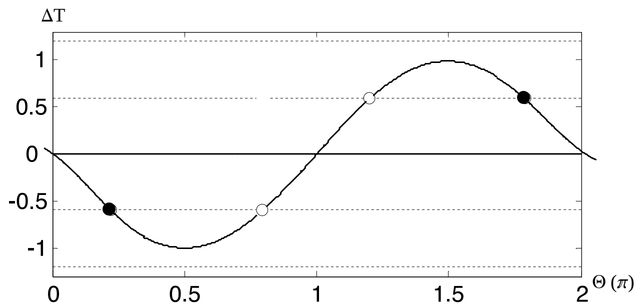
## Relating coupling strength ($K$) to the PRC




The Kuramoto model's sinusoidal phase-coupling corresponds to a PRC that is a flipped sinusoid. To obtain the Kuramoto model's coupling strength, $K$, we must multiply the PRC's maximum advance/delay, $\Delta T_{max}$, by the network's total spike rate.

Consider only the $j^{th}$ oscillator's effect on the $i^{th}$ oscillator:

$$\dot{\theta}_i = \ldots + \frac{K}{N}\sin[\theta_j - \theta_i] + \ldots$$

If the $i^{th}$ oscillator's phase is $\Theta$ when the $j^{th}$ oscillator's phase is 0 — which, by definition, is when it spikes — then we have $\theta_i = \Theta + \theta_j$, or $\theta_j - \theta_i = -\Theta$. This assumes that the phase-difference remains constant throughout that cycle. In which case, the total change in the $i^{th}$ oscillator's phase over the complete cycle — which, by definition, is the PRC — will be:

$$2\pi\frac{\text{PRC}[\Theta]}{T} = \int_0^T \dot{\theta}_i \, dt = \int_0^T \frac{K}{N}\sin[-\Theta]\, dt = -T\frac{K}{N}\sin[\Theta]$$

where the PRC is in seconds while the phase is in radians, thus the $2\pi/T$ conversion factor. Hence, the Kuramoto model's "PRC" is a flipped sinusoid.

The PRC's maximum delay/advance, $\Delta T_{max}$, is related to the coupling strength, $K$, by

$$2\pi\frac{\Delta T_{max}}{T} = T\frac{K}{N} \Leftrightarrow K = N\frac{2\pi}{T}\frac{\Delta T_{max}}{T}$$

In the presence of inhibition ($g$), the epression we obtained for $\Delta T_{max}$ in Lab 4 (Phase Response) can be rewritten as:

$$\Delta T_{max} = \frac{\tau_m A_I}{r - r_{th}} \text{ where } r_{th} = \frac{2}{3}(1+g)^{3/2}$$

And the expression we gave for the neuron's spike rate in Lab 2 (A Spiking Neuron) can be rewritten as:

$$\frac{1}{T} = \frac{3^{7/6}}{2\pi\tau_m}(r - r_{th})^{2/3} \Leftrightarrow \frac{1}{r - r_{th}} = \left(\frac{3^{7/6}\,T}{2\pi\tau_m}\right)^{3/2}$$

Hence we can express $\Delta T_{max}$ as:

$$\Delta T_{max} = \tau_m A_I\left(\frac{3^{7/6}\,T}{2\pi\tau_m}\right)^{3/2} = \left(\frac{T}{2\pi}\right)^{3/2}\frac{3^{7/4}}{\sqrt{\tau_m}}A_I$$

Substituting this expression into our expression for $K$ above yields:

$$K = 2\pi N \frac{1}{T^2} \left(\frac{T}{2\pi}\right)^{3/2} \frac{3^{7/4}}{\sqrt{\tau_m}} A_I = \frac{3^{7/4} N}{\sqrt{2\pi \tau_m T}} A_I$$

Hence, we must multiply inhibition's synaptic strength ($A_I \propto \Delta G$) by the square-root of the average firing rate — and the number of active neurons ($N$) — to convert it into the Kuramoto model's coupling strength (K).

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