Boundary conditions for LES away from the wall

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Artificial boundary conditions for LES away from the wall have been developed with the hope of avoiding the problem of grid refinement in the wall region of the LES. In the particular example of channel flow, the main idea is to replace the natural no-slip boundary conditions (at $y = 0$) by artificial boundary conditions at $y = y_1 > 0$. The one-point statistics (mean velocity and turbulence intensities) of the flow at y_1 are supposed to be provided externally. In practice, this information could be obtained from a RANS for the same flow. However, it is known that supplying only the one-point statistics of the velocity field is not sufficient for obtaining a reasonable core flow. The method developed here consists of building two-point statistics at the artificial boundary by using information from the core flow at $y = y_2 > y_1$. In particular, the time evolution of the velocity fields at $y = y_1$ and $y = y_2$ are assumed to be self-similar with a time scale ratio determined dynamically during the simulation. Encouraging results for the channel flow at $Re_\tau = 1000$ have been obtained when the domain removed from the simulation $(0 < y < y_1)$ contains half of the grid points used in "full domain" LES of the channel flow.

1. Introduction

The grid refinement required in the near wall region has severely slowed the development of large-eddy simulation (LES) for flows of practical interest. Several techniques aimed at keeping the grid coarse in the near wall region have been investigated. Most of them supply artificial boundary conditions, either at the physical wall or inside the flow. In the latter case, the boundary conditions must compensate for the total absence of knowledge of the dynamics inside the unresolved wall region. In this preliminary study, we will only consider this type of off-wall boundary condition.

Previous studies (Baggett, 1997) have shown that providing the correct one-point statistics at the artificial boundary is not sufficient. Imposing only the mean velocity values and the mean turbulent stresses at the artificial boundary has been shown to lead to very poor results even in the simple geometry of the channel flow. Some information regarding the structure of turbulence should be imposed at the artificial boundary as well. In other words, at least the two-point statistics should have a reasonably correct value at the artificial boundary.

It is sometimes considered that going further than the second order statistics and trying to impose, for instance, third order moments of the velocity fluctuation is

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not necessary. As noted by Jiménez & Vasco (1998), this statement is debatable since some of the third order statistics of the velocity fluctuations correspond to the energy flux through the boundary. It might turn out that imposing this energy flux could be as important as imposing the wall stress. However, imposing the energy flux at the boundary is certainly quite difficult.

The purpose of this study is to investigate some new and very simple ideas for extrapolating from the core flow some information on the two-point statistics that should be imposed at the boundary. The artificial boundary conditions that we consider here only impose the first and second order statistics of the velocity fluctuations. In this first stage, we only consider LES of the channel flow for which we have a reference LES at Reynolds number $Re_\tau = 1000$ (Kravchenko, Moin & Moser, 1996).

2. Artificial boundary conditions

The underlying idea is to use some scaling law for reconstructing the velocity field at a certain distance from the solid boundary from the known velocity field in the core flow. For this reason, in this first study, we have focused on the channel flow for which a logarithmic profile is known to exist. In this domain, the size of the structures is supposed to grow linearly with the distance to the wall. In the channel flow at $Re_\tau = 1000$, both $y_1^+ = 100$ and $y_2^+ = 200$ are in the log-layer. We have thus considered several possibilities for reconstructing the velocity at y_1 from the velocity at y_2. First, we have considered the possibility of imposing a linear scaling law on the characteristic length scale for the velocity. However, we found that imposing the time scale of the velocity fluctuation is much easier and leads to better results. In practice, we have thus first assumed that the typical time-scale of the velocity fluctuation δv_i also follows a linear law in the log-layer. This can be expressed by:

$$\frac{1}{\langle(\delta v_i)^2\rangle} \left\langle \left(\frac{\partial \delta v_i}{\partial t} \right)^2 \right\rangle \propto y^{-2} \ . \tag{2.1}$$

This assumption is reasonable but it only relates statistical quantities at different values of y^+. The main assumption of our approach is to use this relation for connecting every point in the artificial boundary (y_1) with a point in another plane (y_2) which lies within the computed part of the flow:

$$\frac{1}{\sqrt{\langle(\delta v_i(y_1))^2\rangle}} \frac{\partial \delta v_i(y_1)}{\partial t} = \gamma_i \frac{1}{\sqrt{\langle(\delta v_i(y_2))^2\rangle}} \frac{\partial \delta v_i(y_2)}{\partial t} \tag{2.2}$$

where the γ_i should be equal to $y_2/y_1 = 2$ if the scaling law (2.1) were correct. In practice, $v_i'(y_1) \equiv \sqrt{\langle(\delta v_i(y_1))^2\rangle}$ is unknown and has to be provided as part of the boundary conditions while $v_i'(y_2) \equiv \sqrt{\langle(\delta v_i(y_2))^2\rangle}$ is directly measured from the computed part of the flow. Hence, the second order statistics must be provided at the boundary, as expected. The condition (2.2) will thus read:

$$\frac{\partial \delta v_i(y_1)}{\partial t} = \gamma_i \frac{v_i'(y_1)}{v_i'(y_2)} \frac{\partial \delta v_i(y_2)}{\partial t} \ . \tag{2.3}$$

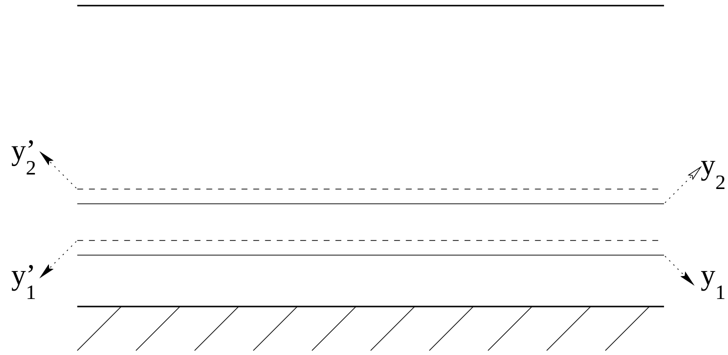

FIGURE 1. Schematic representation of the plane y_1, y_1', y_2, and y_2'.

In actual simulations, the condition has been implemented as follows:

$$\delta v_i(y_1, t + \Delta t) = \delta v_i(y_1, t) + \gamma_i \, \frac{v_i'(y_1)}{v_i'(y_2)} \left(\delta v_i(y_2, t) - \delta v_i(y_2, t - \Delta t) \right) \ . \tag{2.3}$$

Since the scaling (2.1) is not necessarily correct, we have also considered the possibility of estimating the value of the parameters γ_i during the course of the simulation. This dynamical estimation is done by using two additional planes inside the computed flow (see Fig. 1): plane y_1' just above y_1 and plane y_2' just above y_2 ($y_j' = y_j + \Delta y_j$, where Δy_j is the mesh size at plane j). The γ_i parameters measured for the pair (y_1', y_2') are used for connecting the planes (y_1, y_2).

Clearly, the artificial boundary conditions (2.3) do not determine the mean velocity value (first order statistics) which also needs to be supplied externally. The underlying idea of this approach is to connect the LES with an alternative and cheaper approach for the wall region. For instance, the mean velocity at the boundary $\langle v_i(y_1) \rangle$ and the turbulence intensities $v_i'(y_1)$ could be derived from a RANS. In the tests presented here, we have used the LES value from the other side of the channel when the other wall was treated classically. This is thus an asymmetric simulation. When both walls have been treated with the artificial boundary conditions described before (symmetric simulation), the first order statistics $\langle v_i(y_1) \rangle$ and the second order statistics $v_i'(y_1)$ have been taken from the LES of Kravchenko.

It must be noted that, in their present form, the artificial boundary conditions do not impose the stress $\langle \delta u(y_1) \delta v(y_1) \rangle$. Since this stress is perhaps the most important quantity in the wall region of a turbulent flow, it must be verified *a posteriori* that the predicted value is indeed correct.

The simulations that are presented in this report correspond to $n_x = 48$, $n_y = 65$, and $n_z = 48$. When the artificial boundary conditions are placed at $y^+ = 100$, 25% of the grid points are removed from the simulation in the asymmetric simulation and 50% in the symmetric one.

3. Numerical results

The first results we obtained were very disappointing. Trying to impose the linear law for the typical time scale of the velocity fluctuations ($\gamma_u = \gamma_v = \gamma_w = 2$

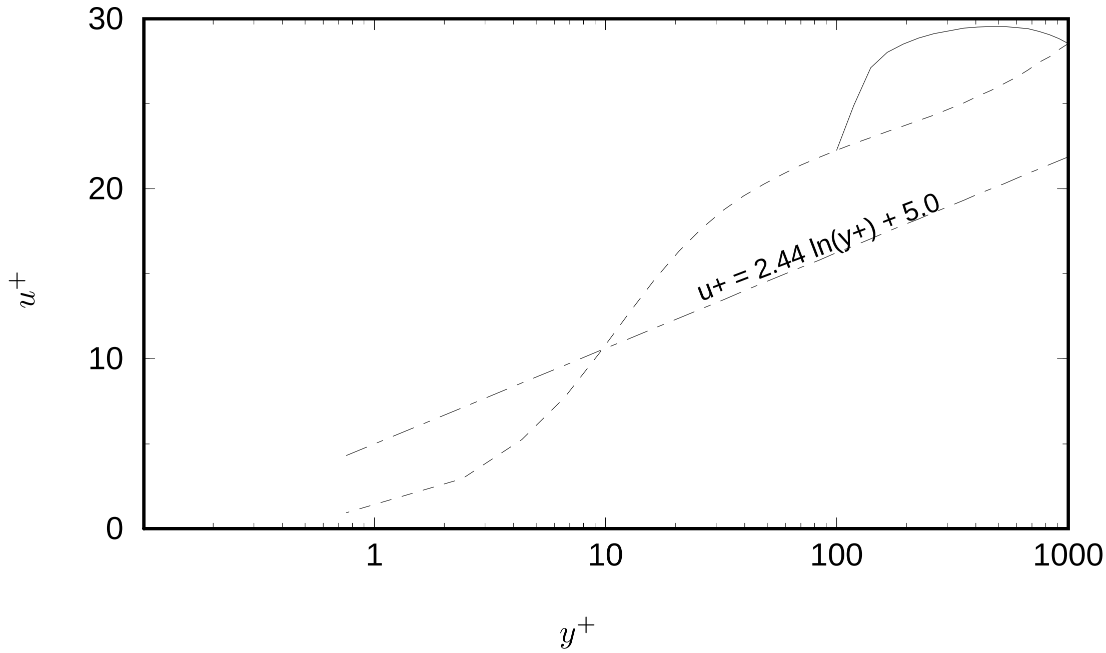

FIGURE 2. Mean velocity profile for the asymmetric computation. $\gamma_u = \gamma_v = \gamma_w = 2$. ——— : lower half, artificial boundary conditions; ---- : upper half, classical boundary conditions. Non-dimensionalization is based on the mean friction velocity.

leads to very poor results for all the relevant quantities. For instance, the mean velocity profile (Fig. 2) for the asymmetric computation was totally different in the upper half and the lower half of the channel. This shows that the artificial boundary conditions (2.3) with $\gamma_u = \gamma_v = \gamma_w = 2$ are not able to correctly mimic the dynamics of the flow between $0 < y^+ < y_1^+$.

This has strongly motivated the use of the dynamic evaluation of the parameters γ_i. As can be seen in Fig. 3a, when measured on the wall with classical boundary conditions, the dynamic values for these parameters are very close to 1.

This result is somewhat puzzling because it shows that the scaling argument used for motivating the artificial boundary conditions is not valid. In particular, the time scales (and the length scales) do not grow linearly in the log-layer of our LES. A possible reason for that is the lack of resolution in our coarse LES at $Re_\tau = 1000$, not only in y, but also in x and z; even on the "resolved" wall, the mean velocity profile does not fall on the curve $u^+ = 2.44 \ln(y^+) + 5.0$, see Figs. 2 and 3. Recall that LES's of the channel flow using structured grids (i.e., uniform Δx and Δz) are often quite coarse in x and z in the log region close to the wall; structures are not completely resolved there. Thus, the grid used here is not sufficient to capture the wide range of scales necessary for the expected scaling $\gamma_i = 2$ to hold. Another possibility is that the proposed scaling $\gamma_i = 2$ should not hold anyway because the dominant integral scale (in the streamwise direction) does not scale with the distance to the wall. This point should certainly be addressed further in a follow up of this work, using both DNS data and resolved LES data such as Kravchenko,

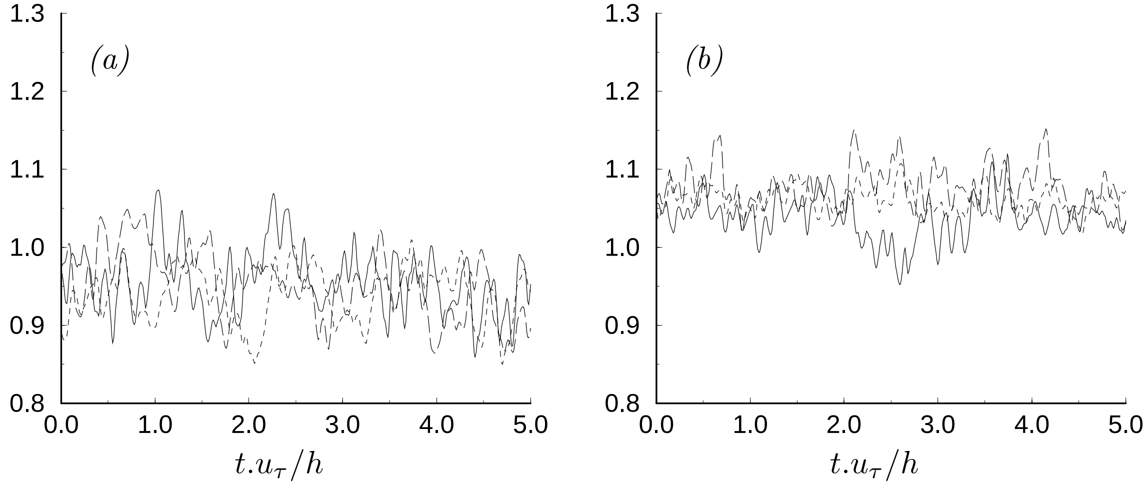

FIGURE 3. Time evolutions for the γ parameters. (a) upper half, classical boundary conditions; (b) lower half, artificial boundary conditions. ——— : γ_u; - - - - : γ_v; — — — : γ_w.

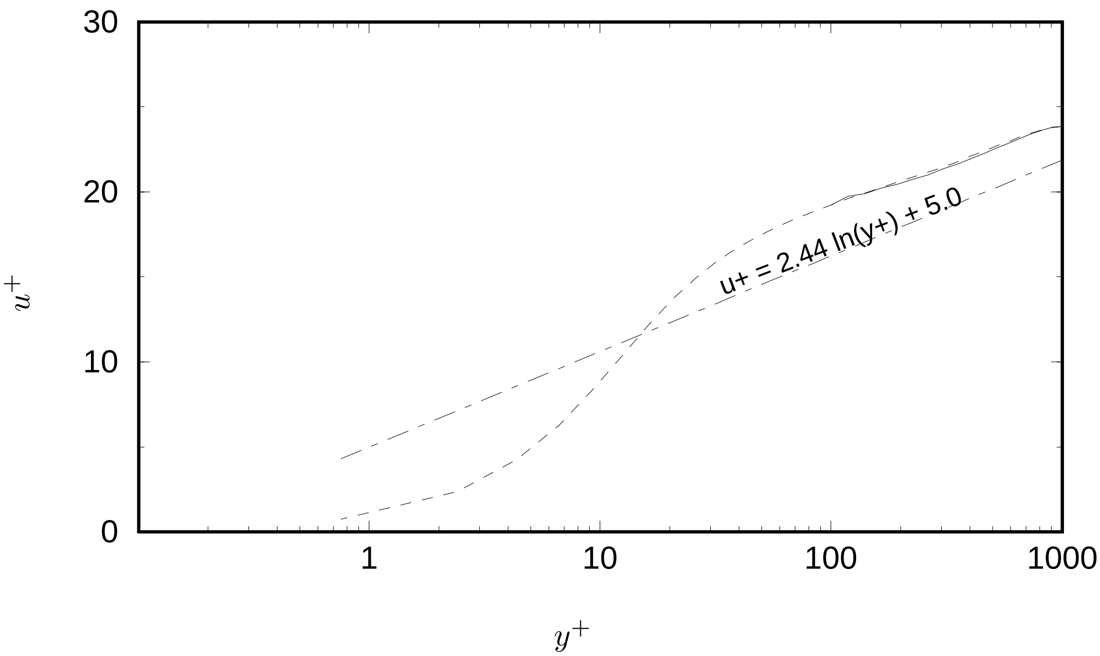

FIGURE 4. Mean velocity profile for the asymmetric computation. γ's computed dynamically. ——— : lower half, artificial boundary conditions; - - - - : upper half, classical boundary conditions.

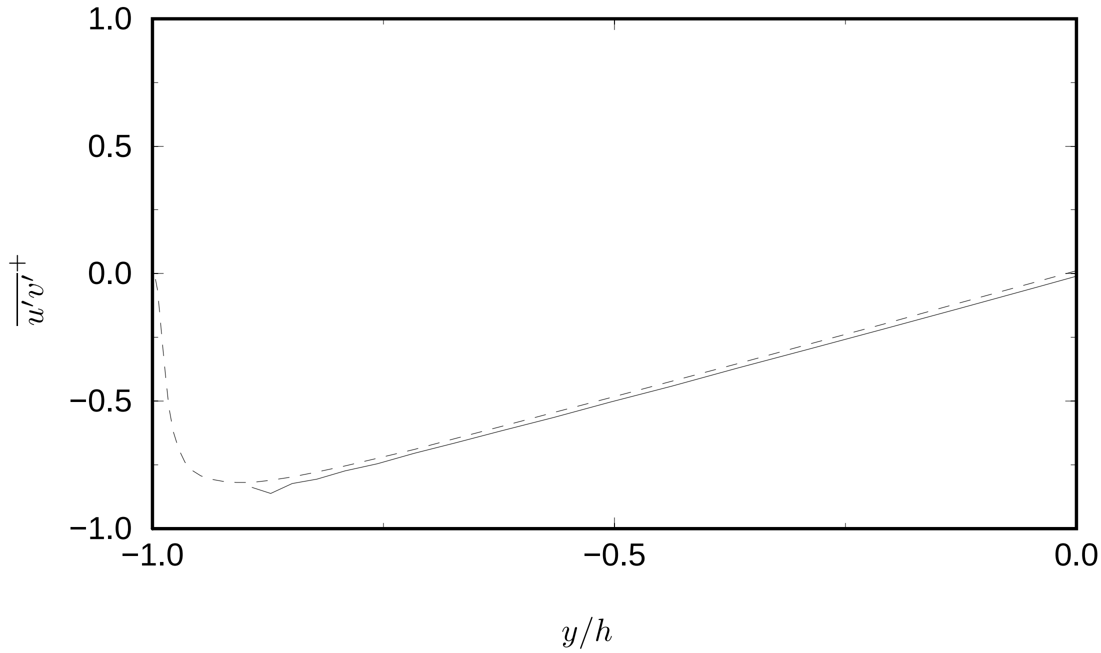

FIGURE 5. Resolved shear stress for the asymmetric computation. γ's computed dynamically. ——— : lower half, artificial boundary conditions; - - - - : classical boundary conditions, upper half.

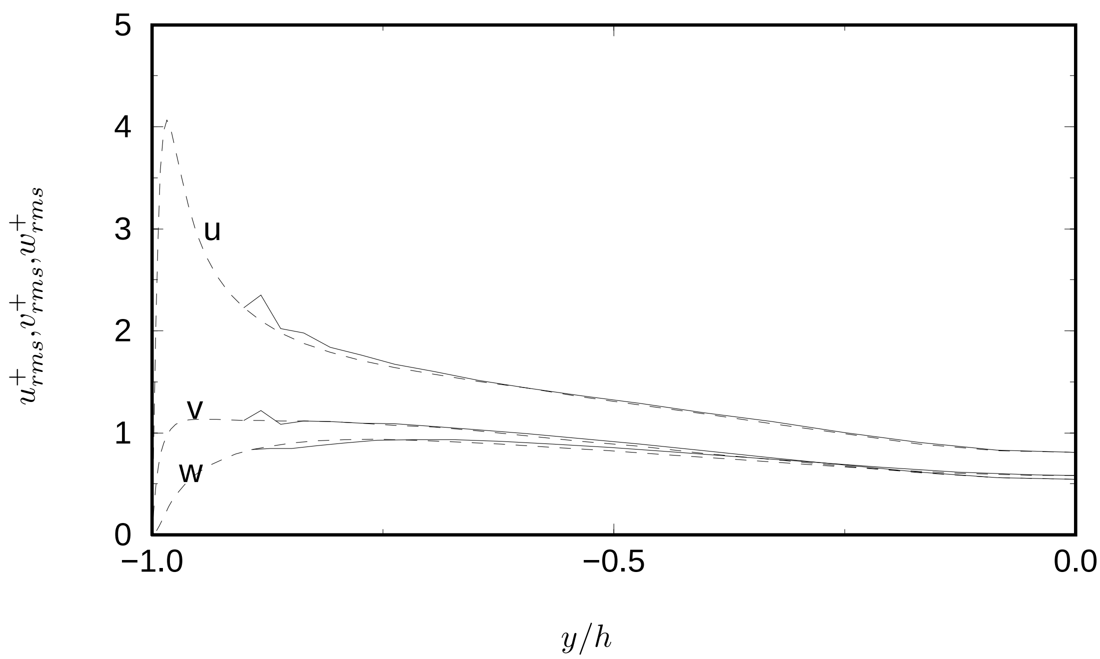

FIGURE 6. Turbulence intensities for the asymmetric computation. γ's computed dynamically. ——— : lower half, artificial boundary conditions; - - - - : upper half, classical boundary conditions.

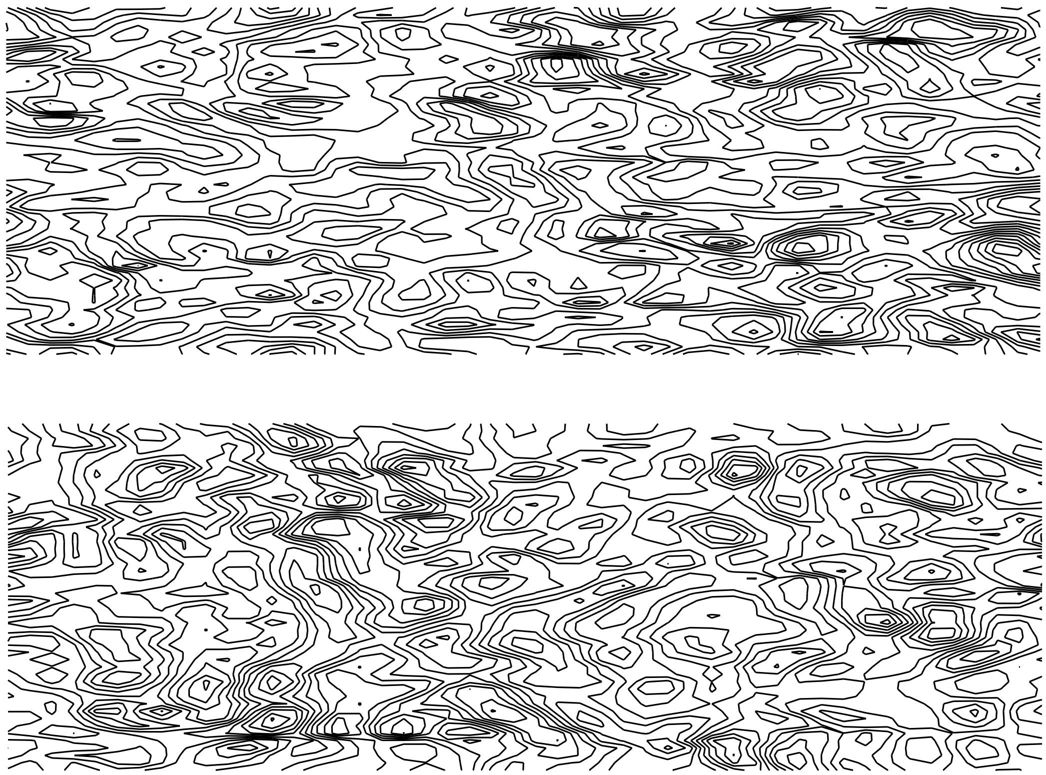

FIGURE 7. Iso-lines of v-velocity in the plane $y = 0.8h$ (top) and $y = -0.8h$ (bottom). γ's computed dynamically. Flow is from left to right.

Moin & Moser (1996).

Proceeding nevertheless, using the dynamic values of the γ_i into the artificial boundary conditions, leads to very interesting results. As shown in Fig. 3b, the computed values for γ_i near the wall with artificial boundary conditions remain close to unity although slightly greater than near the top 'resolved' wall. We present hereafter the results for the first and second order statistics through the channel.

Remarkably, all these quantities are almost symmetric although the boundaries on the two sides of the channel are treated very differently. This shows that the artificial boundary conditions with the dynamic computation of the parameters γ_i give a good representation of the velocity field at y_1. Note that the stress $\langle \delta u \delta v \rangle$ has the correct behavior although it is not prescribed explicitly by the boundary conditions.

The turbulence intensities and the stress show some fluctuations with respect to the expected values very close to the artificial boundary. This is due, in part, to the fact that the velocity fluctuations imposed by the artificial boundary conditions require rather severe fluctuations in the pressure field in order to enforce the

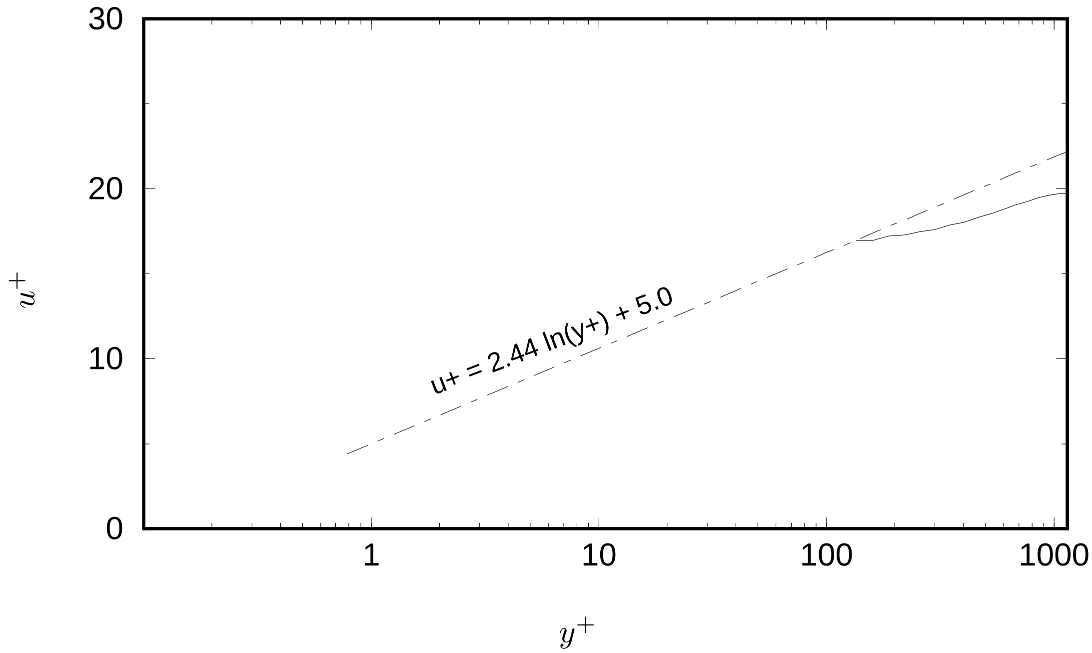

FIGURE 8. Mean velocity profile in wall units for the symmetric computation. γ's computed dynamically. Artificial boundary conditions on both sides.

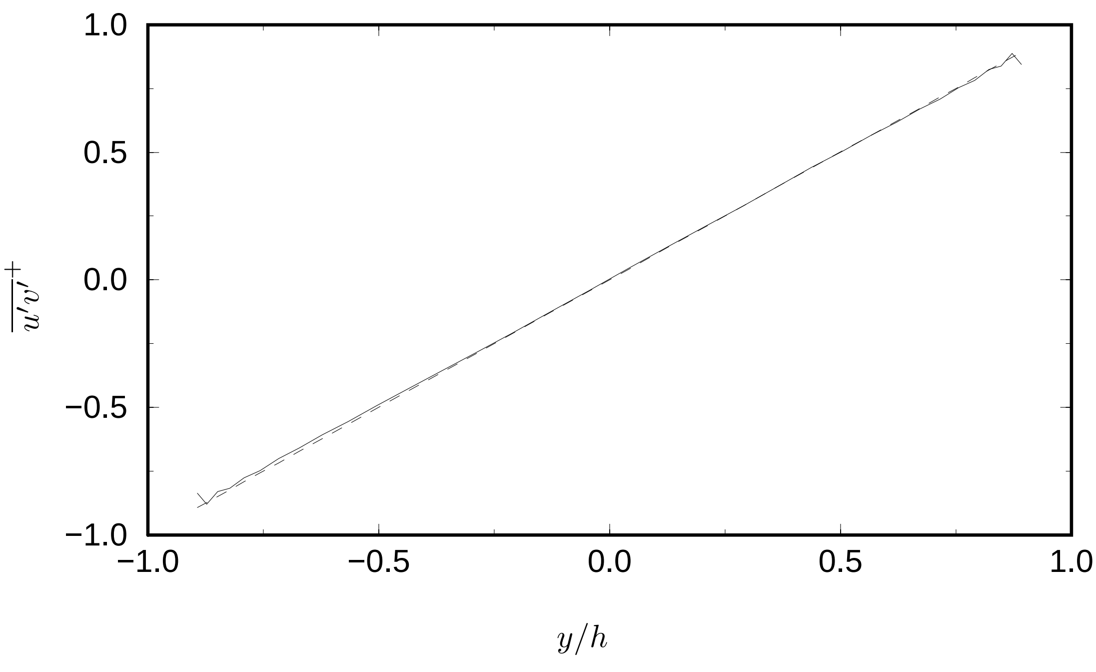

FIGURE 9. Resolved shear stress for the symmetric channel computation. γ's computed dynamically. ——— : artificial boundary conditions on both sides; - - - - : theoretical value.

incompressibility conditions. These pressure fluctuations might then propagate in the near boundary region and affect the velocity field (see Jiménez & Vasco, this volume).

The two-point correlations of the velocity field are not directly imposed by the conditions (2.3), but, of course, the velocity components produced by these boundary conditions are not random and do include some structures (see Fig. 7). However, the two-point statistics show that the turbulence structure is affected by the artificial boundary conditions (too large spanwise correlation near the artificial boundary).

These results remain, however, very encouraging. The fact that the flow remains almost perfectly symmetric even when the walls are treated differently shows at least that this approach should be investigated further. Unfortunately, the next step in the evaluation of the peculiar boundary conditions (2.3) is less conclusive. Indeed, we have tried to use the same conditions on both sides of the channel; the results for the mean profile are, of course, symmetric, but they differ strongly from the reference LES of Kravchenko. In general, it is found that the second order statistics are much better predicted than the mean velocity profile. A possible explanation could be that the energy flux through the boundary is not at all controlled by the conditions (2.3). A badly predicted energy flux could indeed affect the mean profile more than the second order statistics.

4. Conclusion

It is very difficult to draw any definitive conclusion from the the preliminary study presented here. However, we have shown that simple artificial boundary conditions can be built with many desirable properties. In particular, we have developed and partially tested a simple procedure to easily impose the correct amplitude for the first and second order statistics of the velocity field at the artificial boundary, while some information regarding the structure of the turbulent flow is fed to the boundary from the computed neighboring core flow.

This procedure has been very successful when used only on one side of the channel flow. This result is encouraging. Unfortunately, when used on both sides, the obtained mean velocity profile is substantially different from the reference profile (here, the one obtained when running the coarse LES with the classical no-slip boundary condition on both walls). Most probably, in the asymmetric computations, the upper channel with the classical no-slip boundary conditions imposes enough constraint to keep the velocity profile close enough to the reference.

The coarse LES used in this preliminary study is quite poor; the resolution is too coarse for this high $Re_\tau = 1000$ channel. A follow up of this work would certainly require repeating some of the investigations with a better resolution: either rerun the high Re_τ investigations, with and without the approximate boundary conditions, but with finer resolution (possibly requiring embedded grids close to the wall), or run lower Re_τ investigations.

Thus, it remains to be shown that good quality results can indeed be obtained with the type of artificial boundary conditions presented here when the reference numerics are better. In particular, further development could require adaptations that enforce additional constraints on the various fluxes at the artificial boundary (e.g., stress and/or energy fluxes).

REFERENCES

Baggett, J.S. 1997 Some modeling requirements for wall model in large eddy simulation. *CTR Annual Research Briefs 1997,* Center for Turbulence Research, NASA Ames/Stanford Univ.), 123-134.

Jiménez J. & Vasco C. 1998 Approximate lateral boundary conditions for turbulent simulations, in the present volume.

Kravchenko A. G., Moin, P. & Moser, R. 1996 Zonal embedded grids for numerical simulations of wall-bounded turbulent flows. *J. Comp. Phys.* **127,** 412-423.