# On the feasibility of merging LES with RANS for the near-wall region of attached turbulent flows

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## 1. Motivation and objectives

The large number of grid points required in the near-wall region of attached turbulent boundary layers is the chief obstacle to applying large eddy simulation (LES) to many flows of engineering interest. Current subgrid scale models do not accurately model the subgrid scale (SGS) Reynolds stresses (Jiménez & Moser 1998). Thus, if the LES is to include the near-wall region, the filter width has to be such that most of the Reynolds stresses are carried by resolved motions. This requires the filter width to scale as a fixed fraction of the local turbulent integral scales which are proportional to the distance from the wall. Baggett, *et al.* (1997) calculated that the number of grid points required for accurate LES of a turbulent boundary layer scales as $N \sim Re_\tau^2$.

Several approaches have been proposed to alleviate these near-wall resolution requirements, and nearly all of them fall into one of two categories. The first, and most common, approach is to replace the no-slip boundary condition with an approximate boundary condition. Usually, the wall stresses are modeled and the transpiration velocity is set to zero. This approach was introduced by Schumann, who assumed the streamwise wall stress was in phase with the streamwise velocity at the first off-wall grid point (1975). Improvements to the basic idea of Schumann, that the wall stress is a simple deterministic function of the velocity at the first wall point, have been made (Grötzbach 1987, Piomelli *et al.* 1989, Hoffmann & Benocci 1995). More recently, a two-layer approach has been employed in which the three-dimensional unsteady boundary layer equations are integrated on an embedded near-wall grid to estimate the wall stresses (Balaras *et al.* 1996, Cabot 1995, 1996, 1997). There have also been recent attempts to provide boundary conditions which specify the velocities on some plane parallel to the wall, but these have met with limited success (Baggett 1997, Jiménez & Vasco 1998, Nicoud *et al.* 1998).

In the second approach to wall modeling, which we explore in this report, the no-slip boundary condition is applied at the wall, requiring the wall-normal filter width to be refined near the wall. However, the filter width in the directions parallel to the wall is not refined so that the near-wall structures which carry the Reynolds stresses must be accounted for by the SGS model. A simple approach is to supplement the SGS model with a RANS eddy viscosity model in the vicinity of the wall. This idea was originally proposed by Schumann (1975) who used a mixing length eddy viscosity to supplement the Smagorinsky eddy viscosity in the near-wall region. Moin & Kim (1982) and Sullivan *et al.* (1994) have explored similar approaches. More recently, Spalart *et al.* (1998) have advocated the use of the one-equation Spalart-Allmaras eddy viscosity model as an SGS model. The length scale in the destruction term is modified so that the eddy viscosity crosses over from the usual Spalart-Allmaras RANS eddy viscosity near the wall to a proposed LES eddy viscosity, similar to that of Smagorinsky, away from the wall. Spalart, *et al.* call this approach "Detached-Eddy Simulation" (DES) since it is intended to be used in regions, such as separated regions, in which only eddies that are detached from the surface must be resolved for accurate LES of the flow away from the wall.

In this report, we explore the feasibility of using RANS eddy viscosity models to supplement the SGS model in the near-wall region. This exploration is motivated by Durbin's development of the $v2f$ eddy viscosity RANS model which has been shown to give good near-wall predictions in a variety of flows (Durbin 1991, 1995). Unfortunately, as we shall see below, a direct crossover to RANS in the near-wall region is unlikely to be successful in attached turbulent boundary layers. In the next section our numerical experiments in turbulent channel flow are described. In §2.1 two different techniques for adding a RANS eddy viscosity model to a conventional LES eddy viscosity model in the near-wall region are described. In §2.2 we argue that the failure of such models is due to the formation of an artificial near-wall turbulent cycle with streamwise streaks and vortices occurring at scales dictated by the grid resolution and not by near-wall physics. The artificial cycle is worth investigating because it appears to explain the overly large streamwise velocity fluctuations in the near-wall region as well as the overly large additive logarithmic law constants frequently observed in large eddy simulations of turbulent boundary layers and channel flows. Finally, in §2.3 we mention some other wall modeling approaches being explored at CTR and end with some concluding remarks in §3.

## 2. Accomplishments

Turbulent channel flow simulations are a good test bed for studying crossovers between LES and RANS since the near-wall dynamics are similar to those in many flows of engineering interest. The Navier-Stokes equations are discretized with second order finite differences in the spatial dimensions and third order Runge-Kutta/Crank Nicolson in time. The subgrid scale model is the Smagorinsky model with the coefficient determined by the plane-averaged dynamic procedure (Germano *et al.* 1991). Unless otherwise stated, all quantities are nondimensionalized by the friction velocity, $u_\tau$, and channel half-height, $h$, with the usual skin-friction Reynolds number being defined as $Re_\tau = u_\tau h/\nu$.

The computational domain used for all simulations is $[0, 2\pi] \times [-1, 1] \times [0, 2\pi/3]$ with the coordinates representing the streamwise, wall-normal, and spanwise directions, respectively (denoted by $x, y$, and $z$ below). The spanwise width of the domain is probably inadequate to correctly capture the large eddies which shape the wake region near the channel center, but this is of little consequence for the present study since we are primarily interested in the near-wall region. The grid used has 32 uniformly spaced points in each of the periodic horizontal directions and anywhere from 69 to 101 points in a hyperbolic tangent stretched mesh for the wall-normal direction. The stretching is determined so that the first wall-normal grid point is located within 2 wall units ($\nu/u_\tau$) of the wall. In all cases, the mean streamwise pressure gradient is equal to the wall stress, that is, $-\partial P/\partial x = \tau_w = 1$.

### *2.1 Blending SGS and RANS eddy viscosity models*

Two different approaches for incorporating a RANS eddy viscosity in the near-wall region are tested here, the first is designated **Model 1**:

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -\nu_t[\bar{S}_{ij} - (1 - f(y))\langle\bar{S}_{ij}\rangle] - f(y)\nu_R\langle\bar{S}_{ij}\rangle \tag{1}$$

and, **Model 2**:

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -[(1 - f(y))\nu_t + f(y)\nu_R]\bar{S}_{ij}, \tag{2}$$

where $\tau_{ij}$ is the usual SGS stress tensor, $\nu_t$ is the dynamic Smagorinsky eddy viscosity, and $\nu_R$ is the eddy viscosity furnished by an external RANS simulation, which in this case was Durbin's $v2f$ model. The $\langle\cdot\rangle$ denotes a plane average. The "blending" function $f(y)$ facilitates merging the RANS and LES descriptions of the flow. In the case of the second model, $f = 0$ corresponds to the original LES model and $f = 1$ corresponds to a fully RANS model. Generally speaking, $f$ should be a function of the resolution which might be parameterized by the ratio $\Delta/L_\epsilon$, where $\Delta$ is a measure of the filter width and $L_\epsilon$ is an estimate of the turbulent integral dissipation length. However, in the numerical experiments conducted here, $f$ was estimated *a priori* from a mean-flow momentum balance as will be discussed further below.

Model 1 is essentially the model of Schumann (1975) in which the near-wall mixing length eddy viscosity has been replaced with a more general eddy viscosity calculated by the $v2f$ model. The RANS eddy viscosity, $\nu_R$, appears only with the plane averaged resolved strain rate and affects only the mean flow directly. Model 2 is similar to the DES approach (Spalart *et al.* 1998) in which the eddy viscosity parameterizes SGS turbulence away from the wall and all turbulence near the wall. Although, in the DES approach the blending between the RANS and LES regions is accomplished by modifying the dissipation length scale in the transport equation for the eddy viscosity instead of using an explicit blending function $f$.

Both models were tested in LES of turbulent channel flow at $Re_\tau = 1000$ on a $32 \times 69 \times 32$ mesh. The blending function was estimated through a mean momentum balance using Eqs. (1) and (2) for the subgrid scale models, the resolved stress distribution from an LES simulation with no RANS correction, and the target mean velocity profile from a separate $v2f$ RANS calculation. The profiles of $f$ are shown in Fig. 1. Both models produced somewhat improved mean flow results in the sense that the additive constant in the log-law was approximately correct (instead of being over-estimated by nearly a factor of two without any RANS correction). However, other details of the flow were completely wrong. Model 1 yielded near-wall streamwise velocity fluctuations that were much too high. Model 2 did a better job of estimating the magnitude of the near-wall fluctuating velocities, but the peaks were too far from the wall, and in general the viscous and buffer regions were much too thick. For Model 2 the blending function $f$ peaked at a value of approximately 0.25 in the buffer region. Higher values of $f$ (closer to $f = 1$ corresponding to all

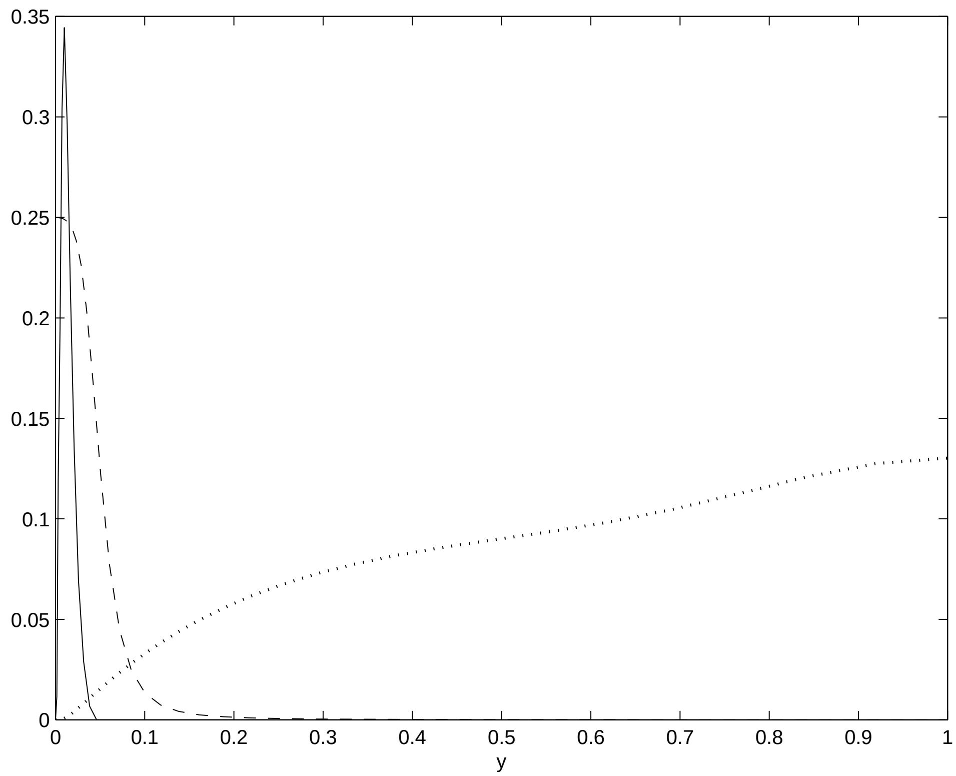


FIGURE 1. Profiles of the blending functions $f$ used for Model 1 (solid line) and Model 2 (dashed line) at $Re_\tau = 1000$. The $v2f$ eddy viscosity is shown as the dotted line.

RANS eddy viscosity) lowered the additive constant in the logarithmic region and further thickened the buffer region.

Closer inspection of the results showed that in both cases the near-wall region contained streamwise vortices and streaks whose horizontal dimensions were much too large. For instance, the streamwise streaks were observed to have a mean spanwise spacing of nearly 260 wall units instead of the physical spacing of 100 wall units. The persistence of this artificial near-wall turbulent cycle makes it unlikely that simple crossovers from LES to RANS models in the near-wall region can be used for LES of turbulent boundary layers as will be discussed in the next section.

### *2.2 Artificial near-wall turbulence*

The near-wall turbulence cannot be completely parameterized by a RANS model because the near-wall region would be effectively laminar and there would be no fluctuating velocities to provide boundary conditions to the turbulent logarithmic region of the outer LES. If the strength of the near-wall RANS contribution is reduced, then only resolved Reynolds stress or viscous stress can balance the difference between the pressure gradient and the contribution from the RANS eddy viscosity. Of course, instabilities ensure the existence of turbulent motions unless the effective Reynolds number, due to the addition of RANS eddy viscosity, is very low. However, as we shall see below, the turbulent motions supported by the simulation are

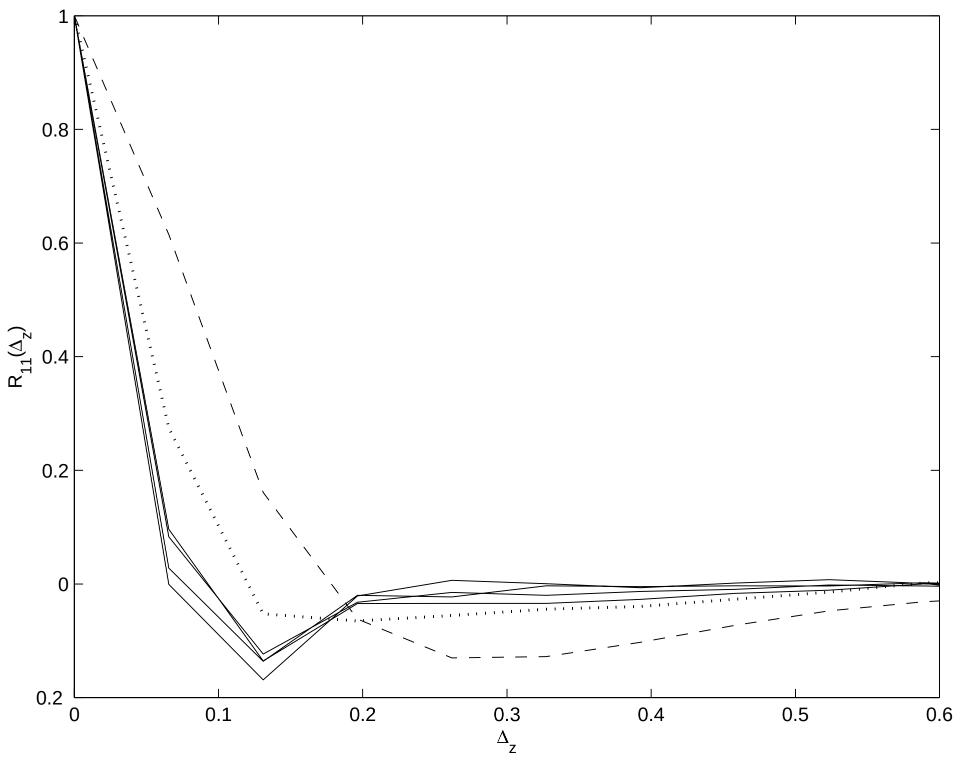


FIGURE 2. Spanwise correlation of streamwise velocity at $y^+ = 10$. There a 4 solid curves corresponding to $Re_\tau = 1000, 2000, 4000$ and $Re_\tau = 1000$ with Model 1 in §2.1. The dotted line corresponds to $Re_\tau = 1000$ with Model 2 in §2.2. Note the slightly larger streak spacing. The dashed line is $Re_\tau = 200$ and the streak spacing is $\Delta_z^+ \approx 105$.

artificial.

The no-slip boundary condition produces a near-wall viscous region in which the mean pressure gradient is balanced only by mean viscous stress so that $\partial \langle U \rangle / \partial y = Re_\tau$ at the wall. The core flow is fully turbulent outside the buffer region so that mean shear scales approximately like $1/y$. In the intermediate buffer region the mean shear has to be reduced by wall-normal streamwise momentum transport, i.e. mean Reynolds stress, to couple the core flow to the wall. In the absence of an SGS model that carries a significant amount of the Reynolds stresses, only resolved motions can contribute the necessary Reynolds stresses. In simulations with a no-slip boundary condition and coarse horizontal resolution, with or without a RANS model contribution, the near-wall region develops an artificial self-sustaining turbulent process consisting of streamwise vortices and streaks with horizontal dimensions dictated by the discretization (the effective horizontal filter width) to carry the necessary Reynolds stresses.

The existence of the overly large streamwise streaks and vortices was confirmed by flow visualizations. Further evidence is offered in Fig. 1 where the spanwise correlation of the streamwise velocity is plotted for several different simulations at

$y^+ = 10$. The location of the first minimum in the spanwise correlation is a measure of the mean distance between the centers of adjacent high and low speed streaks. For Reynolds numbers greater than at least $Re_\tau = 1000$, without the addition of a near-wall RANS model, the streak spacing is independent of the Reynolds number. Visualizations of the near-wall streamwise vorticity showed that the near-wall streamwise vortices had the smallest possible spanwise extent that could be supported on the grid; that is, they were essentially always two grid points wide. When Model 1 was used to blend in a near-wall RANS model, the near-wall turbulent cycle was essentially the same as with no RANS model, only slightly weaker since the wall-normal shear is slightly reduced by the additional mean forcing term. The dotted line in Fig. 1 shows the effect of Model 2 on the spanwise streak spacing. The net effect of Model 2 is to increase the overall viscosity in the neighborhood of the wall, thus decreasing the effective Reynolds number of the artificial near-wall turbulence, leading to larger streaks and a mean-flow profile corresponding to a lower Reynolds number simulation.

As the Reynolds number increases, the viscous stress contribution decreases and the combined SGS and resolved Reynolds stresses must peak closer to the wall. However, since the near-wall streamwise vortices and streaks are constrained to have horizontal dimensions dictated by the grid, their wall-normal to horizontal aspect ratio must increase as the whole process moves closer to the wall. The resulting artificial near-wall turbulence is less effective at providing wall-normal momentum transport as is demonstrated in Fig. 2 where the correlation coefficient of the streamwise and wall-normal velocity fluctuations is plotted for several Reynolds numbers. Since the correlation between $u'$ and $v'$ is decreasing as the Reynolds number increases, the mean shear increases to increase the viscous stress. Some of this mean shear is rotated towards the wall by the streamwise vortices, resulting in the overly large peak in $u'$ and an increased value of the mean Reynolds stress $-\langle u'v' \rangle$. The correlation curves in Fig. 2 also show that the correlation increases adjacent to the wall for all but the $Re_\tau = 200$ simulation, which serves as another indication of the incorrect physics. Moreover, the correlation curve (the dotted line in Fig. 2) for the simulation at $Re_\tau = 1000$ with Model 2 shows higher correlation indicating a lower effective Reynolds number.

Adding in a RANS eddy viscosity through an SGS model like Model 1 can only help the situation slightly. Since the RANS eddy viscosity only affects the mean flow, the near-wall region will still have the artificial near-wall turbulence, which remains effectively unaltered. While it may be possible to find a blending function $f$ which produces a reasonable mean-flow profile with Model 1 in spite of the artificial near-wall turbulence, it seems unlikely that $f$ could be found *a priori* to produce a predictive representation of the near-wall region. In the other approach to blending in a RANS eddy viscosity, as in Model 2 the effect is to lower the effective Reynolds number of the artificial near-wall turbulence resulting in *larger* streamwise vortices and streaks and mean flow profiles corresponding to lower Reynolds numbers. Again, it might be possible to choose a blending function $f$ which produces a mean flow profile with the right gross characteristics, but it seems unlikely that this

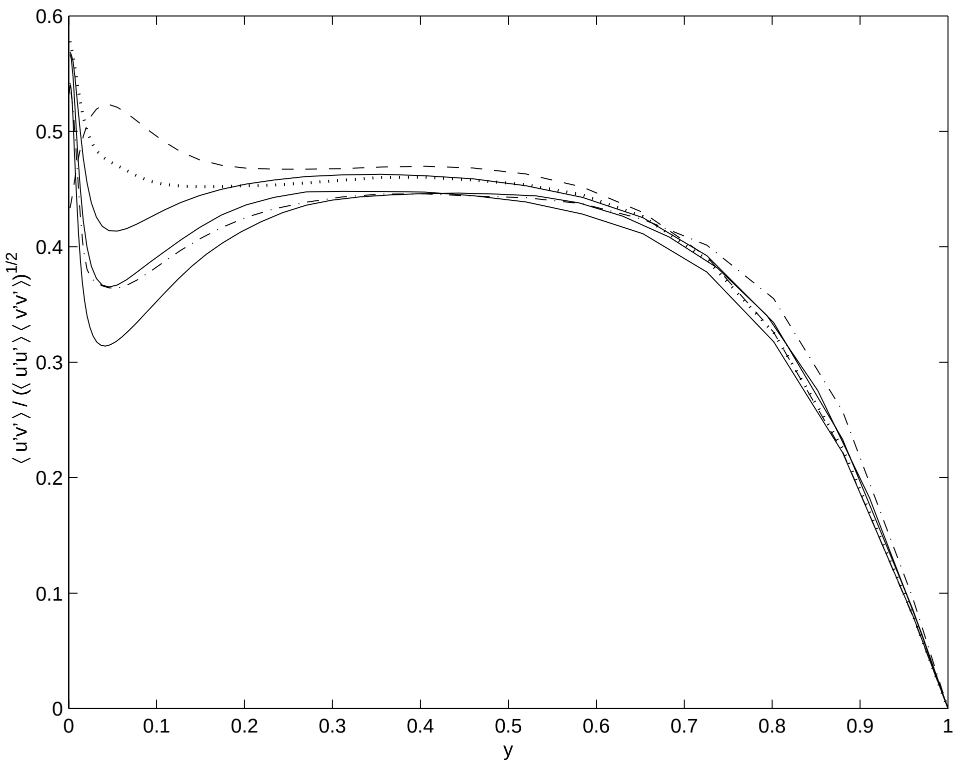


FIGURE 3. Correlation coefficient of the wall-normal and streamwise velocity fluctuations. The three solid lines, in order of decreasing correlation, correspond to LES with no near-wall RANS model at $Re_\tau = 1000, 2000, 4000$, respectively. The dashed line is $Re_\tau = 200$. The dash-dotted line and the dotted line are at $Re_\tau = 1000$ with Models 1 and 2,respectively.

can be done in an *a priori* manner. The overall trend of increasing near-wall eddy viscosity is to increase the dimensions of the near-wall turbulent cycle which should be accounted for entirely by the subgrid scale motions at high Reynolds numbers. The only other alternative is to increase the near-wall RANS eddy viscosity until the flow is effectively “laminar” there, but this will result in an artificial transition region between the laminar-like near-wall flow and the turbulent core flow.

The persistence of this artificial near-wall turbulence explains some common problems encountered in LES of attached turbulent boundary layers. The overly large peak in $u'$ and the overly high value of the additive constant in the logarithmic region are both determined by the inefficient wall-normal transport of streamwise momentum by the distorted near-wall turbulence. Interestingly, without any RANS correction near the wall, the value of additive constant in the logarithmic region increases linearly with the Reynolds number; see Fig. 4. Increasing the near-wall eddy viscosity only compounds the problem.

### *2.3 Other wall modeling work at CTR*

Unless vastly improved, fully anisotropic SGS models are found for the near-wall region, attempts to enforce the no-slip boundary condition are likely to fail

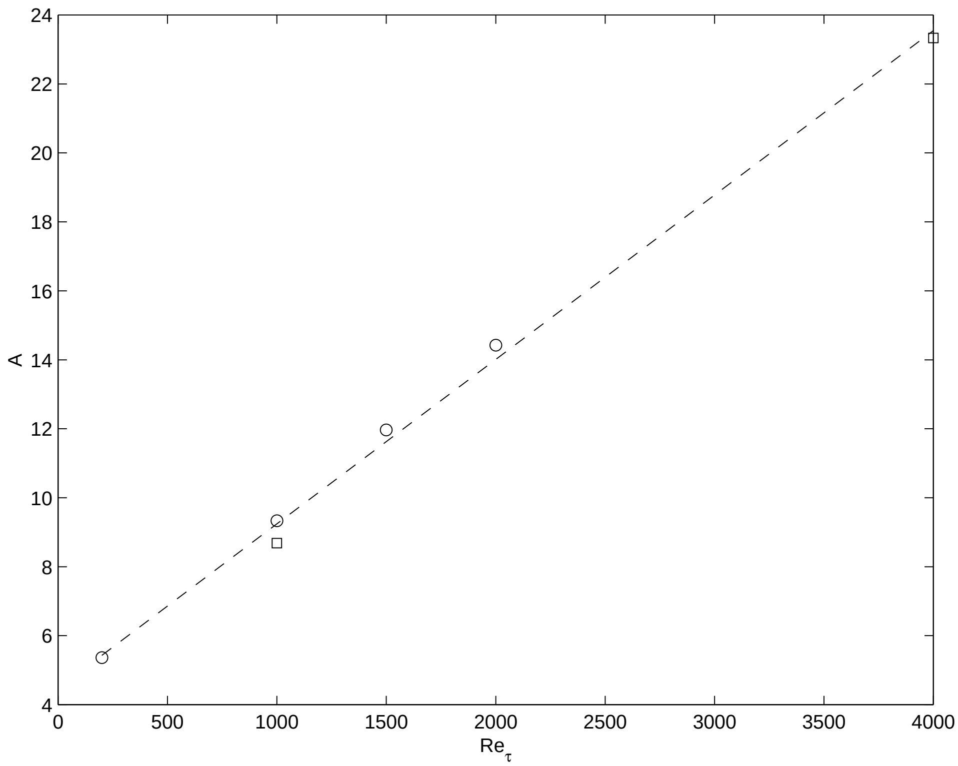


FIGURE 4. Value of the log-region intercept (determined from the mean velocity at 150 wall units) $A$ in $U = \log(y^+)/\kappa + A$ as a function of Reynolds number for LES simulations without any near-wall RANS corrections. The squares indicate simulations with 101 wall-normal points and the circles correspond to 69 wall-normal points. The dashed line is the least squares fit of the data. For $Re_\tau = 200$, we have $A \approx 5.4$.

unless the LES has resolution similar to that of a direct numerical simulation in the near-wall region. This is because the outer flow is coupled to the wall through the buffer region which develops an artificial near-wall turbulent cycle to exchange streamwise momentum with the wall. The source of the problem is the no-slip boundary condition which can only be "seen" by a properly resolved inner flow. Usually this is addressed by providing approximate wall stress boundary conditions (with zero transpiration velocity).

Most of the current wall stress models assume that the first off-wall grid point of the LES is in the logarithmic region. Thus the first grid point has to be located at some minimum distance from the wall which depends on the Reynolds number. This problem can be removed, thus allowing near-wall grid refinement without a no-slip boundary condition, by neglecting the viscosity in the outer LES equations and integrating them down to and even on the wall. The horizontal velocities are allowed to slip on the wall and the zero transpiration condition is maintained so that the turbulent length scales are set by the blocking effect of the wall. The inviscid LES equations at the wall require the surface values of the SGS stress tensor, which are provided by an auxiliary model that must incorporate the effects of the unresolved near-wall turbulence (and hence, the effects of viscosity). The inviscid approximation is only expected to be valid at high Reynolds numbers where the wall-normal extent of the viscous and buffer regions is very small compared to the LES region of interest. In separated regions, where the local Reynolds numbers are relatively low, it may be necessary to include viscous terms so that the no-slip boundary condition can be applied. The issue of changing the boundary conditions in this zonal manner still must be addressed.

We are currently developing this framework in LES of turbulent channel flow and in the separated boundary layer computations reported by Cabot elsewhere in this volume. The surface values of the SGS stress tensor are given by integrating the three-dimensional unsteady boundary layer equations, with a mixing length eddy viscosity, which are driven by the wall slip velocities as in the two-layer wall-modeling approach mentioned in the introduction. Efforts are also underway to replace the wall-damped mixing length eddy viscosity in the boundary layer equations with a model similar to Durbin's $v2f$ model.

## 3. Future plans

The results presented in this report indicate that a direct blending of RANS and LES eddy viscosities as a way of merging RANS and LES regions is unlikely to work in near-wall regions where it is important to have good boundary conditions for not only the mean flow, but also the fluctuating velocities. This is because the outer flow is coupled to the wall through a physically incorrect buffer region. However, merging LES and RANS may certainly be possible in flows where the details of the near-wall fluctuations are not important for the LES of the outer flow as is envisioned by Spalart *et al.* (1998) in their DES approach. However, there is still the issue of generating high Reynolds number turbulent inflow conditions at the entrance to the separated region for which no satisfactory solution is presently available.

It seems that the no-slip boundary condition must be abandoned for high Reynolds number LES. Instead, as mentioned in §2.3, we are currently testing the inviscid LES core flow approximation with estimates of the surface SGS stresses coming from the integration of separate three-dimensional unsteady boundary layer equations that represent the near-wall region.

Two other directions with regards to the near-wall sublayer equations are also being explored. The first direction is to replace the wall-damped mixing length eddy viscosity in the three-dimensional near-wall boundary layer equations with the more general eddy viscosity generated by a coupled $v2f$ RANS simulation (Durbin 1991). In the second direction under investigation, the three-dimensional boundary layer equations are reduced to a simplified set of one-dimensional ordinary differential equations in the wall-normal direction at each location where a wall-stress estimate is required (W. C. Reynolds, private communication, 1998).

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