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**Requirements and Design for a Special Gyro
for Measuring General Relativity Effects from an
Astronomical Satellite**

By

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1. Introduction

L. I. SCHIFF has pointed out [1] that the principal experimental evidence to support the General Theory of Relativity beyond the Equivalence Principle is the measured precession of the perihelion of the orbit of the planet Mercury. (Other measured relativistic effects, including the red shift, can be explained on the basis of the Special Theory together with the Equivalence Principle, and do not constitute a test of the General Theory.)

Professor SCHIFF has suggested [2, 3, 4] that if an extremely accurate gyroscope could be constructed, it might be used in an experiment which would constitute the first laboratory check of the General Theory. In the SCHIFF experiment the spin axis of a free gyro would be monitored over long periods of time while the gyro was being transported through the earth's gravity field, either in an earth-fixed laboratory or in a satellite. Transport of the gyro through the earth's gravitational field would produce a relativistic precession of the gyro coordinate system through the same mechanism that imparts a precession to Mercury's perihelion as the latter is transported through the sun's gravitational field. For a satellite in a reasonably low-altitude orbit, this "major" relativity effect would be about 7 arc seconds per year. It is interesting to note that the major relativistic precession is independent of the gyro spin speed: the same coordinate precession would occur with a non-spinning mass. (Spinning the rotor, of course, greatly reduces its response to non-relativistic disturbance forces.)

In addition to the above "major" effect, the satellite gyro would experience an additional, much smaller relativity effect, known as the LENSE-THIRING precession, of about 0.1 arc second per year due to rotation of the earth, which alters slightly the earth's gravitational field. The LENSE-THIRING effect is without any experimental check.

FAIRBANK [5] has described plans to carry out the SCHIFF experiment using cryogenic techniques, and several preliminary experiments are now being conducted. The design goal is to develop a very special gyro which, when operating in a special satellite environment, would have a drift rate, due to non-relativistic effects, of less than 0.05 arc second per year (about 1.7×10^{-9} deg. per hour). The gyro would be so arranged that the angle between its spin vector and the optical axis of an astronomical telescope in the satellite could be measured to an accuracy of about 0.01 arc second (after smoothing of the data). The telescope, in turn, would be used to monitor the direction to a fixed star. With such an apparatus the "major" general relativity effect could be detected to about 1 per cent accuracy, while the LENSE-THIRRING precession could be detected to about 50 per cent accuracy.

To achieve this accuracy, the proposed gyro would have to exhibit drift performance better than the current "state of the art" by a factor of the order of 10^6 . Even so, the experiment is not considered infeasible, because it may be possible to take advantage of two very special circumstances to improve the performance of the gyro by this order of magnitude. These are, first, that the forces required to support the gyro in a satellite can be made lower by at least a factor of 10^7 , so that drift due to support forces should be reduced correspondingly; and second, that in this experiment a preponderant amount of averaging may be applied to attenuate the non-relativistic effects.

2. Experimental Arrangement

The general arrangement of apparatus in a satellite vehicle is shown in Fig. 1. The principal elements are the spinning gyro and a telescope, which is fixed in the vehicle, together with a means of measuring the angle between the gyro spin axis and the telescope axis. The experiment would be operated in a vacuum. In addition, it may be desirable to operate at cryogenic temperature, which would be provided by the liquid helium flask, radiating heat outward

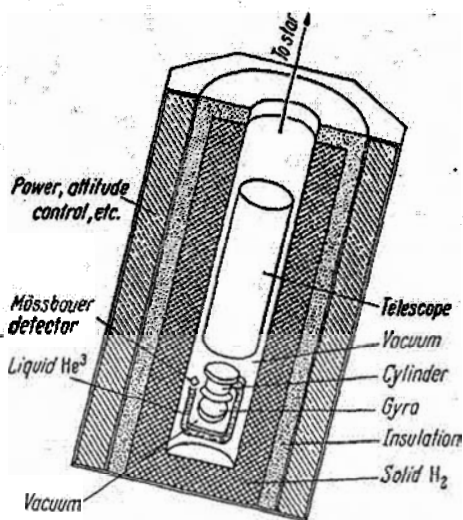


Fig. 1. Arrangement of experiment in Orbiting Astronomical Observatory

to a solid hydrogen enclosure, for example. A superconducting magnetic shield surrounds the experiment, to protect it from external electromagnetic disturbances.

The Vehicle

The Orbiting Astronomical Observatory, of which a number will be launched by the United States National Aeronautics and Space Administration in the coming five years, is in many ways an ideal vehicle in which to carry out the SCHIFF experiment. Designed specifically to house experimental astronomical equipment, the OAO provides an open tube 40 inches in diameter and 9 feet long running through its center. This space is completely at the disposal of the experimenter. The precision attitude control system can control the vehicle orientation either using its own star tracking telescopes or using a signal provided by the experimental telescope. In the latter case the system is to align the vehicle to the telescope signal to an accuracy of 0.1 arc second.

Three possible means for supporting the gyro rotor include using electric fields, making the rotor superconducting and using magnetic fields, and controlling the path of the vehicle so that it follows the orbit of the rotor without contact. Of the three means, the electrostatic is the most highly developed, and a number of experimental gyros have been built which exhibit very low drift rates. The performance of such gyros should be greatly enhanced in the low- g , high-vacuum environment of a space vehicle. The superconducting gyro is inherently much simpler and more elegant, the configuration being passively stable. The supporting magnetic field would be furnished by current in superconducting coils so that no power would be required. The development of such gyros is currently in a much earlier stage. The third technique, of "servoing" the path of the vehicle to follow that of the rotor, would seem to be the ideal way of avoiding supporting forces altogether. Some of the errors produced by the supporting forces are examined in the succeeding section.

Methods of Measurement

Two angles must be measured, one between the telescope axis and the line of sight to a reference star, and the other between the spin axis of the rotor and the telescope axis. The first measurement is a "standard" astronomical one.

To measure the direction of the spin axis of a spherical rotor, optical techniques may be used. These involve either viewing patterns or marks on the sphere through special window shapes by optical means, and converting to electrical signals which may be analyzed to determine rotor orientation, or grinding an optical flat on the expected spin axis.

A more exotic technique using the MÖSSBAUER phenomenon has been proposed by FAIRBANK and BOL [5]. This scheme is indicated in Fig. 1 and is shown in some additional detail in Fig. 2. A spinning cylinder rotates in synchronism with the sphere, as shown. A tiny amount of radioactive material is carried on the sphere and a suitable absorber material is carried as a coating on the flange of the cylinder. A non-spinning detector is located behind the flange. As the sphere and cylinder spin together, the emitter and the absorber are carried around in circular paths.

If the spin axes of the sphere and cylinder are coincident, then the circular paths are in parallel planes, and there is never any relative velocity between the emitter and the absorber. Under these conditions, as MÖSSBAUER discovered, radiation from the emitter will be absorbed by the absorber, and the detector will read null. However, if there is

a finite angle between the axes of the circular paths of the emitter and absorber, then they will be in non-parallel planes, and the emitter will have a linear velocity relative to the absorber which will be sinusoidal at the spin frequency. This relative velocity produces a DOPPLER shift in the frequency of radiation arriving at the absorber, so that the absorber fails to absorb all of the energy, and part of it is read by the detector. The absorption band is very narrow, so that extremely small relative velocities can be detected in this manner. Experiments (described below) indicate that the angle between spin axes can be read with adequate precision with this technique.

Once the angle is known between the spin axes of the rotor and cylinder, the angle between the cylinder axis and the telescope axis can be read optically, utilizing the flat surface of the cylinder.

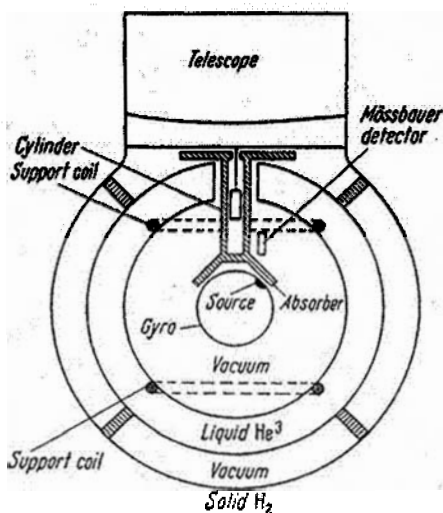


Fig. 2. Some details of a proposed configuration

Environmental Control

Whichever method of support and of readout is employed, it appears desirable to employ a cryogenic environment for a number of reasons, including those of dimensional stability, the availability of magnetic

shielding by use of a superconducting material, reduction of the noise level in sensing equipment, and alleviation of outgassing problems (all gases except helium become solid at liquid helium temperature).

It is anticipated that the apparatus for the experiment may not be started up until the vehicle orbit is established, to avoid the necessity for supporting the rotor at $1g$, and for a number of other reasons.

Rotor Configuration

It seems clear from the accuracy requirements that the rotor must be supported without any mechanical contact (with either solid or gaseous material). A spherical shape then seems indicated because of its simplicity and the precision with which such a shape can be produced. Moreover a spherical shape is more easily supported with precision by electrostatic or magnetic fields. A possible modification of the spherical shape might be to undercut the equator by an appropriate amount so that, when the rotor is spinning, the centrifugal bulging will produce a perfectly spherical shape.

While a spherical external shape seems clearly desirable, the question remains whether to make the rotor isoinertial or to give it a preferred axis. Three considerations involved in this decision are: (1) the relation between the spin axis (which can be measured) and the momentum vector (which is the quantity of interest) during polhode-type motions of the rotor, (2) limitations on readout techniques, and (3) motions produced by gravity gradient torques.

To study the polhode motions of the rotor, suppose that the motions have first been damped to the point where the spin axis is rotated by a small angle γ from the major axis of the rotor, as indicated in Fig. 3. The resulting precession of the rotor about the fixed angular momentum vector, H , is elegantly represented by the rolling-cone image of POINSON, shown in Fig. 3 (for the case of equal major moments of inertia). The "space cone" remains inertially fixed while the "body cone" rolls around it. The body cone is rigidly fixed in the rotor, as indicated, so that the total angular velocity of the body, Ω , is always at the line of tangency between the two cones. Thus the angular velocity vector precesses around the angular momentum vector with a cone angle, α , shown in Fig. 3.

The value of α is readily obtained for the general case from the usual calculation, in which p, q, r are components of angular velocity, and A, B, C are moments of inertia:

$$\cos \alpha = \frac{A p^2 + B q^2 + C r^2}{\sqrt{A^2 p^2 + B^2 q^2 + C^2 r^2} \sqrt{p^2 + q^2 + r^2}} \quad (1)$$

Consider first the case of a strongly preferred axis. Suppose, for example, that the minor axes have equal moments of inertia such that

$B = C = A/2$. Then, for the severe case that the angle γ (Fig. 3) between the major axis and the spin vector is $\gamma = 45^\circ$, it will be found from (1) that $\alpha = 18\frac{1}{2}^\circ$, so that an extremely large error is made in using the spin axis as an indication of the direction of the momentum vector. More favorably, if γ has been made initially very small (by damping for a sufficiently long time) then $p \approx (1 - (\gamma^2/2)) \Omega$ and, with $\cos \alpha \approx 1 - \alpha^2/2$, α is given by:

$$\alpha \approx \frac{\gamma}{2}. \quad (2)$$

That is, for a rotor having a preferred axis the error made in using the spin axis as an indication of the direction of the momentum vector will be of the order of the angle between the spin axis and the major axis of the rotor.

If, in the other extreme, it is attempted to make the rotor isothermal, and if this is done within an error ϵ —e.g., $B = C = (1 - \epsilon)A$ —then Eq. (1) can be converted to the following:

$$\cos \alpha = \frac{1 - \epsilon \left(\frac{q^2 + r^2}{\omega^2} \right)}{\sqrt{1 - (2\epsilon - \epsilon^2) \left(\frac{q^2 + r^2}{\omega^2} \right)}}$$

or, with $\cos \alpha \approx 1 - \frac{\alpha^2}{2}$, and $\frac{(q^2 + r^2)}{\omega^2} = \sin^2 \gamma$,

$$\alpha = \epsilon \frac{\sin 2\gamma}{2}. \quad (3)$$

Considering again the severe case that the initial spin direction is 45°

from the major axis of inertia, the angle between the spin vector and momentum vector is found to be

$$\alpha = \frac{\epsilon}{2} \quad (4)$$

while, more favorably, if initial damping has produced near-alignment of the spin vector along the major axis of inertia to within a small angle γ , then the angle between the angular velocity and momentum vectors is given by

$$\alpha \approx \gamma \epsilon. \quad (5)$$

Comparison between expression (2) and (5) shows that the angle between the angular velocity and momentum vectors can be made

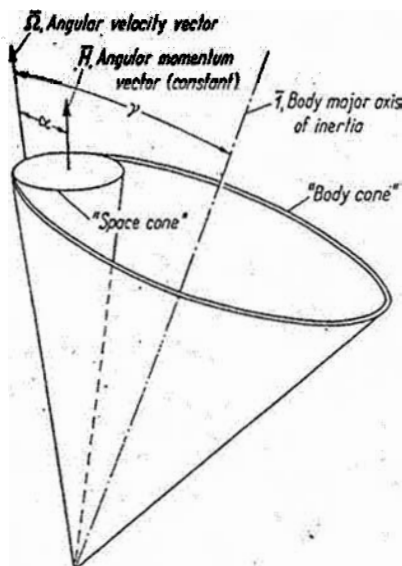


Fig. 3. Poinsot representation of polhode motion of rotor

better by the factor $\epsilon/2$ by making the moments of inertia equal within error ϵ . This represents one important advantage of using an isoinertial rotor.

Gravity gradient torque produces substantial drift of the rotor when the moments of inertia are very different, and this represents an even more compelling reason for using an isoinertial rotor. The effect of gravity gradient torque will be calculated in the next section.

Note that with an isoinertial rotor the readout method must identify the spin axis directly, rather than any axis fixed in the body. Either optical methods (using a random pattern) or the MÖSSBAUER technique may be employed for this purpose.

3. Some Estimates of System Errors

Supporting Fields

There are a number of mechanisms by which the supporting field can produce precession of the momentum vector. The most common is a rotor mass unbalance, in which the resultant of the support forces does not pass through the mass center of the rotor. The support force and gravity force then form a couple which can produce precession. For a spherical rotor shape, another mechanism involves non-symmetry of the field with respect to the rotor such that the upper and lower support forces individually have different lines of action, so that a couple is produced which, again, causes precession.

Such supporting-field anomalies control the drift performance of current gyros. But, in every case where a disturbing torque is produced by the supporting field, the satellite environment should contribute an improvement factor of at least 10^7 simply because the supporting field required in a satellite at, say, 500 miles altitude will be lower by this factor. The most elegant method of carrying out the SCHIFF experiment would be to avoid support forces altogether simply by servoing the path of the vehicle so that it never touches the rotor, which would be in its own free-fall orbit. Problems associated with creating the free-fall environment for the rotor are discussed in detail in [6].

Trapped Flux

A number of other mechanisms of rotor torquing will be important, including magnetic moment due to interaction between flux trapped in the rotor and the ambient magnetic field.

The recent discovery of FAIRBANK and DEEVER [7] that trapped flux in a superconductor is quantized, leads to the possibility that initial cooling of the rotor can be performed in a sufficiently small ambient

field that the trapped flux may be made precisely zero. Typically, for a sphere 1 cm in diameter the field would have to be made less than about 10^{-7} gauss.

Gravity Gradient

The gradient of the earth's gravity field produces a torque on a non-isoinertial body at a distance R from the earth's center given by the relation (c.f. [9], Eq. (40))

$$M = \frac{3g_e R_e^2}{R^3} \mathbf{1}_R \times \bar{\mathbf{I}} \cdot \mathbf{1}_R, \quad (6)$$

in which g_e is free-fall acceleration at the earth's surface, R_e is earth radius, $\mathbf{1}_R$ is the unit vector along R , and $\bar{\mathbf{I}}$ is the moment-of-inertia diadic. Thus, for a sphere having spin-axis symmetry ($B = C < A$), the magnitude of the torque will be

$$M = 3 \frac{g_e}{R_e} \left(\frac{R_e}{R} \right)^3 (A - B) \frac{\sin 2\theta}{2}, \quad (7)$$

where θ is the angle between the sphere major axis and radius vector R .

The sphere spin axis will be essentially fixed in inertial space. Suppose, for example, that it is normal to the earth's spin axis, so that, as the sphere is transported around in orbit, Fig. 4, two orthogonal components of the gravity-gradient torque acting on the sphere will be given by the following modification of (7):

$$\left. \begin{aligned} M_x &= -3 \frac{g_e}{R_e} \left(\frac{R_e}{R} \right)^3 (A - B) \sin i \cos i \sin^2 \beta, \\ M_y &= 3 \frac{g_e}{R_e} \left(\frac{R_e}{R} \right)^3 (A - B) \cos i \sin \beta \cos \beta, \end{aligned} \right\} \quad (8)$$

in which i is the inclination of the orbit with respect to the earth's equator and β is location along the orbital path. (A coin-shaped body is used in Fig. 4 as a model of the non-isoinertial, symmetrical rotor, for emphasis.)

Component M_y is much less important than M_x because its average value is zero. M_x has (because of the $\sin^2 \beta$ term) an *average value*, which will, in turn, produce a steady component of gyro precession given by $\dot{\phi} = M/h$, in which $h = A \Omega$:

$$\dot{\phi}_{av} = \frac{3}{2} \frac{g_e/R_e}{\Omega} \left(\frac{R_e}{R} \right)^3 \left(1 - \frac{B}{A} \right) \sin i \cos i. \quad (9)$$

In expression (9), $g_e/R_e = (1.24 \times 10^{-3})^2$. (This corresponds to the familiar 84.4 minute period of a SCHULER pendulum.) For an orbit at an altitude of 500 miles, $\left(\frac{R_e}{R} \right)^3 \approx 0.7$. $\dot{\phi}$ will be largest for $i = 45^\circ$, which is approximately the orbit inclination most likely to be used in

the experiment. Using these values and a nominal spin speed of $\Omega = 10^3$, Eq. (9) becomes

$$\dot{\phi}_{av} = \frac{3}{2} \times 10^{-9} \left(1 - \frac{B}{A}\right). \quad (10)$$

Since the experiment requires that the gyro drift rate be less than 10^{-14} rad/sec., it is seen that the moments of inertia must be matched to one part in 10^5 ! That is, a preferred axis cannot be used.

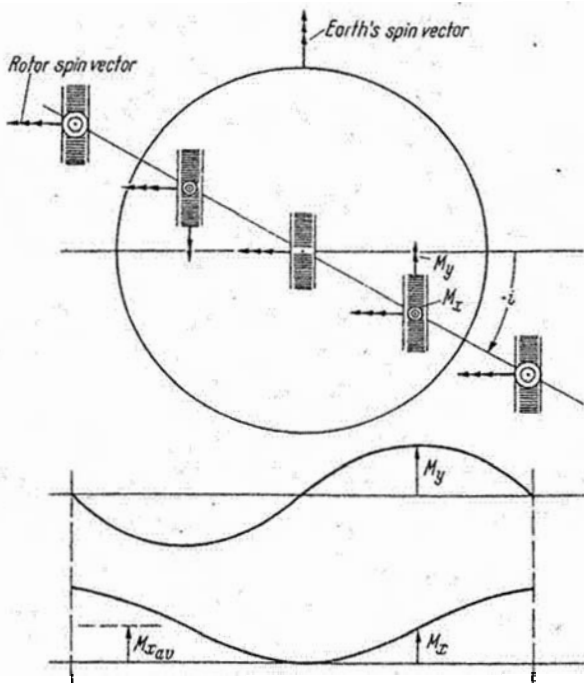


Fig. 4. Gravity gradient torque on an inertially oriented body has an average value, in general.

4. Preliminary Experiments

A number of preliminary experiments are indicated prior to the full scale SCHIFF satellite experiment described above. Some of these will now be described.

The use of the MÖSSBAUER phenomenon to check the angle between the spin axes of two rotors has been tested at very low speed in an experiment by M. BOL [5]. The rotating members in this case were disks attached to the spindles of a jeweler's lathe, rotating in synchronism at 6 rpm. An emitter was attached to one disk, and an absorbing material to the other. By varying one spindle angle slowly back and forth through null and noting the variation in the amount of radiation detected behind the absorber, BOL was able to establish the null angle to better

than one minute of arc. If this data can be extrapolated to ultimate rotor speeds of 10^3 or 10^4 radians per second, then detection by this means should be possible to accuracies of the order of .04 to .004 arcsecond.

Experimental work is underway to study the magnetic support and control of a spinning, superconducting cylinder, for possible use with the MÖSSBAUER technique. Following this, the problems of mating the cylinder with a spherical rotor will be studied.

Development of the electrostatically supported gyro [8] was begun in 1952 by NORDSIECK, its inventor, and has proceeded quite far in Prof. NORDSIECK's laboratory and at several others, as is well known. The development of cryogenic gyros is not so far along, and several fundamental problems must be solved before this method will be successful, including those of trapped flux and of the non-ideal characteristics of superconducting materials. These problems are currently receiving strong attention in several laboratories.

Independent of, and prior to the SCHIFF experiment, it is planned to operate a satellite vehicle having a free-fall-trajectory orbit by servoing the vehicle trajectory to follow that of an internal proof mass. This project is described in Ref. [6] and would have a number of scientific uses in addition to the SCHIFF experiment. LANGE [6] has shown that the acceleration of such a vehicle could be maintained to within about $10^{-12}g$, (earth's gravity). The satellite could thus be used also to provide a direct, instantaneous measurement of atmospheric drag, to study the higher harmonics of the earth's gravitational field, and to perform certain point-mass relativity experiments.

Finally, it may be possible to perform a special version of the SCHIFF experiment in an earthbound laboratory at the equator, by doing a very large amount of averaging of gyro drift produced by non-relativistic disturbances. In this special case the gyro relativity effect would be about a single axis and could be measured by noting precisely the time at which the gyro spin axis passed through the zenith each day. Such an earthbound experiment would provide valuable experience with a complete experimental system, prior to performing measurements from a satellite. Moreover, should sufficient accuracy be obtained, it might be possible to measure the largest of the earth-spin-vector motions described by Prof. GRAMMEL in his introductory remarks to this Symposium.

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Discussion

E. T. BENEDIKT: The above paper deals with an experimental demonstration of a general relativistic effect which, according to a nomenclature introduced by T. LEVI-CIVITA can be referred to as "geodetic precession". This effect obtains because the relations defining transfer parallelism (in the sense of LEVI-CIVITA) are not exact differentials.

This result can be verified (as suggested in the above paper) by experimentation with a system of two planetoids¹ occupying initially identical (in practice, nearly identical) positions in space, and which orbit under the (exclusive) action of a gravitational field in such a manner as to occupy coincident positions at some successive instants (in an actual situation, one of the planets could be the Earth, whereas the planetoid could be an artificial space vehicle). If two initially parallel directions are gyroscopically maintained in the planetoids, it ought to be found that parallelism will generally no longer subsist upon reunion of the planetoids.

In addition to the above effect, another effect can (in principle) be expected due to the action of the (non-random) interstellar magnetic field, whose existence can be incontrovertibly inferred from observations of trajectories of cosmic rays and interplanetary particles. According to a theory advanced in 1918 by H. WEYL, the existence of an electromagnetic field produces a "torsion" of (four-dimensional) space time². Analytically, this implies that the quadratic expression of the square of the (infinitesimal) distance between two neighboring points of space time is no longer homogeneous, but of the form

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta + \gamma_\lambda dx^\lambda \quad (\alpha, \beta, \lambda = 0, 1, 2, 3).$$

In the above expression x^α are the generalized coordinates of an event in space time, i.e., $x^0 = ict$, where c is the speed of light, t the time and x^k ($k = 1, 2, 3$) are ordinary curvilinear coordinates; $g_{\alpha\beta}$ are the gravitational potentials, and γ_λ are quantities proportional to the components of the four-dimensional electromagnetic vector potential; or more explicitly γ_0 and the γ_k 's are respectively proportional to the electrostatic potential and the components of the magnetic

¹ A planetoid is a celestial body of mass sufficiently small so a snot to sensibly modify the surrounding gravitational field.

² This theory has been subsequently rejected by WEYL himself; however it is being recently revived in various contributions by L. MORZ.

vector potential. An (in principle) experimentally verifiable consequence of this theory consists in what, by analogy with the terminology employed above, could be denoted the "geodetic expansion (or contraction)", that is, the non-integrability of length. With reference to the actual situation considered previously, two standards of length carried by the two planetoids and initially identical (by measurement) would generally no longer be found identical upon subsequent reunion.

In the particular situation in which only a (static) magnetic field is present, the change in length (in either planetoid) would result proportional to the flux of the above field across a surface bounded by its orbit (assumed to be closed). The numerical value of this change in length cannot be predicted on the basis of WEYL's theory, inasmuch as the coefficient of proportionality between the metric coefficients γ_{ij} and the components of the electromagnetic vector potential cannot be determined within framework of the above theory¹. At any rate, inasmuch as noticeable effects arising from the orbital motion of the Earth have not been observed, this coefficient of proportionality, and hence the geodetic contraction must be extremely small. Its empirical verification—if at all humanly feasible—would obviously involve the use of large magnetic fluxes, i.e. large magnetic fields and/or particle orbits of large radius.

As shown by the numerical table below, the magnetic fluxes across the orbits of respectively a terrestrial satellite and an interplanetary space vehicle are far greater than the flux obtaining within a typical terrestrial instrument such as a cyclotron. This result can be illustrated more concretely by considering that an ion, moving with a speed comparable to that of light in a cyclotron of linear

Experimental situation	Radius of orbit	Area of orbit	Magnetic Field	Magnetic flux
Terrestrial laboratory (cyclotron)	1.5×10^2 cm	$.7 \times 10^5$ cm ²	10^3 Gauss	$.7 \times 10^8$ Gauss cm ²
Terrestrial satellite	6.3×10^8	1.3×10^{18}	.5	$.7 \times 10^{13}$
Interplanetary space vehicles	1.5×10^{13}	$.7 \times 10^{27}$	10^{-5}	$.7 \times 10^{22}$

dimensions of the order of 3 in., has to orbit for 1 to 3 months in order to encircle a flux comparable to that of the interplanetary field across a typical interplanetary orbit. Therefore, if there is a chance at all to detect a geodetic elongation effect, experimentation employing interplanetary vehicles appears to be the most appropriate. It is of course quite unlikely that a change in length obtained by such means can be demonstrated by direct measurement. It should therefore be quite interesting to speculate on possible indirect effects which a geodetic expansion would produce in high precision mechanical instruments, such as gyroscopes.

In conclusion, it is of interest to note that astronomical engineering may play a role in the field of basic scientific experimentation, besides its more popularly known applications to applied science and long range warfare.

R. H. CANNON, JR.: Dr. BENEDICT has suggested one of a number of additional relativistic effects which might be checked by means of an accurate orbiting gyroscope. The author is grateful for his comments and the additional detail contained in his discussion.

¹ An attempt to estimate the above coefficient is presently being made by the author of this note.