Lab 6

February 11, 2010

Attention: Controlling Synchrony

In Lab 5, we studied many interconnected inhibitory neurons and determined that synaptic rise-time sets the network period. In this lab we study the onset of synchrony, exploring the effect of increasing inhibitory strength.

We will drive a population of 256 interneurons with constant input current and have them inhibit each other; the strength of inhibition determines whether or not synchrony occurs. We will model the neurons in the network as phase-coupled oscillators (a method introduced by Kuramoto). When coupling is weak, the oscillators run at their natural frequencies. When coupling is strong, as would happen when attention is present, the network synchronizes.

6.1 Reading

- B. Daniels. Synchronization of Globally Coupled Nonlinear Oscillators: the Rich Behavior of the Kuramoto Model. *Ohio Wesleyan Physics Dept., Essay*, pp. 7-20, 2005.

6.2 Prelab

1. Phase-Coupled Oscillators

   Modeling each neuron as an oscillator, we can describe the rate at which the kth oscillator's phase changes by:

   $$\dot{\theta}_k = \omega_k + \frac{K}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \sin(\theta_n - \theta_k) \tag{6.1}$$

   where ω_k is its natural frequency, K is the degree of coupling, and N is the number of oscillators.

   (a) Show that this equation can be rewritten as:

   $$\dot{\theta}_k = \omega_k + Kr \sin(\psi - \theta_k) \tag{6.2}$$

   Where ψ and r are defined by:

   $$r\, e^{i\psi} = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} e^{i\theta_n} \tag{6.3}$$

   The identity $2i \sin x = e^{ix} - e^{-ix}$ may be helpful.

   (b) Sketch the relationship between ω_k and θ_k in steady state ($\dot{\theta}_k = 0$) on the range $0 < \theta_k < 2\pi$.

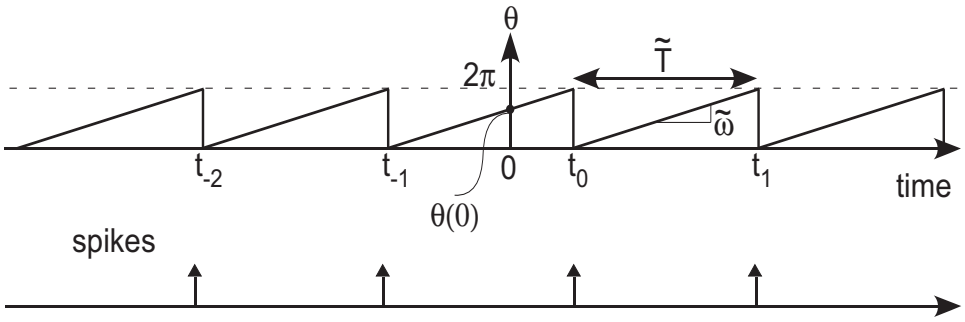

Figure 6.1: The phase of a spiking neuron over time.

2. Dealing with Spiking Neurons

   Vector strength (VS) of a population of neurons is given by

$$\mathrm{VS} = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} e^{i2\pi \frac{t_{n0}}{\tilde{\mathrm{T}}}} \tag{6.4}$$

   where t_{n0} is the spike time of the n^{th} neuron in the interval $[0, \tilde{\mathrm{T}}]$ (refer to Figure 6.1).

   (a) By relating t_{n0} to $\theta_n(0)$ (defined in Figure 6.1), show that

$$\mathrm{VS} = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} e^{-i\theta_n(0)} \tag{6.5}$$

   (b) From the expression derived above, how is VS related to r?

3. Predicting K_c

   The Kuramoto model predicts that $K_\mathrm{c} = \frac{2}{\pi g(0)}$, where $g(0)$ is the peak of the frequency density function $g(\omega)$. We could obtain $g(0)$ from the peak of a histogram of the measured neuron frequencies. But this is problamatic: if the bins are too small, the count per bin will be too low; on the other hand, larger bins will smoothen the peak. A better strategy is to avoid binning altogether and use the cumulative distribution defined as:

$$G(\omega) = \frac{1}{N}\mathbf{count}(\{\omega_1, \omega_2, \ldots, \omega_N\}, \omega) \tag{6.6}$$

   where **count**(*list*,*val*) returns the number of elements in *list* less than or equal to *val* and N is the total number of firing neurons.

   Assuming $g(\omega)$ is gaussian, sketch $G(\omega)$ for very large N. Show graphically how you would determine $g(0)$ from this plot.

6.3 Setup

As in previous labs, there will be a folder on the Desktop; this one is named **Attention Lab**. This folder contains the instrument control program to acquire and view the interneuron membrane potential and spikes in real-time. The TA will instruct you on the use of the software.

Before each test edit the contents of *parameters.txt*. In this lab, the parameters of interest are:

- Input current (I_{IN})
- Leak conductance (G_{lk})
- Inhibitory rise-time (T_{r})
- Inhibitory spread (λ_{I})
- Inhibitory conductance amplitude (G_{I})

As you increase the input current, leak conductance, rise-time, and spread biases, I_{IN}, G_{lk}, T_{r}, and λ_{I} increase exponentially. As the inhibitory conductance amplitude bias is increased, G_{I} decreases exponentially. Other biases can be changed dynamically while running the program (press the $F1$ key for help). These can be used to further explore synchrony, but they are not required in the lab.

6.4 Experiments

In the first experiment, we will explore the amount of inhibition necessary to synchronize a population of inhibitory interneurons. In the second experiment, we will examine the phase of the individual neurons within the population. Specifically, we will study how the natural frequency of each neuron affects its synchronized phase.

Experiment 1: Synchrony Onset

In this experiment, we will

- Observe the amount of inhibition required for the network to synchronize

Disconnect the interneurons from each other by setting the inhibitory spread to 0.750V. Leave the other biases at default levels ($G_{\mathrm{lk}} = 0.0$; $T_{\mathrm{r}} = 2.286$, $G_{\mathrm{I}} = 1.97$). Adjust I_{IN} to get a mean network frequency of about 40 Hz. Note this level of I_{IN} and use it for subsequent experiments.

Globally connect the interneurons by setting the inhibitory spread bias to 1.750V. Vary the inhibitory synapse's strength (20-30 values). Be sure that synchrony is not seen at the highest voltage, but is seen at the smallest. The total range should be about 300 mV. For each strength, take data for about one second.

As was done in Lab 5, compute the vector strength (VS) for the entire network at each G_{I} value. In addition, measure the total number of active neurons, N_{f}, and the average firing rate, μ_{f}, at each G_{I} value. We approximate the coupling between the neurons as:

$$K_{\mathrm{approx}} = \alpha_{\mathrm{c}}\, N_{\mathrm{f}}\, \mu_{\mathrm{f}}\, (5.0 \times 10^{-9})\, e^{0.7\,(2.5 - V_{\mathrm{B}})/0.0256} \tag{6.7}$$

where V_{B} is the G_{I} voltage bias value and α_{c} is a proportionality constant. Set $\alpha_{\mathrm{c}} = 1$ for this experiment. Plot the calculated VS vs the K_{approx} value. Fit the following equation to your data

$$r = \sqrt{1 - \frac{K_{\mathrm{c}}}{K}} \tag{6.8}$$

What value of K_{c} did you find?

Experiment 2: Frequency–Phase Relationship

In this experiment, we will

- Establish how the phases of individual neurons relate to their natural frequencies

Using your data from Experiment 1, pick a G_{I} value with a high VS and a relatively large number of active neurons (those with a frequency greater then 4 Hz). For this value, find the phase of each neuron and plot its sine against that neuron's natural frequency (measure this frequency at a G_{I} value just before the estimated K_{c} point to closely match the average frequency of the neurons when they are synchronous). Only plot neurons that are not drifting (use the VS of each neuron to determine whether or not it can be included). From a linear fit determine the value of K.

Using the measured K value and Equation 6.7 find a corrected value of α_{c} such that $K_{\mathrm{approx}} = K$. Report the value of α_{c} found.

We would like to relate the value of K_{c} to an intrinsic network property. Determine $g(0)$ from the neurons' natural frequencies using the method you proposed in the prelab. Use this to calculate K_{c}. How close is this calculated value to the number derived from the previous experiment? Can you think of any reasons for the deviation?

Experiment 3: Full K Measurement (Extra Credit)

Using the same method described above, calculate K values for all G_{I} values. Plot this vs the K_{approx} values and fit a linear equation to the data. Is the data linear as assumed? How close was the calculated α_{c} value to the slope of the fit? On a new figure, plot VS against K, and fit this using Equation 6.8. How do the new plot and K_{c} value compare to the previous results?