A Spiking Neuron

Like biological neurons, the model neuron we characterize in this lab has a repertoire of (model) ion-channel populations. Here we focus on the population of sodium channels that fire action potentials, or spikes. These channels produce an all-or-none upswing in the membrane voltage through positive feedback (regenerative action) activated when a voltage threshold is exceeded.

We observe the model neuron's response to constant input current, studying the dependence of:

- The membrane voltage's time-course on the input-current level
- The neuron's spike-rate on the input-current level

2.1 Reading

The following book chapter, located on the class website, explains how to analyze model neurons similar to the one used in this lab:

• E. M. Izhikevich. Dynamical systems in neuroscience: The geometry of excitability and bursting. MIT Press, 2007, Chapter 3, pp. 53-82.

2.2 Prelab

This prelab focuses on analysis directly applicable to the experiments. Try to develop intuition for how the neuron behaves.

- 1. An Integrate-and-fire Neuron
 - (a) Consider a simple neuron with membrane potential V, described by:

$$\tau_{\rm m} \dot{V} + V = \frac{I_{\rm IN}}{g_{\rm lk}} \tag{2.1}$$

where $I_{\rm IN}$ is the input current and $\tau_{\rm m} = C_{\rm m}/g_{\rm lk}$ is the membrane time-constant (Figure 2.1). This neuron is declared to spike when $V = V_{\rm th}$. To obtain a simpler description of how V reaches $V_{\rm th}$, we use dimensionless variables. Rewrite the equation, substituting $x = \frac{V}{V_{\rm th}}$ and $x_{\infty} = \frac{I_{\rm IN}}{V_{\rm th}g_{\rm lk}}$. The minimum input that results in a spike occurs when $x_{\infty} = x_{\rm th}$. Determine $x_{\rm th}$ and sketch \dot{x} versus x for $x_{\infty} = x_{\rm th}$.



Figure 2.1: This integrate-and-fire neuron model takes an input current, $I_{IN}(t)$, and produces an output voltage, V(t). It models the neuron's membrane with a capacitor C_m and its leak with a conductance g_{lk} .

- (b) Sketch x(t) when x_{∞} is slightly larger than x_{th} .
- (c) We can solve for the interspike interval by summing the times (dt) each increment (dx) takes, up to the spike threshold (x = 1):

$$T = \int_0^T dt = \int_0^1 \frac{dx}{\dot{x}} = \tau_{\rm m} \int_0^1 \frac{dx}{x_{\infty} - x}$$
(2.2)

Solve this integral and find the frequency f (inverse of T).

- 2. A Neuron with Positive Feedback
 - (a) Real neurons rely on positive feedback, which takes time to develop, influencing the spike rate. The model that we will study is described by:

$$\tau_{\rm m} \dot{V} = -V + \frac{I_{\rm IN}}{g_{\rm lk}} + \frac{1}{3} \frac{V^3}{V_{\rm th}^2}$$
(2.3)

where the rightmost term represents the influence of the sodium current. Rewrite this equation in dimensionless form, substituting $x = \frac{V}{V_{th}}$ and $r = \frac{I_{IN}}{V_{th}g_{th}}$.

- (b) Although this equation is more complex than the one describing the integrateand-fire neuron, we can use the same techniques we used before. The minimum input that results in a spike occurs when $r = r_{th}$ —the value of r required to shift the phase-plot high enough so that its minimum is on the x-axis. Determine r_{th} and sketch \dot{x} versus x for $r = r_{th}$. (Hint: At x's minimum, $\ddot{x} = 0$.)
- (c) Sketch x(t) when r is slightly greater than r_{th} . There should be an inflection point where $\ddot{x} = 0$; find x at this point.

2.3 Setup

The setup is nearly identical to the last lab. There will be a folder on the Desktop named **Neuron Lab**. This folder contains the instrument control program to acquire and view the neuron membrane waveform and spikes in real-time. The TA will instruct you on the use of the software.

The main difference is the contents of *parameters.txt*. In this lab, the parameters of interest are:

- Input current $(I_{\rm IN})$
- Leak conductance (g_{lk})
- Neuron x address
- Neuron y address
- Time

As described in the Synapse Lab, the input current and leak conductance bias voltages determine on-chip currents exponentially. The leak conductance increases exponentially as you increase its bias. Conversely, he input current decreases exponentially as you increase its bias. Notice that there is no threshold (V_{th}) bias; it has been fixed at a reasonable value. The neuron x and y addresses select which neuron receives input current and is observed; each can range between 0 and 31. As with the previous lab, setting time to zero will run the program continuously.

2.4 Experiments

In the first experiment, we will record the silicon neuron's membrane waveform in response to subthreshold current inputs. In the second experiment, we will measure the neuron's spike rate in response to suprathreshold currents. In both experiments, the PC will step the current from zero to the level set by the input current bias. It injects a constant current by stimulating the neuron's excitatory synapse at a high rate (see Lab 1). The synapse excites the neuron constantly when the interspike interval is shorter that its cleft's pulse-width.

Experiment 1: Membrane Waveform

In this experiment, we will

- Determine the threshold voltage.
- Calculate the membrane time-constant.

We will calculate the neuron's threshold by finding the inflection point in its membrane voltage, which occurs at the threshold voltage (V_{th}) . We will also find the waveform's initial slope, which will give an estimate of the membrane time-constant.

Open parameters.txt. Choose a neuron, selecting x and y addresses with any criteria you want as long as both are between 0 and 31. Both biases in the parameter can be adjusted as the program runs. Select a leak conductance that yields clear exponentially decreasing growth (near 0.1V). Then refrain from changing the leak conductance bias so that $\tau_{\rm m}$ remains constant.

To find the neuron's inflection point, stimulate it with an input current that is just high enough to cause a spike. Then, fit the membrane waveform near the inflection point (between about 20% and 80% of the interspike interval) with a cubic polynomial. Solve for the fit's value at its inflection point. This value is $V_{\rm th}$; use it to normalize the measured membrane potential, converting it into dimensionless units (this corresponds to x in the prelab).

To calculate the membrane time-constant, vary the input current (10-20 values), starting from a small value and increasing until the membrane potential approaches threshold. Save the membrane waveform at each bias voltage. Remember to rename the output file each time you change the bias, so that it is not overwritten. Normalize the membrane waveforms by V_{th}. The normalized waveform's initial slope is equal to $x_{\infty}/\tau_{\rm m}$ and its steady-state value is equal to x_{∞} . Calculate $x_{\infty}/\tau_{\rm m}$ for each waveform and plot it versus x_{∞} . Fit this plot in its linear region to obtain $\tau_{\rm m}$.

Experiment 2: Frequency-Current Behavior

In this experiment, we will

• Determine the neuron's spike rate for various constant input currents.

Measure the spike frequency for several values of input current (10-20), whose normalized value we approximate as:

$$r = r_{\rm th} \, e^{(V_{\rm IN0} - V_{\rm IN}) \, \kappa / 0.0256} \tag{2.4}$$

where $V_{\rm IN}$ is the input current bias (in volts), $V_{\rm IN0}$ is the largest bias value (smallest input current) that causes the neuron to spike and κ is a fit parameter determined for the operating point. Generally, a value of 0.7 works well to estimate κ . How we estimate this current from chip data is beyond the scope of this lab; however, if you are interested, feel free to ask the TA for details.

Plot the f versus r curve using log-log axes. Fit the plot with the following equation:

$$f = \frac{\sqrt[6]{3} r^{2/3}}{\tau_{\rm m} r_{\rm th} \,\pi} \tag{2.5}$$

valid for $r \gg 1$. Be sure to check the quality of the fit. If it is poor, κ may need to be adjusted; keep this value between 0.6 and 2. How does the fitted value of $\tau_{\rm m}$ compare to the one found in Experiment 1?

2.5 Postlab

A fit of the positive-feedback neuron model to the membrane waveform of a layer 5 pyramidal neuron in rat neocortex yields r = 1.09 and $\tau_{\rm m} = 4.3$ ms (Figure 2.2). Based on these parameter values, what value does the model predict for the minimum input current required to elicit a spike? The input current was 40pA (Hint: This value corresponds to r = 1.09).



Figure 2.2: A neocortical layer 5 pyramidal neuron's phase-plot is well fit by the equation derived in Prelab Question 2a (solid line). The membrane potential was normalized by the value at its inflection point, determined by fitting a cubic polynomial (*inset*). Data taken from http://vesicle.nsi.edu/users/izhikevich/publications/dsn/index.htm.