and Kegelmen (1990). The dominance of the inviscid inflectional instability in the present flow motivates the use of the incompressible inviscid stability equations as the system model.

A comparison between the prediction of linear theory and the nonlinear evolution of the jet shear layers is given by Morris *et al.* (1990) and Viswanathan & Morris (1992) for planar and round jets respectively. These works derive systems comprised of parabolic equations for the mean flow development that depend upon the local instability eigenproblem for the forcing. Solution of this eigensystem is generally the most computationally-intensive aspect of generating the solution to the evolution model for the jet. The need for very rapid solutions for real-time systems modeling motivates our new approach to approximate solution of the disturbance eigenvalue problem.

3. Linear stability analysis

Motivated by the need for rapid evaluation, piecewise linear and quadratic approximations of the velocity profiles are used to simplify the inviscid linear disturbance equations. In addition, mathematical solutions are somewhat easier to obtain for the temporal evolution problem. All the work described here will use the temporal analysis combined with Gaster's relation to approximate the behavior of the spatially evolving problem. The approach so describing an inflectionally dominated flow is given by Drazen and Reid (1981). A few examples of the piecewise linear analysis for planar flows will be presented before describing a piecewise quadratic approximation that will be used as an approximation in the round jet geometry.

3.1 The piecewise linear approximation for planar geometries

The simplest example of the piecewise linear approach is a three-segment representation of the plane shear layer. This approach has been shown to provide consistent and reasonable characterization of the stability behavior of this flow, and it compares well with a more accurate (but more time consuming) analysis with a hyperbolic tangent representation of the mean velocity profile. The characterization of the disturbances is given by the exponential coefficient for temporal disturbance growth rate and phase speed and is a function of the disturbance wavenumber. As shown in Fig. 1, the phase speed computed using such an approach varies from U to $3U$, where U is the velocity of the slow stream. The complex wave speed comes as complex conjugate pairs, and the actual phase velocity is the mean of the two branches plotted. The imaginary part of the wave speed (which in product with the wavenumber gives the exponential growth rate) vanishes at a wavenumber slightly greater that 0.3.

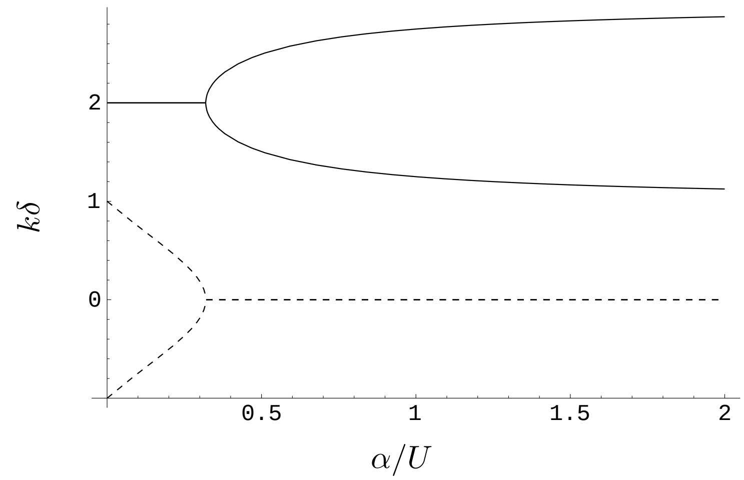

FIGURE 1. Phase speed ——— and growth rate - - - - for piecewise linear planar shear layer.

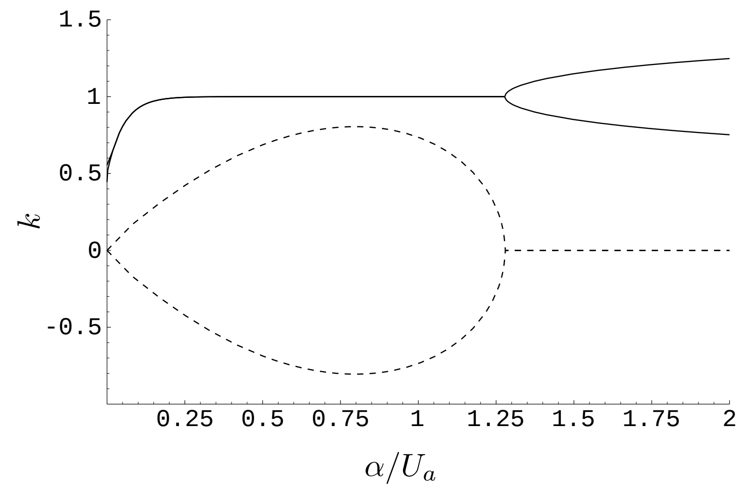

FIGURE 2. Phase speed ——— and growth rate - - - - of a sinuous mode in a piecewise linear planar jet.

The next piecewise linear flow considered is the planar jet. Figure 2 shows the Gaster transformed and scaled spatial growth rate and phase speed versus wavenumber for the sinuous mode of a planar jet having a potential core of 18 jet radii (r_o) long, a shear layer of 1 unit width, and a free-stream velocity equal to 1/3 of the jet velocity $1.5U_a$, where U_a is the average of the jet and co-flow velocities. Note that in this case the long wavelength (low wavenumber) disturbances have a phase speed of the free-stream speed while shorter disturbances have a phase speed of the mean shear layer speed. Figure 3 shows the behavior of the varicose mode. Note that the varicose mode has a phase speed equal to that of the jet centerline for long wavelength disturbances and the same phase speed as the sinuous mode for shorter wavelength disturbances. When the shear layers are thin and separated by a large region of potential flow, the stability of the planar jet is nearly the same as that of the planar shear layer except at very long wave lengths. At smaller separation,

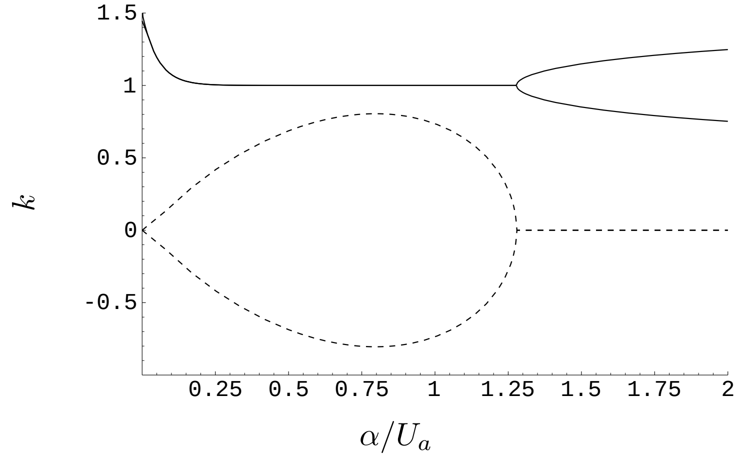

FIGURE 3. Phase speed ——— and growth rate - - - - of a varicose mode in a piecewise linear planar jet.

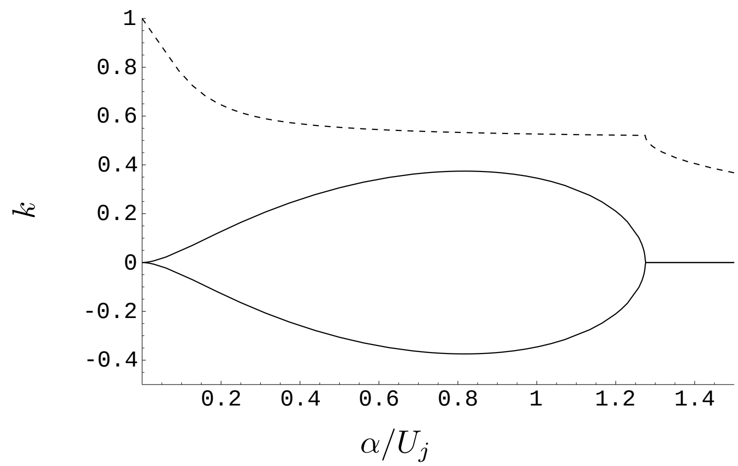

FIGURE 4. Phase speed ——— and growth rate - - - - of a varicose mode in a piecewise linear planar jet.

disturbances of the two shear layers exhibit a strong coupling. These behaviors for both the varicose and the sinuous modes are characteristic of the actual physical system.

3.2 The piecewise quadratic approximation for cylindrical geometries

A piecewise representation of the mean velocity profile simplifies the round jet stability problem to Bessel's equation. The solutions are constrained by requiring finite levels in the inner potential region and solutions that vanish at infinity in the outer potential region. These inner and outer solutions (in terms of Bessel functions) are coupled by matching conditions. The matching is achieved by a combination of the inner and outer Bessel solutions (a linear combination of Bessel functions is a valid solution within the finite thickness shear layer). This problem was formulated by Pal (1998). This approach results in an involved complex-valued quadratic dispersion relation that was solved using Mathematica.

The behavior of the axisymmetric disturbances for a round jet with a potential

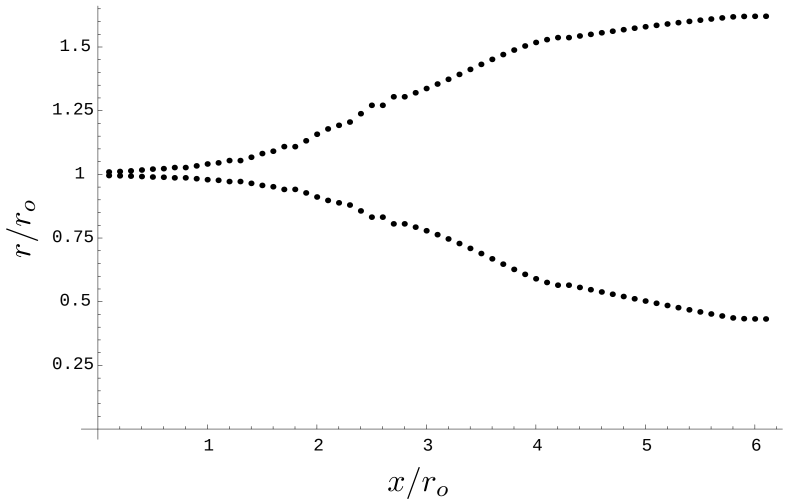

FIGURE 5. Inner and outer shear layer edges.

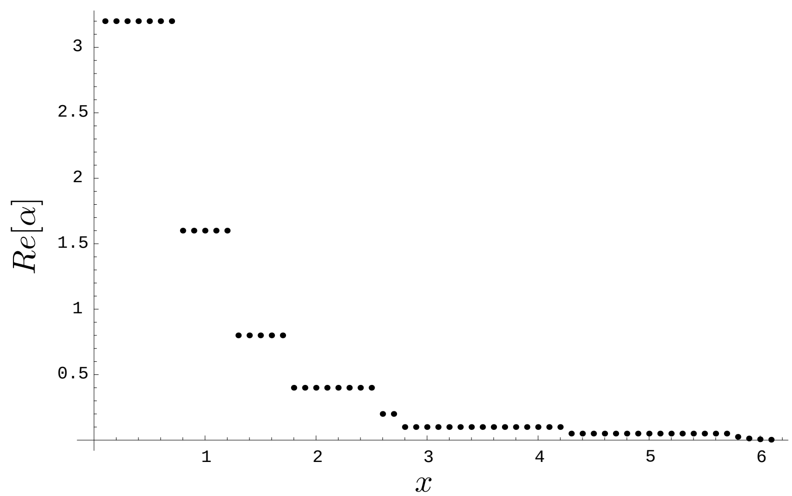

FIGURE 6. Locally dominant instability wave number.

core diameter of $18r_o$ and jet velocity U_j with zero free stream velocity is given in Fig. 4. Note that the appropriately-scaled growth rate and phase speed behave in a manner which is qualitatively similar to the varicose mode of the analogous planar jet.

4. Prediction of jet spreading

The formulation of Viswanathan and Morris (1992) was implemented using the piecewise quadratic stability formulation. An example of the predicted evolution of the shear layer edges using only the $n = 0$ axisymmetric disturbance is shown in Fig. 5. Figure 6 shows the locally dominant wavenumber versus downstream distance in jet radii. It is assumed that, when a disturbances saturates (due to the thickening of the shear layer), the sub-harmonic will become dominant and evolve until saturation, and so on. The results shown in Figs. 5 and 6 are for an initial shear layer thickness of 1% of the jet radius.

5. Conclusions

An approximate analytic solution to the appropriate linear stability problem, and the role of this solution in the evolution of the round jet, has been investigated. When fully implemented, such a formulation may model the physical behavior of the jet with sufficient accuracy to be used as a state model in a feedback control setting. Once programmed in an efficient manner, the computational expense of this model should be manageable. The use of such a model as a state estimator in a feedback control framework thus appears promising though the "real" problem prediction must include at least the $n = \pm 1$ modes in addition to the $n = 0$ mode given here. In the case of a thin shear layer, the $n = \pm 1$ modes behave similarly to the $n = 0$ mode and may be analyzed with the $n = 0$ analysis. However, the solution for $n = \pm 1$ is required as the shear layer thickness becomes significant relative to the jet radius. For the mixing problem it is likely that the solution near the end of the potential core will need to be calculated. A transformation analogous to the Squire reformulation (of the three-dimensional planar stability problem into an equivalent two-dimensional problem in the transformed variables) will be pursued in the round jet problem.

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