Evaluation of noise radiation mechanisms in a turbulent jet

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Data from the direct numerical simulation (DNS) of a turbulent, compressible (Mach=1.92) jet has been analyzed to investigate the process of sound generation. The overall goals are to understand how the different scales of turbulence contribute to the acoustic field and to understand the role that linear instability waves play in the noise produced by supersonic turbulent jets. Lighthill's acoustic analogy was used to predict the radiated sound from turbulent source terms computed from the DNS data. Preliminary computations (for the axisymmetric mode of the acoustic field) show good agreement between the acoustic field determined from DNS and acoustic analogy. Further work is needed to refine the calculations and investigate the source terms. Work was also begun to test the validity of linear stability wave models of sound generation in supersonic jets. An adjoint-based method was developed to project the DNS data onto the most unstable linear stability mode at different streamwise positions. This will allow the evolution of the wave and its radiated acoustic field, determined by solving the linear equations, to be compared directly with the evolution of the near- and far-field fluctuations in the DNS.

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1. Background

Jet noise prediction is a particularly difficult problem because the complexity of turbulent flow permits only approximate experimental and theoretical description of the acoustic sources. Meanwhile, aeroacoustic theory requires such a description as input and attempts to predict jet noise *a posteriori* via solution of a linear wave equation for the radiated sound. Despite many years of investigation of acoustic sources in turbulent flows, fundamental questions persist which cannot be addressed without access to very detailed measurements of the near field turbulence.

Fortunately, computer power has increased to the point where it is now possible to simulate low Reynolds number turbulent jets without modeling approximations. Because this approach provides full knowledge of the jet flow, it permits a detailed examination of the acoustic sources.

Data from a Direct Numerical Simulation (DNS) of a Mach 1.92 (Freund, Lele & Moin 1998) jet is used here to investigate several issues related to the mechanisms of sound generation in turbulent jets:

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- What is the relative contribution of large and small scales to the acoustic sources terms in turbulent jets?
- What role do linear instability waves play in the noise produced by supersonic turbulent jets?

In order to investigate the first question, we use the DNS data to compute the source terms for Lighthill's acoustic analogy. The solution to the acoustic analogy with these sources is then found and compared to the directly computed acoustic field. The results of this calculation are presented in Section 2. Once the efficacy of the acoustic analogy is established, the turbulent source terms can be investigated to understand how different scales of motion contribute to the overall acoustic field– we make some preliminary observations regarding the structure of the source in Section 2. The relevance of different scales, and how accurately significant scales may be represented on a hypothetical coarser mesh, has important implications for subgrid scale modeling in LES applications.

In Section 3, we investigate the radiation of acoustic waves by instability waves in supersonic jets. While it has been shown (*e.g.* Tam 1995) that the linear model correctly predicts certain trends in turbulent mixing noise in supersonic jets, it relies on an assumed distribution (and amplitude) of turbulence fluctuations at the nozzle exit, and detailed measurements and comparisons with the theory have not yet been made. In Section 3 we present a framework that we have developed to test some of the underlying assumptions of the linear theory and to attempt to validate its predictions against the directly computed acoustic field.

2. Acoustic analogy

An important issue in using an acoustic analogy to predict jet noise is the degree to which an acoustic analogy can predict the sound field given an "exact" representation of the source terms from numerical simulation as input. This depends, on one hand, to what extent discretization errors may affect the acoustic sources and, on the other hand, how reliable are the approximations made in separating source terms from propagation terms in the acoustic analogy. These issues were addressed in detail for vortex pairing in a two-dimensional mixing layer (Colonius, Lele & Moin 1997), but it is unclear to what extent the conclusions may be extended to a fully turbulent flow. To address these issues in the Mach 1.92 turbulent jet, we first consider the well-known equation of Lighthill (1952):

$$\ddot{\rho} - a_o^2 \rho_{,jj} = T_{ij,ij}, \tag{1}$$

where,

$$T_{ij} = \rho u_i u_j + (p - a_o^2 \rho)\delta_{ij} \tag{2}$$

with viscosity neglected. It is straightforward to compute the right-hand side of the equation and then solve Eq. (1) for the radiated acoustic field.

2.1 The Mach 1.92 turbulent jet and post-processing of DNS database

In this section we briefly discuss some of the issues pertaining to how the DNS database for the Mach 1.92 turbulent jet was analyzed. DNS of the fully compressible, turbulent round jet were performed by Freund (1998) using a method which relies on 6th-order-accurate compact finite difference schemes in the axial and radial directions, Fourier spectral differentiation in the azimuthal direction, and 4th-order-accurate explicit Runge-Kutta time advancement. The computational domain extends to 31 jet radii downstream and 12 radii in the radial direction, and thus includes the near acoustic field. 640 x 270 x 128 nodes were used in the axial, radial, and azimuthal directions, respectively. The Reynolds number based on diameter was 2000, and the temperature ratio $T_j/overT_\infty$ was 1.12.

In order to simulate the turbulent jet inflow (thereby eliminating the need to simulate the nozzle as well as the jet), data from a turbulent streamwise periodic jet simulation (Freund, Moin & Lele 1997) was fed into the spatial computation. The simulations used to generate the inflow turbulence had a streamwise period of 21 radii. The amplitudes at the inflow were "jittered" by up to 5% to break any periodicity of the data.

The full instantaneous flow field was archived at roughly 7000 times (at increments of 10 computational time steps). For practical reasons, the data was saved on a mesh which consisted of every other computational node. Even at 1/8 of the full resolution, this yields a database nearly a quarter of a terabyte in size. A principal limitation in post-processing is the total amount of data that may be stored on disk and read into memory.

A large number of terms are involved in computing Lighthill's source term in cylindrical coordinates (the transformations are straightforward but tedious, and will not be reproduced here). Such terms involve second spatial derivatives of the computational data, and a problem was encountered in computing source terms smoothly through the coordinate singularity at $r = 0$. In this report we only consider predictions for Lighthill's equation for the axisymmetric ($m = 0$) mode, for which there were no accuracy issues near the centerline. However, we have recently devised a way to compute the sources on an interpolated Cartesian mesh, which appears to alleviate the centerline difficulty–an example of the source computed in this way is given in Section 2.3.

2.2 Numerical solution of the wave equation

In order to determine the acoustic field which results from a particular measured source term, Eq. (1) must be inverted. A variety of methods are available for this task. From a theoretical point of view, the simplest method would be to convolve the source with the Green's function for the three-dimensional wave equation. However, the resulting integrals depend on the source at retarded times. Given the enormous amount of data which would have to be sorted and interpolated for every such combination of source and observer location, this approach was deemed impractical. Instead we employ the less computationally intensive method of solving the wave equation directly in the time domain using finite differences. We discretize the wave equation with the same 6th-order-accurate compact finite difference scheme used in the jet computations, and integrate forward in time with a 4th order Runge-Kutta time algorithm. The source is Fourier transformed in the azimuthal direction since the wave equation is linear and each azimuthal mode may, in turn, be found independently. The computational grid is chosen to be identical to that on which

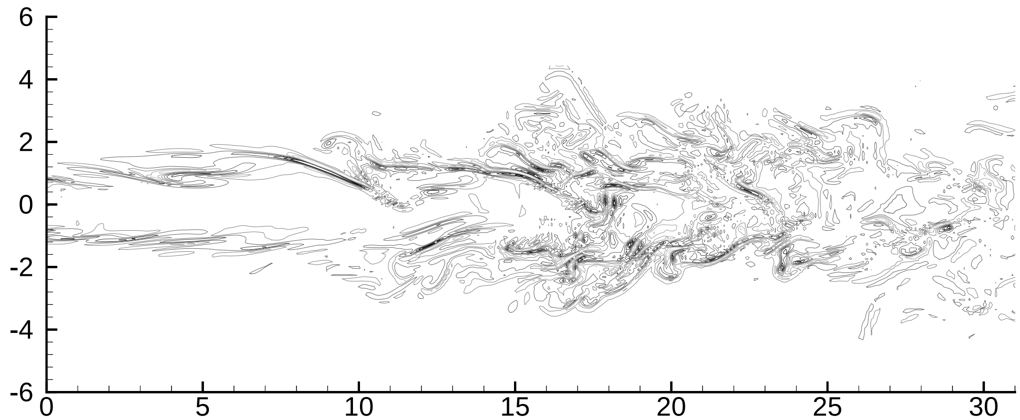

FIGURE 1. Contours of the instantaneous $T_{ij,ij}$ in the plane $\theta = 0, \pi$.

the source data was saved except that in the radial direction we interpolate the source (using a 6th-order-accurate compact interpolation scheme) to a staggered mesh where accurate differencing through the polar coordinate singularity may be achieved (Mohseni & Colonius 1997). One-dimensional characteristic boundary conditions are used together with a buffer region near the computational boundary where the damping terms are added to the wave equation (Freund 1997).

The source terms at a given instant in time are found by performing a cubic spline interpolation of the sources saved at discrete intervals in time. In order to avoid sharp initial transients produced by turning on the source at $t = 0$, we ramp-up the forcing over a time period which is long compared to the dominant frequencies in the acoustic field.

2.3 Results

Figure 1 shows an instantaneous view of the Lighthill source term, $T_{ij,ij}$, at the plane $\theta = 0, \pi$ through the jet. It is clear that, instantaneously, small-scales are dominant contributors to the source. As discussed below, one cannot conclude that these are also dominant contributors to the acoustic field since the source field is comprised of fluctuations whose wavenumber/frequency characteristics prohibit radiation to the far field.

Nevertheless, it is interesting to see how accurately the acoustic field predicted from the full instantaneous source agrees with the directly computed field. The source field was computed (for the $m = 0$ mode only) for 500 of the archived DNS fields and input into the wave equation solver. For the axisymmetric mode in the acoustic field, the instantaneous prediction at the final time is compared in Fig. 2 with the directly computed acoustic field. The quantitative agreement is quite

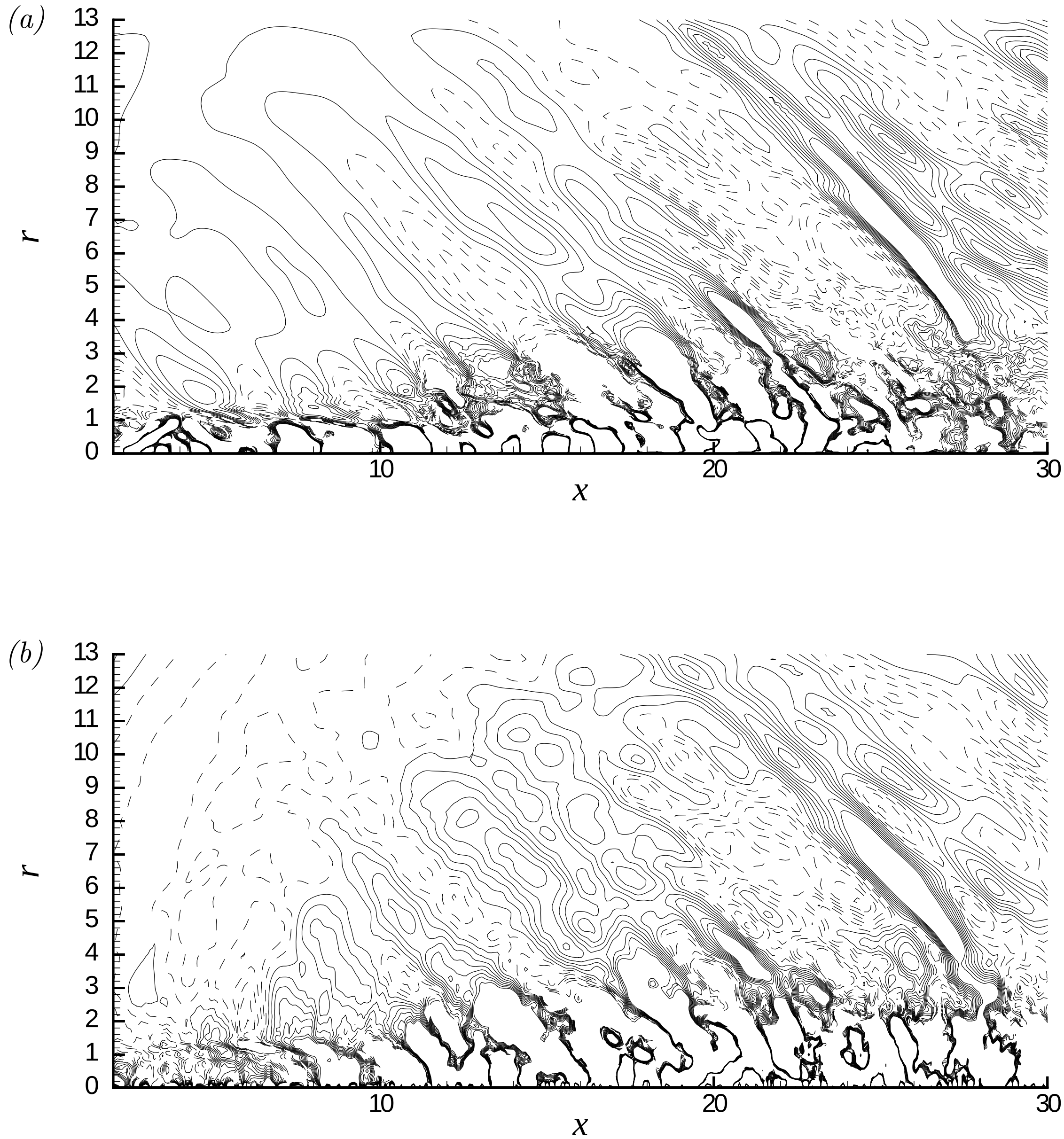

FIGURE 2. Contours of the instantaneous dilatation for the $m = 0$ mode, directly computed via (a) DNS and (b) by the solution of Lighthill's equation. Contour levels (both figures), 15 contours between -0.02 to 0.02, negative contours are dashed.

good, with the Lighthill solution accurately reproducing the major features of the acoustic field such as the strong Mach wave radiation at angles between 45 and 55 degrees from the jet axis. Further work is needed to compute the solution for other azimuthal modes.

Once the acoustic analogy has been validated in this way, we may proceed to analyze the sources to understand and, ultimately, model in a simple way the physics of sound generation by the flow. An important first step in this process is to (approximately) decompose the source term into its radiating and non-radiation components. As noted above, small scales dominate the instantaneous value of the source; however, physical space and real time are not the most natural space in which to examine the acoustic sources. This is because a large portion of the source is composed of wavenumber/frequency combinations which are not able to radiate to the acoustic field. It can be shown that only those wavenumber/frequency combinations whose phase speed is supersonic with respect to any ambient flow may radiate to the far field (Crighton 1975).

An exact decomposition into wavenumber/frequency space requires strict periodicity (or infinite extent) of the source. For spatially evolving flows on a finite computational domain, this condition is not met, and further information regarding the decay of the acoustic sources downstream of the computational boundary must be supplied in order to accurately transform the source in the axial direction. This can be done in a semi-analytical way if the acoustic source is simple enough (*e.g.* Colonius *et al.* 1997, Mitchell, Lele & Moin 1995, Wang, Lele & Moin 1996). It is likely, however, that such methods will fail for a fully turbulent source, and thus we seek an alternative. We anticipate using approximate band-pass filters which can be constructed in real space (*e.g.* Lele 1992) for non-periodic data.

3. Linear stability theory

For jets with sufficiently high Mach number (essentially with a convective Mach number greater than 1), it is know that linear instability waves directly radiate sound to the far field (*e.g.* Tam & Burton 1984). This observation gives rise to an alternative to the acoustic analogy approach in this case. The near field and far field are constructed simultaneously as a solution to linear equations, and the flow acts, in essence, as an amplifier of some prescribed disturbances, with the amplification and eventual decay giving rise to the production of acoustic waves.

Such linear modeling has been shown to produce trends in the radiated sound pressure level similar to noise measurements at various frequencies and at angles where the acoustic field is thought to be dominated by contributions from the large scale structures (Tam 1995). However, the model cannot, without further ad-hoc assumptions, predict the amplitude of the acoustic radiation (devoid of any measurements of the turbulence incident from the nozzle), and a detailed computational or experimental verification of the linear modeling has not previously been attempted.

In application of the linear model, the mean jet flow (from experiment or RANS modeling) is assumed known, and initial amplitude for the eigenfunctions (at a particular frequency) are specified according to an assumed frequency spectrum. Typically only the most unstable mode is considered. The evolution of the mode in the slightly non-parallel mean flow is then found by marching downstream at a particular frequency. Unstable modes eventually stabilize and decay as the mean flow spreads. The envelope function, which describes the growth, saturation, and decay of the mode, can then be used to compute the radiated acoustic field using the method of matched asymptotic expansions. For details, the reader is referred to Tam & Burton (1984).

We believe that the DNS data for the Mach 1.92 jet will be useful in testing the validity of the linear theory. We present here a framework that we developed to compare the linear stability predictions with the (nonlinear) DNS solution. Though straightforward, the calculations are laborious and have not yet been completed. We hope to give the results in a future publication.

At streamwise locations throughout the jet, we transform the DNS data into frequency space and wavenumber space in the azimuthal direction. These fluctuations are then the initial condition for the instability wave calculation. To determine the initial amplitude of the most unstable wave, we observe that an arbitrary fluctuation field can be completely decomposed into a discrete set of unstable waves plus a continuous spectrum of stable modes of the linearized equations (*e.g.* Drazin & Reid 1981). Furthermore, if the adjoint eigenfunctions corresponding to the most unstable mode can be computed, the amplitude of the most unstable mode can be found *without* computing the entire continuous spectrum of stable modes.

This approach may be unfamiliar to some readers, so we briefly discuss the derivation of the equations. We have a (locally) parallel base flow with streamwise velocity, $\bar{u}(r)$, density $\bar{\rho}(r)$, and pressure $\bar{p}$ =constant. Let $\phi(x,r,\theta,t) = \{u,v,w,p\}^T$ be the velocity components (streamwise, radial, and azimuthal) and pressure, and let $\tilde{\phi}(x,r,\theta,t)$ be the solution to the adjoint equations. Then, substituting $\phi(x,r,\theta,t) = \phi(r)e^{i(\alpha x+n\theta-\omega t)}$ into the Euler equations linearized about the base flow, we obtain:

$$\begin{aligned} i(\alpha\bar{u}-\omega)\bar{\rho}u + \bar{\rho}\frac{\partial\bar{u}}{\partial r}v + i\alpha p &= 0 \\ i(\alpha\bar{u}-\omega)\bar{\rho}v + \frac{\partial p}{\partial r} &= 0 \\ i(\alpha\bar{u}-\omega)\bar{\rho}w + \frac{in}{r}p &= 0 \\ i(\alpha\bar{u}-\omega)\frac{p}{\gamma\bar{p}} + i\alpha u + \frac{1}{r}\frac{\partial(rv)}{\partial r} + \frac{in}{r}w &= 0. \end{aligned} \tag{3}$$

The corresponding adjoint equations may be found in a straightforward way:

$$\begin{aligned} i(\tilde{\alpha}\bar{u}-\tilde{\omega})\bar{\rho}\tilde{u} + i\tilde{\alpha}\tilde{p} &= 0 \\ i(\tilde{\alpha}\bar{u}-\tilde{\omega})\bar{\rho}\tilde{v} + \bar{\rho}\frac{\partial\bar{u}}{\partial r}\tilde{u} + \frac{\partial\tilde{p}}{\partial r} &= 0 \\ i(\tilde{\alpha}\bar{u}-\tilde{\omega})\bar{\rho}\tilde{w} + \frac{i\tilde{n}}{r}\tilde{p} &= 0 \\ i(\tilde{\alpha}\bar{u}-\tilde{\omega})\frac{\tilde{p}}{\gamma\bar{p}} + i\tilde{\alpha}\tilde{u} - \frac{1}{r}\frac{\partial(r\tilde{v})}{\partial r} + \frac{i\tilde{n}}{r}\tilde{w} &= 0. \end{aligned} \tag{4}$$

where $\tilde{\phi}(x,r,\theta,t) = \tilde{\phi}(r)e^{-i(\tilde{\alpha}x+\tilde{n}\theta-\tilde{\omega}t)}$.

One can easily check that the following Lagrange identity is satisfied

$$i(\tilde{\omega}-\omega)Z(r) - i(\tilde{\alpha}-\alpha)\{\tilde{u}Z(r)+p\tilde{u}+\tilde{p}u\} - i(\tilde{n}-n)\frac{1}{r}(p\tilde{w}+\tilde{p}w) = -\frac{1}{r}\frac{\partial}{\partial r}(r(\tilde{v}p-v\tilde{p})), \tag{5}$$

where $Z(r) = \bar{\rho}(u\tilde{u} + v\tilde{v} + w\tilde{w}) + \frac{p\tilde{p}}{\gamma\bar{p}}$.

For the spatially evolving instability waves we have $\tilde{\omega} = \omega$ and $n = \tilde{n}$. Therefore if ϕ_i and $\tilde{\phi}_j$ are the eigenfunctions to the flow equations (3) and the adjoint equations (4) corresponding to the eigenvalues α_i and α_j, we have the *biorthogonality* relation

$$[\phi_{\alpha_i}, \tilde{\phi}_{\alpha_j}] = 0 \qquad \text{if} \quad \mathrm{i} \neq \mathrm{j} \tag{6}$$

where

$$[\phi, \tilde{\phi}] = \int_0^\infty (\bar{u}Z(r) + p\tilde{u} + u\tilde{p}) r dr. \tag{7}$$

Let us denote the DNS data (transformed into frequency and azimuthal wave space at a particular streamwise location) by ϕ_{DNS}. Then we expand ϕ_{DNS} in terms of the eigenfunctions:

$$\phi_{DNS}(r) = \sum_{j=1}^{N} c_j \phi_j(r) + \text{Integral over continuous spectrum} \tag{8}$$

where N is the number of discrete spatial modes. Now multiply both sides of Eq. (8) by the adjoint eigenfunction $\tilde{\phi}_{\alpha_i}$, and, using the biorthogonality relation (6), we obtain

$$c_i = \frac{[\phi_{DNS}, \tilde{\phi}_{\alpha_i}]}{[\phi_{\alpha_i}, \tilde{\phi}_{\alpha_i}]} \tag{9}$$

Of particular interest will be the evolution of the most unstable wave as a function of streamwise location in the jet, and the acoustic field predicted from the (matched asymptotic expansion of the) linear equations, as compared to the acoustic field from the DNS.

4. Closing remarks

We have analyzed data from the DNS of a turbulent Mach=1.92 jet to address several issues related to noise generation mechanisms. As a first step towards understanding the turbulent sources of sound, we have computed the full Lighthill source terms and found solutions of the wave equation with these sources. We have found a good agreement for the $m = 0$ mode, paving the way for a detailed analysis of the sources to determine how various scales in the near field turbulence contribute to the radiated acoustic field.

For supersonic jets, we have also developed a framework to test the validity of the well-known linear stability wave model (*e.g.* Tam 1995) of sound generation by the large-scale turbulence. By using the adjoint to the linear stability equations, we are able to compute the projection of the DNS data onto the most unstable mode at different streamwise positions. The amplitude of the stability wave thus set, we may proceed to compute the evolution of the wave and its radiated acoustic field by solving the linear equations and compare the resulting acoustic field with the DNS. We hope to present the results of the analysis in a future publication.

REFERENCES

Colonius, T., Lele, S. K. & Moin, P. 1997 Sound generation in a mixing layer. *J. Fluid Mech.* **330,** 375-409.

Crighton, D. G. 1975 Basic principles of aerodynamic noise generation. *Prog. Aerospace Sci.* **16,** 31-96.

Drazin, P. & Reid, W. 1981 *Hydrodynamic Stability.* Cambridge University Press.

Freund, J. B. 1997 Proposed inflow/outflow boundary condition for direct computation of aerodynamic sound. *AIAA J.* **35,** 740-742.

Freund, J. B. 1998 On the role of linear instability modes in a turbulent annular mixing layer. *Submitted to Phys. Fluids.*

Freund, J., Lele, S. & Moin, P 1998 Direct simulation of a Mach 1.92 jet and its sound field. *AIAA/CEAS 98-2291.*

Freund, J., Moin, P. & Lele, S. 1997 Compressibility effects in a turbulent annular mixing layer. *Technical Report TF-72,* Department of Mechanical Engineering, Stanford University.

Lele, S. K. 1992 Compact finite difference schemes with spectral-like resolution. *J. Comput. Phys.* **103,** 16–42.

Lighthill, M. J. 1952 On sound generated aerodynamically. I: General theory. *Proc. Royal Soc. London A.* **211,** 564-587.

Mitchell, B. E., Lele, S. K. & Moin, P. 1995 Direct computation of the sound generated by subsonic and supersonic axisymmetric jets. *Technical Report TF-66,* Department of Mechanical Engineering, Stanford University.

Mohseni, K. & Colonius, T. 1997 Numerical treatment of polar coordinate singularities in finite difference and pseudo-spectral schemes. *Division of Engineering and Applied Science, Caltech.*

Tam, C. K. 1995 Supersonic jet noise. *Ann. Rev. Fluid Mech.* **27,** 17-43.

Tam, C. K. W. & Burton, D. 1984 Sound generated by instability waves of supersonic flows. Part 1. Two-dimensional mixing layers, Part 2. Axisymmetric jets. *J. Fluid Mech.* **138,** 249-295.

Wang, M., Lele, S. K. & Moin, P. 1996 Computation of quadrupole noise using acoustic analogy. *AIAA J.* **34,** 2247-4344.