Preliminaries: Notation

Notation: Let's set some standard notation.
We let $\mathbb{Z}$ be the set of integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
We let $\mathbb{Q}$ be the set of rational numbers: $\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$.
We let $\mathbb{R}$ be the set of real numbers.

Notation:
The symbol $\in$ means "is an element of."
The symbol $\notin$ means "is not an element of."
In the context of a set, the symbol $\mid$ means "such that."

Example 1: Let $A = \{2, 3, \text{Drake}\}$.
Notice that $2 \in A$ and $\text{Drake} \in A$, but $4 \notin A$.

Example 2:
(a) $\left\{x \in \mathbb{Z} \mid x^2 = 25\right\} = \{5, -5\}$
(b) $\left\{x \in \mathbb{Z} \mid x^2 = -1\right\} = \emptyset$

Example 3: Intervals
Open intervals: $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$
Closed intervals: $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$
Half-Open: $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$
Half-Open: $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

Note: We use parentheses for $-\infty$ or $+\infty$.
i.e.: We write $(-\infty, 5]$, but never $[-\infty, 5]$.

Notation:
The symbol $\implies$ means "implies." ($A \implies B$ means "If A, then B.")
The symbol $\iff$ means "if and only if."

Example: Consider:

$$x^2 = 9 = x = \pm 3.$$

This statement is false! However, the following statement is true:

$$x^2 = 9 \implies x = \pm 3.$$

Preliminaries: Functions

A *function* is an input-output rule, with the requirement that: For every input, there exists exactly one output.

◦ Domain of a function: The set of inputs.

◦ Range of a function: The set of resulting outputs.

N.B.: Some functions are described by formulas, but not all.

Again: Not all functions are formulas.

Example: Consider the "age" function: i.e., the function which inputs a person and outputs their age in years. For example:

$$\text{Age(Drake)} = 30 \qquad\qquad \text{Age(Yeezy)} = 40.$$

Exercise 1: Find the domains of the following functions:

$$f(x) = \sqrt[4]{2x+6}, \qquad g(x) = \frac{\sin x}{x^2 - x}, \qquad h(x) = \log_2(x-5).$$

Exercise 2: Sketch the graph of the following function:

$$f(x) = \frac{x^3}{x}$$

Exercise 3: Sketch the graph of the following function:

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Z} \\ 0 & \text{if } x \notin \mathbb{Z}. \end{cases}$$

Such a function is called a *piecewise function.*

Example: The absolute value function is a piecewise function:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Transformations of Functions

1. Shifts
Vertical Shifts: $y = f(x) + k$ (shift k units up)
Horizontal Shifts: $y = f(x - k)$ (shift k units right)

2. Dilations
Vertical Dilation: $y = cf(x)$ (dilate in y-direction by factor of c)
Horizontal Dilation: $y = f(\frac{x}{c})$ (dilate in x-direction by factor of c)

If $0 < c < 1$, then the dilation is a "shrink."
If $c > 1$, then the dilation is a "stretch."

3. Reflections
Reflect in x-axis: $y = -f(x)$
Reflect in y-axis: $y = f(-x)$.