The Leverage Ratchet Effect

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Abstract

Firms' inability to commit to future funding choices has profound consequences for capital structure dynamics. With debt in place, shareholders pervasively resist leverage reductions no matter how much such reductions may enhance firm value. Shareholders would instead choose to increase leverage even if debt levels are already high and new debt must be junior to existing debt. These asymmetric forces in leverage adjustments, which we call the *leverage ratchet effect*, cause equilibrium leverage outcomes to be history-dependent. When forced to reduce leverage, shareholders are biased toward selling assets relative to potentially more efficient alternatives such as pure recapitalizations.

1. Introduction

In this paper we show that the inability of firms to commit to future funding choices has profound and previously unexplored consequences for understanding capital structure outcomes and dynamics. Once debt is in place, shareholders will resist any form of leverage reduction no matter how much the leverage reduction may increase total firm value. At the same time, shareholders would generally choose to increase leverage even if any new debt must be junior to existing debt. The resistance to leverage reductions, coupled with the desire to increase leverage creates asymmetric forces in leverage adjustments that we call *the leverage ratchet effect*.

We first study shareholders' attitudes toward one-time changes in leverage accomplished by buying back or issuing debt of various seniorities. We show that the leverage ratchet effect is

always present, even in the presence of other frictions which make the current level of debt excessive. We proceed to examine, in a model with standard tradeoffs, the equilibrium dynamics that result when creditors anticipate and take into account the leverage ratchet effect. Finally, we explore how the leverage ratchet effect plays out when shareholders can change leverage by buying and selling assets as well as buying and selling securities. This analysis shows that the leverage ratchet effect has important implications for the interaction of investment and funding decisions.

Because capital structure decisions are made in environments with uncertainty, the asymmetries associated with the leverage ratchet effect can have significant consequences for the dynamics of leverage and firm value. To see this, consider a firm with some debt in place. A negative shock to the value of that firm's assets increases its leverage, but because of the leverage ratchet effect shareholders will not voluntarily reduce leverage to its original level. Of course, a positive shock to the value the firm's assets reduces leverage. In this case, by contrast, shareholders will generally have incentives to increase leverage, possibly even beyond its original level. Exogenous changes in tax rates, bankruptcy costs, and other parameters that affect funding will also tend to induce asymmetric responses when shareholders have control over leverage decisions.[1] Active leverage reductions will not generally take place even if they would increase firm value, while leverage increases are to be expected. Understanding how the asymmetries implied by the ratchet effect play out requires a dynamic model in which creditors understand shareholder incentives and anticipate their actions. Analyzing these equilibrium dynamics is one of the key contributions of this paper.

The simplest manifestation of the leverage ratchet effect, as noted in Black and Scholes (1973), arises in the frictionless setting of Modigliani and Miller (1958). To see the effect in this simple setting, consider a firm with one class of debt with total face value equal to D_H and assume the debt is risky, i.e., there is a positive probability of default. Will shareholders be willing to use cash or issue equity to buy back some of this debt and reduce the total amount owed to D_L? If shareholders issue new shares to buy back debt, the remaining debt becomes less risky and more valuable, because with less debt the likelihood of default generally decreases and the amount each creditor recovers in the event of default generally increases. Denoting by q_H and q_L respectively the per unit market values of debt in the high and low debt capital structures respectively, we have $q_L > q_H$. With perfect markets the total value of the firm, V, will not change with the change in capital structure. Thus, denoting by E_H and E_L the values of equity in the high and low debt capital structures, we have

$$V = E_H + q_H D_H = E_L + q_L D_L$$

In buying back debt, the shareholders must pay $q_L(D_H - D_L)$, which means they end up with

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[1] Evidence of such an asymmetric response in the context of tax rate changes is provided in recent work by Heider and Ljungqvist (2015).

$$E_L - q_L\left(D_H - D_L\right) = \left(E_L + q_L D_L\right) - q_L D_H = V - q_L D_H$$
$$= E_H - (q_L - q_H)D_H$$

Hence, shareholders are strictly worse off when $q_L > q_H$. Intuitively, by reducing leverage, shareholders transfer wealth to existing creditors.[2] Conversely, if shareholders can raise new debt of equal seniority in order to fund a payout for themselves, wealth is transferred in the other direction and shareholders benefit at the expense of existing creditors.

Whereas this simple version of the leverage ratchet effect is well known from Black and Scholes (1973), it may appear to depend on there being (i) no benefit to firm value from a leverage reduction, and (ii) a single class of debt (which is diluted by a leverage increase). We show instead that the effect does not depend on these assumptions but is in fact quite pervasive. In particular, we show that even when a leverage reduction would alleviate frictions such as bankruptcy or agency costs, shareholders resist leverage reductions *no matter how large the potential gain to the total value of the firm*. Intuitively, the benefits from lower bankruptcy or agency costs accrue to creditors, and shareholders are unable to capture these benefits because they must pay the higher post-recapitalization price for the debt they buy. Moreover, we find that the leverage ratchet effect and other agency problems of debt mutually reinforce each other. Specifically, higher leverage intensifies shareholders' desire to choose excessively risky investments, and at the same time the shareholders' ability to shift risk strengthens their resistance to leverage reductions.

With respect to shareholders' incentives to increase leverage, we show that unless the tax benefit of debt has been fully exhausted, shareholders can gain from a one-time debt issuance *even when new debt must be junior* (and thus there is no mechanical dilution of existing creditors). These incentives arise even when, due to other frictions, the new debt would destroy firm value, as would happen, for example, when deadweight bankruptcy costs deplete the assets significantly. Intuitively, by increasing debt shareholders impose an externality on existing senior creditors, who are harmed by the higher likelihood of incurring bankruptcy costs as well as intensified shareholder-creditor conflicts. Thus, the inability to commit makes leverage choices based on static tradeoff theory unstable.

We proceed to develop a framework for studying the equilibrium implications of the leverage ratchet effect, namely shareholders' asymmetric attitudes to leverage adjustments. To do so, we present a dynamic model in which shareholders cannot commit to future funding choices and creditors anticipate that funding decisions will be made according to shareholders' preferences. We do this first for a model without shocks, one in which the value of the real assets of the firm and other parameters do not change. We then introduce shocks and examine how these affect the dynamics of leverage choice.

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[2] We assume throughout our analysis that creditors are small and dispersed so that conflicts of interest cannot be dealt with by collective bargaining, in which case the price at which debt is repurchased is at least its marginal value if retained. Note, however, that in this example equity holders would still lose even if debt could be repurchased at the original price q_H. In that case, the loss to shareholders is $(q_L - q_H)D_L$, the gain to just the remaining creditors from the reduced dilution of their claim in default.

Throughout, we assume the existence of standard frictions such as taxes, bankruptcy costs and agency costs. In the dynamic model, the initial cost of debt reflects creditors' anticipation of the leverage ratchet effect. Therefore, leverage may start at a level lower than would have been predicted by a standard static tradeoff theory. However, we also show that, following shocks, it tends to ratchet upwards and continues to do so even when leverage is already above the level that would maximize total firm value.

Whereas countervailing forces from covenants, asset growth or short debt maturities may reduce the effects of the leverage ratchet, these forces are unlikely to eliminate the effect completely. Restrictive covenants and significant reliance on short-term debt can be costly. In fact, we show that the leverage ratchet effect remains important even with short-term debt. While leverage falls when debt matures, shareholders respond by increasing leverage more aggressively prior to maturity.

Our analysis suggests that the static tradeoff theory of capital structure is unlikely to explain the capital structure of firms. The leverage ratchet effect implies that leverage begets more leverage. Because past leverage decisions distort future leverage choices, capital structure becomes history-dependent.

In our initial analysis of the leverage ratchet effect and equilibrium leverage dynamics, we only consider leverage changes brought about by buying and selling equity and debt securities in markets. In reality leverage can also be adjusted in ways that involve the sale or purchase of assets. Suppose, for example, that the firm must reduce its leverage because of covenants or regulations, but shareholders can choose to accomplish the leverage reduction by (i) selling assets and using the proceeds to reduce debt levels; (ii) issuing new equity to buy back existing debt; or (iii) issuing new equity to buy additional assets.

We first provide an important equivalence result, showing that shareholders are indifferent among all the possible ways to adjust leverage if assets are homogeneous, there is one class of debt outstanding, and sales or purchases of assets do not, by themselves, generate or destroy value for shareholders. If any of the key conditions of the equivalence result do not hold, however, shareholders will have clear preferences concerning the mode of leverage adjustment. In particular, if the firm has multiple classes of debt, or if assets are heterogeneous, or if the firm has superior information about the quality of the assets it holds, then shareholders will want the firm to reduce leverage by buying back junior debt using the proceeds from selling assets that are relatively safe or deemed to be fairly valued (or overvalued) in the market. Asset sales shift some of the cost of the deleveraging to the remaining senior debtholders, whose claims will be backed by fewer, and possibly riskier, assets. These distributive effects may dominate even if asset sales, possibly occurring at fire-sale prices, are inefficient and reduce the value of the firm.

Our results apply in particular to the failure of firms in distress to voluntarily recapitalize. It is also relevant to banking where leverage is high and creditor monitoring is especially weak due to deposit insurance and other guarantees. Our results have important implications for the design of capital regulation, including whether regulators should specify ratios or direct leverage adjustments.

The paper is organized as follows. Section 2 presents the basic model and considers a "one time" change to the firm's capital structure via a pure recapitalization. We show the forces that lead to the leverage ratchet effect and consider how it is mutually reinforcing with investment-related agency conflicts. Section 3 develops a dynamic equilibrium model of leverage, and demonstrates that firms will limit leverage initially but "ratchet up" indefinitely in response to shocks. In Section 4 we consider alternative ways for a firm to adjust leverage other than pure recapitalization. Section 5 provides concluding remarks and discusses some of the empirical predictions of our model.

Related Literature

As mentioned, the observation that in the absence of frictions shareholders lose by giving up their default option was first made by Black and Scholes (1973). Leland (1994) identifies a similar resistance (which he terms "surprising") in the context of a specific model in which debt is homogeneous and presumed fixed, bankruptcy costs are proportional, and only marginal changes of *pari passu* debt are permitted. Closely related is the result in the sovereign debt literature, e.g., Frenkel et al. (1989) and Bulow and Rogoff (1990), that creditors gain when debt is repurchased by a borrower in the open market. Similarly, it is well-known that shareholders can gain by issuing new debt of equal priority which dilutes existing creditors, though, as noted by Fama and Miller (1972), this effect is easily preventable by strict "me-first" priority rules which require new debt to be junior to existing debt. Our initial results extend these observations by establishing the generality of shareholder resistance to leverage reductions, and desire for leverage increases, even when debt is strictly prioritized and reducing leverage would increase firm value.

Our paper is related to parts of the literature that explore the effects of frictions on investment and capital structure choices. Myers (1977) shows that overhanging debt can lead to underinvestment when new investments are funded by equity or junior debt. In Myers (1977) underinvestment only occurs when the net present value of the new project is insufficiently large for the shareholders' share of the benefits to exceed the wealth transfer to existing creditors. We show that the capital structure distortions from debt overhang can be more severe: shareholders' resistance to leverage reductions is pervasive and persists *no matter how much the leverage reduction would increase the total value of the firm*. We also find that other agency conflicts between shareholders and debt holders intensify shareholder resistance to leverage reduction even though the alleviation of these frictions through leverage reduction enhances total firm value.

Gertner and Scharfstein (1991) explore to what extent financially distressed firms can use workouts that include exchange offers to public debt holders, as well as restructurings of bank debt and infusions of new cash, in order to make use of new investment opportunities. In addition to investment distortions from debt overhang and gambling for resurrection, they note the resistance of shareholders to issuing equity to buy back debt, which Proposition 1 below generalizes. They also discuss the role of debt priority and covenants, and their interaction with investment decisions. This discussion is related to our analysis in Section 4 of leverage adjustments through combinations of transactions in assets and various debt and equity securities.

The resistance of shareholders and managers to issuing new shares to reduce leverage is often explained by reference to asymmetric information along the lines of Myers and Majluf (1984).[3] The Myers-Majluf argument, however, does not explain resistance to reducing leverage through asset sales, earnings retentions or rights offerings, none of which requires new equity issuance or is affected by market undervaluation of the firm's shares.[4] The uniform resistance we see to all forms of leverage reduction by distressed firms and in banking is better explained by the leverage ratchet effects we analyze in this paper. Indeed, Welch (2011) shows that the correlation between equity issuance and capital structure changes is insignificant or even perverse (with firms issuing equity while increasing leverage).

In the literature on dynamic capital structure, it is common to explore shareholders' decisions with respect to payouts and default without allowing changes in capital structure prior to default. Models that allow adjustments often assume that it is prohibitively costly to reduce leverage in distress, or that debt can only be recalled at par or at a premium. In addition, these models generally assume that new debt cannot be issued unless all existing debt is retired first.[5] The assumption that firms are required to repurchase all existing debt before making any changes in their debt level is clearly inconsistent with actual practice. Moreover, this assumption effectively rules out any leverage ratchet effect a priori. Our analysis assumes instead that new debt can be issued and existing debt can be repurchased any time at competitive market prices.[6] To keep our analysis focused on the basic effects of the leverage ratchet, we do not assume, in contrast to much of the literature on dynamic capital structure, that there are exogenously-given transactions costs incurred in issuing equity or in making changes in capital structure.

Dangl and Zechner (2016) analyze the choice between long- and short-term debt in a model with shocks to asset value, observing that with long-term debt shareholders do not have incentives to reduce leverage when the firm has poor performance. In their model, when debt is short term the firm effectively commits to reducing leverage and starting afresh each time the debt matures. An important assumption in Dangl and Zechner (2016) is that covenants prohibit the issuance of any new debt that would increase the total face value of debt outstanding, which also reduces the

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[3] See for example Bolton and Freixas (2006) and Kashyap et al. (2011).

[4] Indeed, Myers and Majluf (1984) emphasize that, with the information asymmetries they consider, raising funds by retaining earnings should be preferred to new borrowing. Further, when leverage reductions are imposed by regulation, adverse selection becomes irrelevant. The expected "dilution costs" (losses due to undervaluation) for the shareholders when the firm has above-average return prospects should be offset by the expected gains when the firm's prospects turn out to be especially poor. In other words, removing discretion also mitigates any negative signal associated with recapitalizations (see Admati et al (2013, Section 6) and Kashyap et al (2011, p. 10)). Of course, managers may still protest increased equity requirements in an attempt to show that their firm is undervalued in the market, whether true or not.

[5] See, e.g., Fischer et al (1989), Leland (1998), Goldstein et al (2001), Strebulaev (2007), and Titman and Tsyplakov (2007). Hennessey and Whited (2005) allow only one-period riskless debt.

[6] Some papers make the assumption that new debt can be issued pari passu with existing debt, which can help overcome the underinvestment problem identified in Myers (1977). E.g. in Hackbarth and Mauer (2011), shareholders can commit to a funding mix that includes prioritization as a way to address investment distortions arising from conflicts of interest. In the spirit of Myers (1977), we assume for most of our analysis that violating the seniority of existing creditors is not possible.

scope for the leverage ratchet effect. We illustrate in Section 3.4 that in a dynamic model with shocks and no commitment, the ratchet effect remains important even if debt maturity is short.

Brunnermeier and Oehmke (2013) show that shareholder incentives may lead to a "maturity rat race," since shortening the maturity structure of a firm's liabilities dilutes the longer-term creditors. The key assumption in Brunnermeier and Oehmke (2013) is that, although the firm can commit to a total amount of debt, it cannot commit to a particular maturity structure of that debt. They observe that this inability to commit is especially applicable to financial institutions. Our paper is similarly based on borrower-creditor conflict of interest and on the difficulty of making binding commitments. Unlike Brunnermeier and Oehmke (2013), however, our focus is on the firm's leverage choices rather than on the maturity structure of a fixed amount of debt.

Bizer and DeMarzo (1992) demonstrate that in the presence of agency costs of debt, lack of commitment leads borrowers to choose excessive leverage. They consider a risk-averse borrower or sovereign, rather than a corporation, but the desire to increase leverage once existing leverage is in place mirrors our finding regarding shareholders' desire to increase leverage over time. We also use solution methods developed in Bizer and DeMarzo (1994) for our analysis of the dynamic equilibrium. DeMarzo and He (2016) extend our results by developing a methodology to solve for a time-consistent no-commitment dynamic leverage policy in a Leland-style diffusion model of the firm's cash flows. They solve in closed form the pricing and rate of issuance of the firm's debt, which interacts with the rate of asset growth and debt maturity to generate a mean-reverting leverage process. The level of debt evolves as smooth average of the firm's historical profits, and as a result of the leverage ratchet, the tax benefits of debt are effectively dissipated through increased default risk.

Our analysis here indicates that high leverage resulting from a leverage ratchet effect can become *privately* costly when viewed from the combined perspectives of all creditors and shareholders. Because of the collateral harm of financial fragility in banking, the results of this paper therefore strengthen our conclusion in Admati et al (2013) that effective regulation to reduce leverage can be highly beneficial in an industry like banking.

2. Debt Overhang and Leverage Distortions

In this section, we develop a simple but general reduced-form model to analyze the leverage choices of a firm with outstanding debt already in place. We assume the same frictions that are present in the standard tradeoff theory model and consider the possibility of a one-time leverage adjustment in which equity is swapped for debt or vice versa. Subsequently, in Section 3, we embed this static model in a dynamic setting and examine the leverage ratchet's dynamic consequences, while in Section 4 we consider alternative transactions to adjust leverage involving purchases or sales of assets.

2.1. A Model with Standard Frictions

Consider a firm that has made an investment in risky assets and has funded itself partly with debt. We begin with a simple tradeoff model of capital structure based on taxes and net default costs, which we generalize later as we consider additional frictions. For our basic argument, we make the following assumptions:

Firm Investment: The firm has existing real investments normalized to have initial value 1. Investments have final value $\tilde{x}$, realized in the future ("date 2").

Firm Liabilities: The firm is funded by equity, together with a debt claim with total face value D that is due at date 2 when the asset returns are realized. If $\tilde{x} \geq D$, debt claims will be honored in full. If $\tilde{x} < D$, the firm will be forced to default.

We begin by considering three standard frictions that may affect the payouts of the firm's securities at date 2. These are taxes, bankruptcy costs, and third party (government) subsidies.

Taxes: We assume the firm may incur a corporate tax liability if asset returns exceed required debt payments. The tax payments are given by $t(\tilde{x}, D) \in [0, \tilde{x} - D]$ when $\tilde{x} > D$. We assume that no tax is paid when $\tilde{x} \leq D$. Finally, we assume that the total tax liability is weakly decreasing in D, i.e. $t_D(\tilde{x}, D) \leq 0$.

Default costs (net of subsidies):

If $\tilde{x} < D$, the firm cannot meet its debt obligations and must default unless it receives a subsidy from the government or some other third party.[7] Let $n(\tilde{x}, D)$ be the *net* default costs for the firm, which is the difference between the bankruptcy cost and any third party subsidy. In the event that $\tilde{x} > D$, there are no subsidies nor bankruptcy costs and thus $n(\tilde{x}, D) = 0$. If $\tilde{x} < D$, we assume that $\tilde{x} - n(\tilde{x}, D) \in [0, D]$. Net default costs could be negative (if subsidies exceed bankruptcy costs) but we assume subsidies only protect creditors so that available funds do not exceed D.

With these assumptions, the payoffs on the firm's debt and its equity are as follows: [8]

	If $\tilde{x} < D$	If $\tilde{x} \geq D$
Payoff to Shareholders	0	$\tilde{x} - t(\tilde{x}, D) - D$
Payoff to Debtholders	$\tilde{x} - n(\tilde{x}, D)$	D

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[7] We allow for the possibility of subsidies because of their importance in the recent financial crisis.
[8] There may be other direct effects of leverage on cash flows; e.g., the firm may have to pay higher wages as in Berk, et al. (2010), or have reduced costs due to disciplining as argued by Jensen (1986). We can adjust *t* and *n* to embody other such direct consequences of leverage on free cash flows.

Pricing at Date 1:   All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities at date 1 equal to their expected payoff with respect to a risk-neutral distribution F of the gross return $\tilde{x}$ on the firm's assets, which has full support on $[0, \infty)$, and is independent of the firm's leverage choice.[9] Throughout the paper, when we refer to probabilities, it is with respect to this risk-neutral measure.

Given our assumptions about payouts and pricing, it follows that at date 1 the values of the firm's debt and its equity are:

$$\text{Total value of debt} = V^D(D)$$
$$= \int_D^\infty D \, dF(x) + \int_0^D \left(x - n(x, D)\right) dF(x) \tag{1}$$

and

$$\text{Value of equity} = V^E(D) = \int_D^\infty \left(x - t(x, D) - D\right) dF(x). \tag{2}$$

The total value of the firm is therefore $V^F(D) \equiv V^E(D) + V^D(D)$.

2.2. Resistance to Leverage Reductions

Now suppose the firm considers reducing leverage by buying back a portion of its outstanding debt. For now, we hold fixed the assets of the firm, and assume the cash used for the buyback is raised through a rights offering to existing shareholders, or a market offering of equity or other equity-like securities (such as preferred shares). Alternatively, the firm may use cash on hand that it would have alternatively paid out as a dividend. If the firm plans to buy back debt with a nominal claim equal to $d > 0$, then upon the announcement of the recapitalization, the value of the firm will change from $V^F(D)$ to $V^F(D - d)$.

If the firm is inefficiently over-leveraged, then this change in firm value will be positive. While it is clear that some fraction of this gain will be captured by the firm's debtholders, diminishing the benefit to equity holders, intuition suggests that equity holders should still gain if the benefit of the recapitalization is sufficiently large. In fact, we will show that no matter how large the potential benefit, the debtholders *always* capture more than 100% of the gain to firm value, deterring equity holders from ever *voluntarily* recapitalizing.[10]

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[9] Setting the interest rate to zero is without loss of generality (we can alternatively interpret prices as future values). The existence of fixed pricing kernel F is restrictive, but follows immediately if investors are risk-neutral. Alternatively, one can assume the firm acts as a price-taker with respect to a risk-neutral pricing kernel implied by no arbitrage. Price-taking is reasonable if the firm is small and asset returns are already spanned (e.g. via options markets or asset-backed borrowing, see Hellwig (1981)). Either assumption allows us – as is standard in the corporate finance literature – to ignore any general equilibrium consequences of the firm's security choices.

[10] We take up whether a possibly forced recapitalization is feasible and consistent with limited liability in Section 4.

To understand this result, consider first the pricing of the firm's debt. Equation (1) above implies that, without the buyback, the date 1 market price of debt per unit of nominal face value is equal to:

$$q(D) = \frac{V^D(D)}{D} = 1 - \underbrace{F(D)}_{\text{Probability of Default}} \left(1 - \underbrace{E\left[\left. \frac{\tilde{x} - n(\tilde{x}, D)}{D} \right| \tilde{x} < D \right]}_{\text{Expected Recovery Rate}} \right). \tag{3}$$

That is, the debt price decreases with the probability of default but increases with the expected recovery rate.

Next observe that if the firm wants to buy back a discrete amount of debt in the open market, it cannot do so at the price given in (3). The repurchase price must be such that debtholders are at the margin indifferent between selling debt and holding on to it.[11] The buyback price of the debt must therefore be equal to the market price $q(D-d)$ that prevails at the post-buyback debt level. Thus, upon announcement of the debt buyback, the value of the firm's debt will change to

$$V^D(D-d) + q(D-d)d ,$$

and, because they must pay the cost of the buyback, the value of equity will change to

$$V^E(D-d) - q(D-d)d .$$

Importantly, we assume that both default and debt buyback decisions are made only on the basis of how they affect shareholders' wealth.[12] Therefore, the buyback will be undertaken only if the market value of the firm's equity would increase with the buyback:

$$V^E(D-d) - q(D-d)d > V^E(D). \tag{4}$$

The following proposition shows that, no matter how large the gain to total firm value is, equity holders are always harmed by a recapitalization. As we show in the proof, there are three sources of loss for equity holders, namely: the loss of their default option (default option effect), the reduction in dilution of existing creditors (dilution effect), and the loss of tax shields (tax effect).

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[11] For extensive discussions of this holdout effect, see Frenkel et al. (1989) and Bulow and Rogoff (1990). The holdout effect depends on the assumption that the buyback occurs through the market, as in the Bolivian debt buyback of 1988. The holdout effect can be avoided if the buyback occurs through collective bargaining with *all* creditors, see van Wijnbergen (1991) on the Mexican debt buyback of 1990.

[12] With regard to default, we are assuming equity holders default strategically, that is, when the value of equity just equals zero. If liquidation could occur when the equity value is still positive, default would entail an additional loss that shareholders may seek to avoid. Also, managers' and shareholders' interests need not be aligned if managers experience other losses in the event of default (such as lowered employment opportunities). While potentially important, we do not consider the governance issues associated with the decision to default, issue shares or make a rights offering. (Under U.S. law, a rights offering can be made without shareholder approval, though it may still fail if investors do not find it in their interest to acquire the new shares. In most other countries, rights offerings must be approved by shareholder meetings.)

Proposition 1 (Shareholder Resistance to Leverage Reduction): *Equity holders are strictly worse off issuing securities to recapitalize the firm and reduce its outstanding debt. Losses to equity holders arise from the loss of their default option, the reduction in dilution of existing debt, and higher taxes. The loss to equity holders is increases with debt tax shields and recovery rates.*

Proof: Using (2) and (4), we can write the gain to shareholders from changing from debt D to $D-d$ as

$$
\begin{aligned}
G(D, D-d) &\equiv V^E(D-d) - V^E(D) - q(D-d)d \\
&= \int_{D-d}^{D} (x-D)\, dF(x) \\
&\quad + d \times \big(1 - F(D-d) - q(D-d)\big) \\
&\quad + \int_{D}^{\infty} t(x,D)\, dF(x) - \int_{D-d}^{\infty} t(x, D-d)\, dF(x)
\end{aligned}
\tag{5}
$$

The first term in (5) captures the *default option effect*: the loss of equity's default option given final asset values between $D-d$ and D. This term is strictly negative given our assumption that F has full support.

The second term in (5) captures the *dilution effect*: the portion of the debt repurchase price that compensates debtholders for their share of any recovery in default. From (3), we can see that this term is negative and decreases with the (ex post) expected recovery rate of the debt:

$$
\begin{aligned}
d \times \big(1 - F(D-d) - q(D-d)\big) = \\
- d \times \underbrace{F(D-d)}_{\text{Probability of Default}} \times \underbrace{E\left[\left.\frac{\tilde{x} - n(\tilde{x}, D-d)}{D-d}\right| \tilde{x} < D-d\right]}_{\text{Expected Recovery Rate}} \leq 0.
\end{aligned}
\tag{6}
$$

Note that this term would be zero at the margin if the debt repurchased were strictly junior to any remaining debt, as in that case the expected recovery rate would be zero for the marginal dollar of junior debt. When the debt repurchased has equal priority with the remaining debt, as assumed thus far, and the remaining debt has a positive recovery rate, this term is negative. Note also that default subsidies will raise, and bankruptcy costs will lower, this cost.

Finally, the third term in (5) is the *tax effect*: it is negative because taxes are non-increasing in D.

Thus, combining these three effects, we see that

$$
V^E(D-d) - V^E(D) - d \times q(D-d) < 0
\tag{7}
$$

and hence shareholders always lose from a recapitalization. ∎

An alternative way to understand Proposition 1 is to consider who gains from a reduction in leverage. First, debtholders are fully repaid in some states of the world where the firm would have defaulted (the default option effect). Second, when the firm does default, there are fewer

debt claims and so each receives a larger share of any recovery value (there is less dilution). Shareholders pay the marginal bond holder for these gains, but obtain no benefit from the reduction in bankruptcy costs. Finally, the government enjoys higher tax revenues (the tax effect).

Proposition 1 restates and generalizes observations that have been made elsewhere in the literature. Black and Scholes (1973) note that in a setting with perfect markets shareholders lose from repurchasing debt. Equation (5) shows that this result is due to the loss of the default option and the expectation of a positive recovery rate on the debt. Leland (1994) demonstrates a similar result in the context of a continuous-time tradeoff model with linear taxes and a particular model of default costs. But to the best of our knowledge, the full generality of Proposition 1 has not been clearly articulated nor fully appreciated in the capital structure literature.[13]

We have assumed thus far that the firm has only a single class of debt outstanding. If the firm has several classes of debt, shareholders will naturally find it most attractive to buy back the cheapest, or most junior, class outstanding. These classes differ in their expected recovery rates, but because the expected recovery rate is always non-negative, the above logic still applies and we have the following immediate generalization:

Proposition 2 (Shareholder Resistance to buying back *any* debt class): *Shareholders are strictly worse off issuing equity to repurchase any class of outstanding debt. The loss increases with the seniority of the debt purchased.*

Note that shareholders' resistance to a recapitalization does not depend on the existence of tax benefits of debt. More strikingly, shareholders will resist a recapitalization no matter how large the potential gain to firm value. All of the benefits produced by the debt buyback – which in our model thus far come from reduced bankruptcy costs – accrue to existing debtholders. Shareholders – who must buy back the debt at a market price that reflects the reduced likelihood of default – are unable to appropriate these gains and so will resist a recapitalization.[14]

The observation that shareholders resist a recapitalization even when it would raise the value of the firm stands in contrast to the standard tradeoff theory of capital structure, where firms are presumed to choose their debt levels so as to maximize total firm value given the countervailing

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[13] Indeed, resistance to recapitalization is often justified by appealing to transactions costs or lemons costs associated with equity issues (see e.g., Bolton and Freixas (2006)). Of course, such explanations do not explain the failure of recapitalizations via rights offerings or when firms have cash available to pay out as dividends, whereas Proposition 1 immediately applies.

[14] Our analysis raises potential concerns about the practice of using mark-to-market accounting for a firm's liabilities. Mark-to-market accounting is usually based on the principle that reported valuations should reflect the scope the firm has for its actions in markets, e.g. in selling assets or buying back debt. If the market price of the firm's debt is based on the expectation that there will not be a buyback, this price is not relevant for a buyback. A mark-to-market valuation that is based on this price does not actually reflect the scope for a buyback that the firm has. As a monopsonist or a monopolist in the markets for its own securities, the firm should not take the prices of its securities as given and independent of its own actions. Market prices of the firm's securities depend on its actions, especially actions related to changes in capital structure.

frictions of tax benefits and distress and other costs associated with leverage. In fact, shareholder and firm value maximization coincide only when capital structure decisions are taken *ex ante*, before any debt has been issued. Once existing debt is in place, shareholder-creditor conflicts emerge, with important consequences for how both the asset *and* liability side of the firm's balance sheet will be managed going forward.[15]

Indeed, the consequences of debt overhang for recapitalization are stronger than those for equity-financed investment as described in Myers (1977). When a firm must issue equity to undertake a valuable project, the loss to the shareholders due to the wealth transfer to risky debtholders brought about by the reduction in leverage can be more than offset by the positive net present value (NPV) of the project, a portion of which the shareholders capture. Thus, if the NPV of the project is large enough, Myers's underinvestment problem disappears, and the outcome is efficient. By contrast, no matter how much the debt buyback would increase the total firm value, shareholders resist, and so this manifestation of debt overhang always results in a loss of efficiency.

Matters would be different if there were collective bargaining about the price of debt in the buyback.[16] For example, if debt contracts had collective action clauses, the firm's management, acting on behalf of shareholders, could negotiate a buyback agreement with debtholder representatives. In such negotiations, and with the no-buyback outcome as a default option, debtholders would end up sharing their gains from the buyback with the shareholders. This sharing of gains cannot be achieved in a market buyback. And even in a negotiation, if debtholders are dispersed, holdouts could be likely. In other words, at terms for which shareholders would not resist a recapitalization, we would expect (at least some) debtholders to resist, *precluding a purely voluntary leverage reduction*.[17]

Our results above establish the resistance of shareholders to pure recapitalizations for all equity-based sources of funding. That said, managers may have their own objectives, facing losses not shared by equity holders in the event of default. In that case, managers may pursue a leverage reduction even if it is not in shareholders' interests. Alternatively, the firm may be forced to reduce leverage by covenants or regulation. Our results imply that shareholders will seek to stop such actions where possible.

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[15] This point is central to the literature on dynamic theory of capital structure; see for example Strebulaev and Whited (2012). However, despite its name, this literature is more concerned with the dynamics of default and investment decisions for a given capital structure than with the evolution of capital structure through new issues and repurchases of debt and equity. Moreover, leverage changes are often restricted exogenously; e.g. Bhamra et al. (2010, p. 1499) state "In common with the literature, we assume that refinancings are leverage increasing transactions since empirical evidence demonstrates that reducing leverage in distress is much costlier."

[16] In a different setting the impact of collective bargaining on debt dynamics is also noted by Strebulaev and Whited (2012).

[17] Note that in general it is not sufficient to negotiate solely with creditors whose debt will be repurchased – that is, it is not enough just to overcome the holdout problem – remaining creditors need to share some of their gains as well. See also Mao and Tserlukevich (2015) for reasons why it may be difficult, even with a small number of creditors, to negotiate a lower price for the debt repurchase.

2.3. Investment Distortions and Resistance

So far we have focused on the tradeoff between the tax benefits of leverage and bankruptcy costs. Bankruptcy costs alone, however, are not the only detrimental consequence of leverage for firm value. It is well-known that debt overhang can distort investment via asset substitution (Jensen and Meckling, 1976), or underinvestment (Myers, 1977), and these costs are often presumed to be even more significant than bankruptcy costs in the determination of optimal leverage from the perspective of tradeoff theory.[18] In the following we generalize our analysis to allow for these investment distortions and show an important feedback effect on the leverage decision: Shareholder-creditor conflicts can both raise the benefit of, and increase shareholder resistance to, recapitalizations.

While the avoidance of incremental agency frictions may increase the benefit from a leverage reduction, shareholders will continue to resist any recapitalization despite these potential gains. In fact, because shareholders forfeit their agency *rents* with a leverage reduction, subsequent debt-equity conflicts only *increase* shareholder losses from a recapitalization (relative to a setting in which these conflicts could be avoided via pre-commitment or alternative governance). We illustrate this effect with the following simple example.

Example 1 (Resistance and Agency Costs): Suppose the value of the firm's assets $\tilde{x} = 0$, 50, or 100 with probabilities 25%, 50%, and 25% respectively. For simplicity, there are no bankruptcy costs or taxes. In that case, the value of the firm is $E[\tilde{x}] = 50$ for any level of debt (as we have not yet introduced any frictions). Suppose $D = 60$, in which case $V^E(60) = 0.25(100-60) = 10$. If the firm repurchases debt to reduce leverage to $D = 30$, the price of the debt will be $q(30) = 0.75$ for a total cost of $0.75(30) = 22.5$. Because the new value of equity is $V^E(30) = 0.50(50-30) + 0.25(100-30) = 27.5$, the net gain to shareholders from the transaction is $G(60,30) = 27.5 - 10 - 22.5 = -5$. Although firm value is unchanged, shareholders lose their option to default when the asset value is 50, which has value $0.50(60-50) = 5$.

Now introduce the possibility of asset substitution, by supposing equity holders have the ability to change the distribution of asset returns to $\tilde{x}' = 0$ or 100 with probabilities 65% and 35% respectively. Note that $E[\tilde{x}'] = 35 < E[\tilde{x}] = 50$. Nonetheless, debt overhang causes equity holders to prefer $\tilde{x}'$ to $\tilde{x}$ for $D > 37.5$.[19] Thus, a reduction in debt from $D = 60$ to 30 will increase total firm value by 15; the possibility of asset substitution raises the benefit of a leverage reduction above zero. But because $V^E(60) = 0.35(100-60) = 14$, the gain to equity from a leverage reduction is now $G(60,30) = 27.5 - 14 - 22.5 = -9$. So, while the benefit of leverage reduction has

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[18] Gertner and Scharfstein (1991) discuss both effects in the context of workouts, with excessive risk taking in the form of gambling for resurrection. They highlight the inadequacy of exchange offers in workouts to lead to efficient investment; we show that these potential investment distortions may also exacerbate capital structure inefficiencies.

[19] That is, $D = 37.5$ solves $0.35(100-D) = 0.50(50-D) + 0.25(100-D)$.

increased by 15, the *cost* to shareholders has simultaneously increased by 4 (the value of equity's incremental agency rents from asset substitution). ∎

We can generalize the preceding example to show that *any* shareholder discretion over firm investment will lead to a similar conclusion. For example, suppose the distribution of asset returns may be affected by actions taken by managers acting on behalf of shareholders. We denote these actions by θ, and the resulting asset returns by $\tilde{x}_\theta$, which has distribution $F(x|\theta)$. Suppose the firm also has the opportunity to invest in additional assets a by raising capital k from shareholders (or reducing planned equity payouts). Moreover, suppose these decisions will be made at a later date and conditional on some new information z that is relevant to both asset returns and the profitability of the investment opportunity. Specifically, if we let $k(a, z)$ be the cost of making investment a given information z, then the equity value function *conditional* on the investment policy functions $a(z)$ and $\theta(z)$ can be written as

$$V^E(D, \theta, a) \equiv$$
$$E_z \left[\int_{D/(1+a(z))}^{\infty} \left(x(1+a(z)) - t(x(1+a(z)), D) - D \right) \, dF\left(x|z, \theta(z)\right) - k(a(z), z) \right] \tag{8}$$

where the expectation is over possible information states *z*. Equity holders choose the policies (θ, a) to maximize (8) given outstanding debt *D*, so that $V^E(D) = \max_{\theta, a} V^E(D, \theta, a)$.

In this case, in addition to asset substitution, leverage may lead to underinvestment due to the traditional debt overhang problem identified by Myers (1977). The next result demonstrates that, once again, the incremental underinvestment and risk shifting associated with leverage, while potentially detrimental to total firm value, will only increase the cost to shareholders from a recapitalization.

Proposition 3 (Agency Costs): *Although shareholder-creditor conflicts regarding investment may raise the benefits of a leverage-reducing recapitalization for total firm value, shareholders are still strictly worse off no matter how large these potential gains. Indeed, relative to a setting in which investments were fixed at the optimal policy given lower leverage, shareholder-creditor conflicts only raise the costs of a recapitalization for shareholders.*

Proof: See Appendix. ∎

Technicalities aside, the intuition for Proposition 3 follows directly from shareholders' optimization problem. Define

$$(\theta_0, a_0) \equiv \arg\max_{\theta, a} V^E(D_0, \theta, a) \tag{9}$$

to be the action choices for debt level D_0. If we reduce leverage from $D_1 > D_0$ to D_0, then the change in the value of equity is given by

$$V^E(D_0) - V^E(D_1) = \underbrace{V^E(D_0, \theta_0, a_0) - V^E(D_1, \theta_0, a_0)}_{\substack{< q(D_0)(D_1 - D_0) \\ \text{from Proposition 1}}} - \underbrace{\left(V^E(D_1, \theta_1, a_1) - V^E(D_1, \theta_0, a_0)\right)}_{\substack{\geq 0 \\ \text{incremental agency rents}}} \quad (10)$$

Thus, the increase in the value of equity post-recapitalization is even smaller now than in the setting without incremental agency costs (i.e., if actions were fixed at the low leverage levels via pre-commitment or governance). Agency costs mitigate the decline in the value of equity as leverage increases, as shareholders take actions that transfer wealth from creditors. But this effect implies that equity holders also gain less from a leverage reduction, and they must pay more for the debt in anticipation that such wealth transfers will be diminished. Thus, even though agency costs may raise the cost of leverage, they impede shareholders' incentive to reduce it.[20]

2.4. Ratcheting Incentives

The standard tradeoff theory of capital structure posits that firms choose debt in order to maximize total firm value, trading off tax benefits against investment distortions from distress and agency costs. Once leverage is already in place, however, debt overhang creates a powerful dynamic that will distort shareholder incentives with respect to changes in the firm's capital structure. We will now show that not only do shareholders choose not to reduce leverage, but also that debt tax shields cause shareholders to *always* prefer to *increase* debt by some amount, even if this additional leverage reduces firm value. In other words, leverage begets additional leverage, creating a leverage ratchet effect.

The observation that shareholders can gain by issuing new debt that has equal (or higher) priority to its existing debt is well known, and results from the fact that the new debt will dilute the claims of existing creditors.[21] Fama and Miller (1972) argue that seniority provisions, requiring any new debt to be junior to existing creditors, prevent such dilution.

In the presence of default or agency costs, however, strict priority rules are insufficient to fully insulate senior creditors from the consequences of future debt issues. As demonstrated by Bizer and DeMarzo (1992), sequential borrowing with *junior* debt can still be detrimental to more senior claims because of its influence on subsequent firm actions, such as risk-shifting, underinvestment, or strategic default. In other words, by increasing future agency costs, new junior debt can harm existing senior creditors.[22] Equity holders do not internalize this harm, distorting their decision to engage in additional borrowing, as illustrated in Figure 1. As we show below, this

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[20] We have focused on shareholder-creditor conflicts, but as argued by Jensen (1986) and others, debt may be helpful in reducing shareholder-manager "free cash flow" agency conflicts. Such conflicts could provide another motive for shareholders not to reduce debt.

[21] Even with a single class of debt, however, it is not the case that a shareholder loss from reducing leverage from D_1 to D_0 implies an equivalent gain if leverage is increased from D_0 to D_1, because the price of debt in the latter transaction is generally below that in the first: $q(D_1) < q(D_0)$. Thus it is quite possible that shareholders might lose from either transaction.

[22] Note that new debt changes the equity holders default decision, which harms senior creditors when there are bankruptcy costs. Thus even absent other agency problems, strategic default creates an agency cost of new debt. Brunnermeier and Oehmke (2013) show that a similar effect arises if shareholders can issue new debt with a shorter maturity than existing debt, as its earlier maturity gives it effective seniority.

feedback effect creates an additional agency cost of debt: *existing leverage distorts future leverage decisions of the firm*.
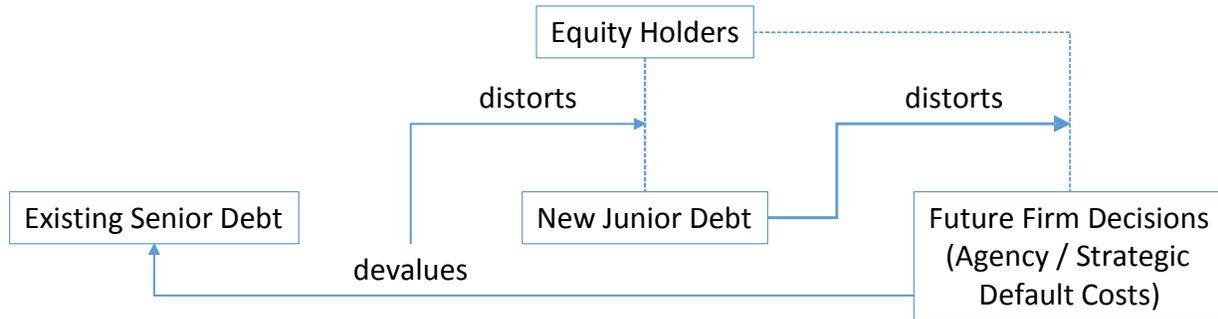

Figure 1: Sequential Borrowing Agency Distortions
Even when new debt must be junior, it distorts future incentives in a way that harms senior creditors. This loss to senior creditors is not internalized by shareholders, distorting incentives with regard to the decision to increase leverage.

To demonstrate the leverage ratchet effect, consider our setting with taxes, default costs, and asset substitution, and suppose that any existing debt is protected so that any new debt issued is junior to all other outstanding debt claims (and hence there is no direct dilution of existing creditors). Let $G(D, D')$ be the gain to shareholders when a firm with existing debt D increases its debt to $D' \geq D$, by issuing new junior debt with face value $D' - D$:

$$G(D, D') = V^E(D') - V^E(D) + (D' - D)q^J(D, D') \tag{11}$$

where $q^J(D, D')$ is the price at which the new junior debt is sold. Then we have the following key result:

Proposition 4 (Leverage Ratchet Effect): *Given initial debt D, suppose the firm has the opportunity to adjust its debt on a one-time basis. Then, the optimal choice $D^*(D) = \arg\max_{D'} G(D, D')$ satisfies:*

- *If the firm has no initial debt, then the amount of debt $D^*(0)$ that maximizes shareholders' gain also maximizes the total value of the firm.*
- *If the firm has outstanding debt $D > 0$, shareholders never gain by reducing leverage. Moreover, if new debt is*
  - *pari passu or senior to claims with a positive expected recovery rate, or,*
  - *junior to existing claims, but the marginal expected tax benefit is positive and the probability of default is continuous at D,*
  then it is always optimal for shareholders to increase leverage by issuing some amount of new debt even if it reduces total firm value; that is $D^*(D) > \max(D, D^*(0))$.

Proof: See Appendix. ∎

The first statement in Proposition 4 is obvious – absent pre-existing debt, shareholders internalize any costs to creditors via the price they will receive for the new debt, and hence will choose leverage to maximize total firm value. This observation is the basis for the standard optimality prediction of the tradeoff theory.

The second statement in Proposition 4, however, makes clear that when new debt dilutes existing claims *or* provides a marginal tax benefit, this prediction must fail if the firm makes new leverage decisions once existing debt is in place. Even if the firm is already excessively leveraged (relative to the tradeoff theory optimum), equity holders will still be tempted to increase leverage by some positive amount.[23]

While the proof of this result in full generality is complicated by technicalities related to differentiability and continuity, the intuition is straightforward. All of standard *costs* associated with leverage in the tradeoff theory – default costs, investment-related agency costs, and even distortions of future leverage choices – are decisions which are *optimized* by equity holders (e.g., equity holders optimally determine when to exercise their put option to default). Thus, although these costs are first order to the firm, by a standard envelope argument they have only a second-order impact on shareholders. Hence, the only first-order effect for shareholders is the incremental tax benefit.

For example, consider the setting of (8) and let $X_{\theta,a}$ and K_a be random variables representing the total asset payoff and investment policy of the firm. Then we can write

$$V^E(D) = \max_{\theta,a} E[(X_{\theta,a} - t(X_{\theta,a}, D) - D)^+ - K_a].$$

Applying the envelope theorem, we compute the derivative at the optimal policy choice (θ^*, a^*). Because the price of new debt is at least equal to the probability of no default, we have [24]

$$\tfrac{\partial}{\partial D} V^E(D) = -E\left[1_{X_{\theta^*,a^*}>D}\left(1 + t_D(X_{\theta^*,a^*}, D)\right)\right] \geq -q^J(D,D) - E\left[t_D(X_{\theta^*,a^*}, D)1_{X_{\theta^*,a^*}>D}\right] \quad (12)$$

Thus from (11),

$$G_2(D,D) \geq -E\left[t_D(X_{\theta,a}, D)1_{X_{\theta,a}>D}\right] > 0. \quad (13)$$

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[23] While there is always *some* amount of debt shareholders would like to issue, this result does not imply that shareholders will gain from issuing *any* amount – if they attempt to issue too much, the price will be sufficiently low to make it unattractive.

[24] We write 1_B to be the indicator variable for the event B, and evaluate the derivative at the optimal policy choice. So $E[1_{X>D}] = \Pr(X > D) = \Pr(\text{no default})$, which equals the marginal value of junior debt unless it has a positive recovery rate (e.g. due to subsidies or a lack of strict priority).

That is, the marginal gain from new incremental debt is (at least) the expected incremental tax shield.[25] Thus, independent of the amount of debt already in place, shareholders always have a positive incentive to increase debt further until its interest tax shield is fully exploited.[26]

We illustrate the result of Proposition 4 in Figure 2, which shows G and V^E for different levels of initial debt D.
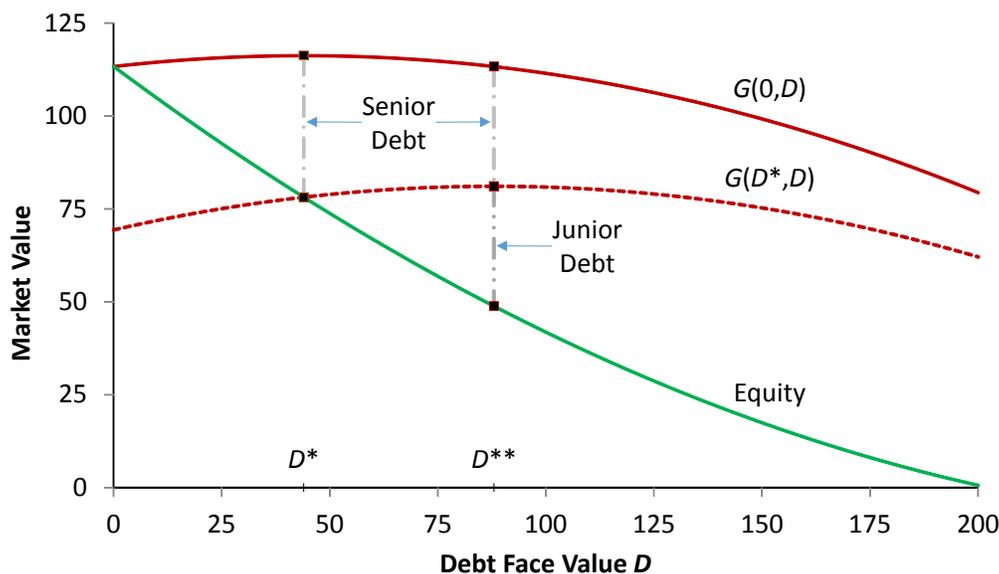

Figure 2: The Leverage Ratchet Effect
Although a new issue of junior debt reduces firm value, it decreases the value of senior debt by even more, leading to a gain for shareholders. At the margin, shareholders benefit from a higher interest tax shield, whereas default and agency costs are second order.
(See Section 3.1 for the specific parameters used here.)

If the firm initially has no debt, equity holders would prefer debt D^* that maximizes total enterprise value (equity plus debt). Once debt D^* is in place however, equity holders face the potential gain $G(D^*, D)$, and so would ideally choose D^{**} if given a one-time opportunity to issue new junior debt. Note that total enterprise value is lower at D^{**}, but equity holders still gain because the value of the senior debt declines. Senior creditors lose because for them, the increased agency and default costs that arise with greater total leverage have a first-order impact on their payoff. Equity and junior creditors ignore the impact of their decisions on senior creditors, and this agency conflict creates the leverage ratchet effect.

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[25] This observation provides some justification for the standard practice in capital budgeting valuation to consider only incremental tax benefits associated with new debt while ignoring the impact of bankruptcy and agency costs. Once existing debt is in place this approach correctly captures the marginal impact to the firm's shareholders.

[26] Only in the special case where the marginal tax benefits are zero, and assuming junior debt is not subsidized in default, is there no incentive to increase (or decrease) debt. If debt provides other benefits, for example by reducing shareholder-manager conflicts, these benefits might also create an incentive for shareholders to increase debt.

3. Leverage Ratchet in Dynamic Equilibrium

Our results thus far demonstrate that under shareholder control, leverage is "irreversible" once put in place and moreover creates a desire for even more leverage. Our analysis, however, has been restricted to one-time static adjustment of the firm's debt level. In a dynamic context, creditors will include the cost of such future distortions in the price they are willing to pay for the debt upfront. Moreover, this price adjustment will naturally affect the firm's optimal initial leverage choice. In this section we develop a simple and tractable dynamic model to explore and highlight the key consequences of the leverage ratchet effect.

We begin by developing a simple, tractable stationary dynamic model of a leveraged firm, in which the firm earns taxable income at a fixed rate until the random arrival of an exit or liquidation event. Because interest on debt is tax deductible, the firm has a tax benefit from leverage, but we assume debt distorts the payoff at exit either due to default or agency costs. We consider the optimal "one-time" debt issuance in Section 3.1, and then illustrate the leverage ratchet effect and derive the equilibrium debt choice when the firm has repeated opportunities to issue debt in Section 3.2. In Section 3.3 we allow for shocks to the income and value of the firm and show how these shocks will lead leverage to increase over time. Finally, in Section 3.4 we discuss the potential role of countervailing forces such as repayment of maturing debt, asset growth, and debt covenants in mitigating the leverage ratchet.

3.1. An Illustrative Example: The Static Benchmark

Suppose the firm generates earnings before interest and taxes at a constant rate y until the arrival at random time τ of an "exit" or liquidation event. The interest rate r, liquidation arrival rate λ, and tax rate t are constant, and the firm issues debt with fixed face value D and constant coupon rate c such that $cD \le y$.[27] The value of the firm in the event of an exit is given by X_θ, which is independent of τ, and where the parameter θ reflects investment or strategy choices that are chosen to maximize shareholder value. In that case, the value of the firm's equity is given by[28]

$$V^E(D) = \max_\theta E\left[\int_0^\tau e^{-rs}(y-cD)(1-t)ds + e^{-r\tau}(X_\theta - D)^+\right]$$

$$= \frac{r}{r+\lambda}\frac{(y-cD)(1-t)}{r} + \frac{\lambda}{r+\lambda}\max_\theta E[(X_\theta - D)^+], \tag{14}$$

for $D \in [0,\bar{D}]$ where $\bar{D}$ is the debt level above which equity holders would choose to immediately default, defined by $V^E(\bar{D}) \equiv 0$.

We complete the model by defining the function

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[27] We take the coupon rate c as given; it is arbitrary for our analysis as the bond price will simply adjust correspondingly. We assume coupons are fully tax deductible, though in practice there may be limits on deductibility if the bond's issuance price is far from par value.

[28] We use the notation $x^+ = \max(x,0)$, and the fact that given arrival rate λ, $E[e^{-r\tau}] = \lambda/(r+\lambda)$.

$$\phi(D) \equiv \max_\theta E[(X_\theta - D)^+] \tag{15}$$

to represent the expected payoff to equity in liquidation. By standard arguments $\phi(\cdot)$ must be nonnegative, strictly decreasing (if $\phi > 0$) and weakly convex in the debt face value D, with $\phi'(0) \geq -1$. For technical convenience we assume X_θ is continuously distributed and ϕ is twice differentiable. Finally, we write $\theta(D)$ to represent the argmax in (15).

We assume that in the event of an exit, debtholders are fully repaid if $X_{\theta(D)} \geq D$, but if $X_{\theta(D)} < D$ the firm will default. As a simplifying assumption, we assume debtholders are unable to recover any of the asset value in default, and so receive a payoff of zero. (While not necessary, assuming zero recovery greatly simplifies our analysis as we do not need to specify or keep track of seniority.) Given a fixed face value $D \in [0, \overline{D}]$, the total value of debt is therefore

$$V^D(D) = E\left[\int_0^\tau e^{-rs} cD \, ds + e^{-r\tau} D 1_{Y_{\theta(D)} \geq D} \right] = \frac{r}{r+\lambda} \frac{cD}{r} + \frac{\lambda}{r+\lambda} D \Pr(X_{\theta(D)} \geq D). \tag{16}$$

From (15) and the fact that Y_θ is continuously distributed, $\Pr(X_{\theta(D)} \geq D) = -\phi'(D)$, and hence the debt has a price per dollar of face value equal to

$$q(D) = \frac{r}{r+\lambda}\left(\frac{c}{r}\right) - \frac{\lambda}{r+\lambda} \phi'(D). \tag{17}$$

We assume the firm exhausts the tax benefits of debt if the coupons cD exceed the cash flows y, or equivalently if $D > D^0 \equiv y / c$. The total value of the firm is then

$$V^F(D) = V^E(D) + V^D(D) = \frac{y(1-t) + t \min(y, cD) + \lambda(\phi(D) - D\phi'(D))}{r+\lambda}. \tag{18}$$

The convexity of ϕ implies that $\phi(D) - D\phi'(D)$ declines with D, and thus (18) captures the standard tradeoff between tax savings and bankruptcy concerns. Solving for the level of debt that maximizes total firm value, an interior solution D^* must satisfy the first-order condition

$$D^* = \frac{tc}{\lambda \phi''(D^*)}, \tag{19}$$

which makes the tradeoff explicit.

Bankruptcy Costs or Agency Costs

Our formulation includes both moral hazard (in the choice of θ) and bankruptcy costs (given the assumed zero recovery rate). We note, however, that the same function ϕ, which indicates the expected payoff to shareholders in liquidation, can also be derived from a pure agency model or a pure bankruptcy cost model, as in the following examples:

Example 2 (Pure Bankruptcy Costs): Consider a pure bankruptcy cost model in which X is uniformly distributed on $[0, \bar{X}]$, and recovery rates are zero in the event of default. Then

$$\phi^{BC}(D) = E[(X-D)^+] = \frac{(\bar{X}-D)^2}{2\bar{X}}. \tag{20}$$

Example 3 (Pure Agency Costs): Suppose shareholders choose the probability θ that the firm has a successful exit with value $g(\theta) = \bar{X}\left(1 - \frac{1}{2}\theta\right)$, and is worthless with probability $1-\theta$. Then

$$\phi^{AC}(D) = \max_{\theta \in [0,1]} \theta(g(\theta) - D)^+ = \frac{(\bar{X}-D)^2}{2\bar{X}}. \tag{21}$$

The following result demonstrates that we can always construct a pure bankruptcy cost or agency cost model to match any payoff function ϕ:

Lemma (Equivalence of Agency and Bankruptcy Cost Models): *Given any $\phi(\cdot)$ from (15), there exists an exit value g in the pure agency model, and a distribution for X in the pure bankruptcy cost model, such that $\phi = \phi^{AC} = \phi^{BC}$.*

Proof: See Appendix. ∎

Throughout this section, we use the payoff function ϕ defined by (20) or (21) to illustrate our results. In that case, from (19), the debt level that maximizes total firm value is given by

$$D^* = \min\left(\frac{tc}{\lambda}\bar{X}, D^0\right). \tag{22}$$

Time Inconsistency: The Leverage Ratchet Effect

Now suppose that the firm is endowed initially with the value-maximizing debt level. Will it stay there? In the absence of binding covenants, the answer is no. Once debt is in place, re-optimization by shareholders leads to an additional increase in leverage.

To see this effect, given initial debt $D \leq D^0$, we can calculate the gain to equity holders from a permanent change to debt level $D + d \leq D^0$ as follows:[29]

$$G(D, D+d) = V^E(D+d) - V^E(D) + dq(D+d)$$

$$= \underbrace{\frac{r}{r+\lambda}\frac{tcd}{r}}_{\text{Incremental Tax Shield}} - \underbrace{\frac{\lambda}{r+\lambda}\left(\phi(D) - \left(\phi(D+d) - d\phi'(D+d)\right)\right)}_{\text{Incremental Agency/Bankruptcy Costs}}. \tag{23}$$

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[29] Recall that because recovery rates are zero, this calculation applies independent of the priority of the new debt. In other words, the incentives we are identifying apply even if existing debtholders can enforce seniority with respect to any new debt.

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The second term on the right-hand side of (23), which is non-negative by the convexity of ϕ, represents the incremental agency or bankruptcy costs borne by shareholders (via the debt price). As in Proposition 4, this term is of the second order at $d = 0$, and we have the following analog to (13):

$$G_2(D,D) = \begin{cases} \dfrac{tc}{r+\lambda} & \text{if} \quad D < D^0 \\ 0 & \text{if} \quad D > D^0 \end{cases}. \tag{24}$$

In other words, the gain from a marginal dollar of new debt is simply equal to the value of its incremental tax shield. The optimal quantity of *new* debt can be found be setting $G_2(D, D+d) = 0$, which implies $d = \min\left(d^*, D^0 - D\right)$ with

$$d^* = \frac{tc}{\lambda\phi''(D+d^*)} = \frac{tc}{\lambda}\bar{X} , \tag{25}$$

where the last equality is specific to the quadratic specification of ϕ in (20).

So, as long as the upper bound for the tax shield has not been reached, the optimal amount of new debt is strictly positive. For example, in Figure 2 of Section 2.4 (which is illustrated using parameters $r = c = 5\%$, $y = 10$, $t = 40\%$, $\lambda = 10\%$, and $\bar{X} = 220$) we have $D^* = d^* = 44$ and $D^{**} = 88$. Indeed, until the tax shield is exhausted, shareholders always want to issue new debt with face value d^*, regardless of how much debt they already have. This finding is illustrated by the dashed straight line (parallel to the 45° line) in Figure 3 below, showing shareholders' desired total leverage for each initial debt level (using the same parameters as above).

Thus, the simple tradeoff level of debt that is obtained by maximizing the value of the firm initially is *not* time consistent. Until the tax shield is exhausted, shareholders would always find it desirable to issue up to d^* in new junior debt given the opportunity to make a one-time change. The optimal incremental debt increases with the tax shield and decreases with the likelihood of liquidation and the intensity of the agency or bankruptcy costs (measured by ϕ'').

3.2. Stable Leverage without Commitment

We have shown that if granted a one-time opportunity to issue new debt, shareholders *always* have an incentive to issue new debt, even if it must be junior to existing claims, until the tax shield is exhausted. Thus, if shareholders are unconstrained in their ability to issue junior debt, and have this opportunity repeatedly, their initial choice of a debt level will not persist. The simple tradeoff approach, determining leverage by value maximization, will not correspond to an equilibrium of the dynamic model.

How will leverage be determined in a dynamically consistent setting? In addressing this question, one must take account of the fact that creditors will recognize the leverage ratchet effect and price the debt appropriately in anticipation of future leverage changes. To analyze this possibility, we consider the following dynamic game. At each time *s*, the firm has existing debt

D_s. It then announces a new quantity of junior debt to issue (or repurchase). The price q_s of the incremental debt is set competitively in the market; in equilibrium, of course, this price will reflect creditors' anticipation of the firm's future leverage choices given the new debt level. While any new debt must be junior to any existing debt, the firm is otherwise unconstrained. Absent commitment, what will be the equilibrium leverage of the firm?

To determine the equilibrium leverage choice, consider first the case in which $D_s \in [D^0, \overline{D}]$. In that case, the firm has exhausted the debt tax shield, and has no incentive to issue additional debt. From our earlier results in Section 2, equity holders will also not choose to repurchase debt. Thus, any debt level greater than or equal to D^0 is "stable" – once it is attained, equity holders will not benefit from any further increase.

Next, suppose D_s is such that equity holders will gain by adjusting debt to D^0. That is suppose $G(D_s, D^0) > 0$. Then it is clear that D_s cannot be a stable outcome, as equity holders would gain by issuing new debt until D^0 is reached. Buyers of the new debt would only be willing to pay $q(D^0)$ since they know the firm will not change debt from that level once it is attained.

Finally, suppose the current debt level D_s is the maximal debt level below D^0 such that $G(D_s, D^0) \leq 0$. Then equity holders could not gain by issuing *any* new debt, since even if they issue only a small amount δ, because $G(D_s + \delta, D^0) > 0$ by the previous logic creditors would anticipate that they would keep issuing until the debt had face value D^0. But at the price $q(D^0)$, issuing new debt would not be profitable for shareholders.

Repeating this logic, we obtain the following construction for an equilibrium. There exists a set of stable debt levels, $D^0 > D^1 > \ldots$, defined recursively by

$$D^{n+1} = \max\{D < D^n : G(D, D^n) \leq 0\}. \tag{26}$$

Note that given our assumptions, G is continuous, so (26) implies $G(D^{n+1}, D^n) = 0$; that is, equity holders are just indifferent between sequential stable debt levels. We then have the following equilibrium, in which the firm's leverage "ratchets up" to the next stable debt level:

Proposition 5 (Stable Leverage Equilibrium): *The following strategies represent a subgame perfect dynamic leverage equilibrium: Given debt D, if $D > \overline{D}$ shareholders default and the price of debt is zero. Otherwise, shareholders immediately increase leverage to the next highest stable leverage level D^n, defined as*

$$D^+(D) \equiv \begin{cases} \min\{D^n : D^n \geq D\} & \text{if} \quad D \leq D^0 \\ D & \text{if} \quad D \geq D^0 \end{cases}. \tag{27}$$

The price of debt is given by $q(D^+(D))$.

Proof: See Appendix. ■ [30]

Proposition 5 implies that only stable debt levels will be observed in equilibrium, and as a result, the price of debt will fall discretely if for some reason a stable debt level is surpassed. This discontinuity in the debt price, which arises as creditors rationally anticipate the leverage ratchet, in turn sustains the stability of the debt. We illustrate this dynamic equilibrium using the setting of Examples 2 and 3 above:

Example 4 (Stable Debt Levels): Let ϕ be as in (20) or (21) with $\bar{X} \geq D^0$. Then for $D \leq D^0$, $G(D - \hat{d}, D) = 0$ implies

$$\hat{d} = 2tc\lambda^{-1}\bar{X} = 2d^*. \tag{28}$$

Therefore, $D^n = D^0 - n\hat{d}$. We illustrate this equilibrium in Figure 3 (for parameters $t = 40\%$, $r = c = 5\%$, $\lambda = 10\%$, $y = 10$, and $\bar{X} = 220$, as given earlier). Note that the tax shield is exhausted with debt level $D^0 = y/c = 200$.

From (25), the debt level D^* that maximizes $G(D, D^*)$ is $D^*(D) = D + 44$. From (28), the debt level $\hat{D}$ that makes equity holder indifferent so that $G(D, \hat{D}) = 0$ is $\hat{D}(D) = D + 88$.[31] The stable debt levels are therefore $D^0 = 200$, $D^1 = 200 - 88 = 112$, and $D^2 = 112 - 88 = 24$. Figure 3 plots D, $D^*(D)$, $\hat{D}(D)$ and finally the stable equilibrium $D^+(D)$ which is a step function showing the jump to the next stable point.

With these parameters, given no initial debt, the firm would choose $D^+(0) = D^2 = 24$, which is lower than the firm value maximizing level $D^*(0) = 44$. Shareholders have no incentive to increase debt above 24 because the debt price they receive will fall from \$0.93 to \$0.66 (per dollar of face value) as creditors anticipate they would continue to increase debt to 112.

If instead cash flows were lower so that $y = 8$, however, then $D^0 = 160$, and $D^+(0) = 72$. In that case, the firm would choose *higher* leverage than the firm value maximizing level (which is still 44). The reason for this reversal is that once D^0 falls to 160, then $160 - 88 = 72$ becomes a stable debt level. But once 72 is stable, given its debt price of \$0.78, it becomes attractive to increase debt to 72 from any lower initial debt level. ■

---

[30] The equilibrium we describe is the unique subgame perfect equilibrium given pure strategies in D. Other equilibria are possible, however, if we allow mixing or non-Markov strategies (e.g. mixing between D^n and D^{n-1}, or moving from one to the other after some period of time, would lead to an increase in all subsequent stable points). That said, if debt quantities are discrete (e.g. there is a minimum increment to the face value of \$1), then the equilibrium we describe is generically unique (since then $G(D^n, D^{n-1}) < 0$).

[31] $\hat{D}(D) = D + \hat{d}$ up to the point $D = D^1$. For $D > D^1$, because there is no tax shield above D^0, $\hat{D}(D) = \min\left(\bar{D}, D + \sqrt{(D^0 - D) \times \hat{d}}\right)$, with $\bar{D} = 201.55$ under the given parameters. Note, however, that the value of $\hat{D}$ in this range has no consequence for the equilibrium.
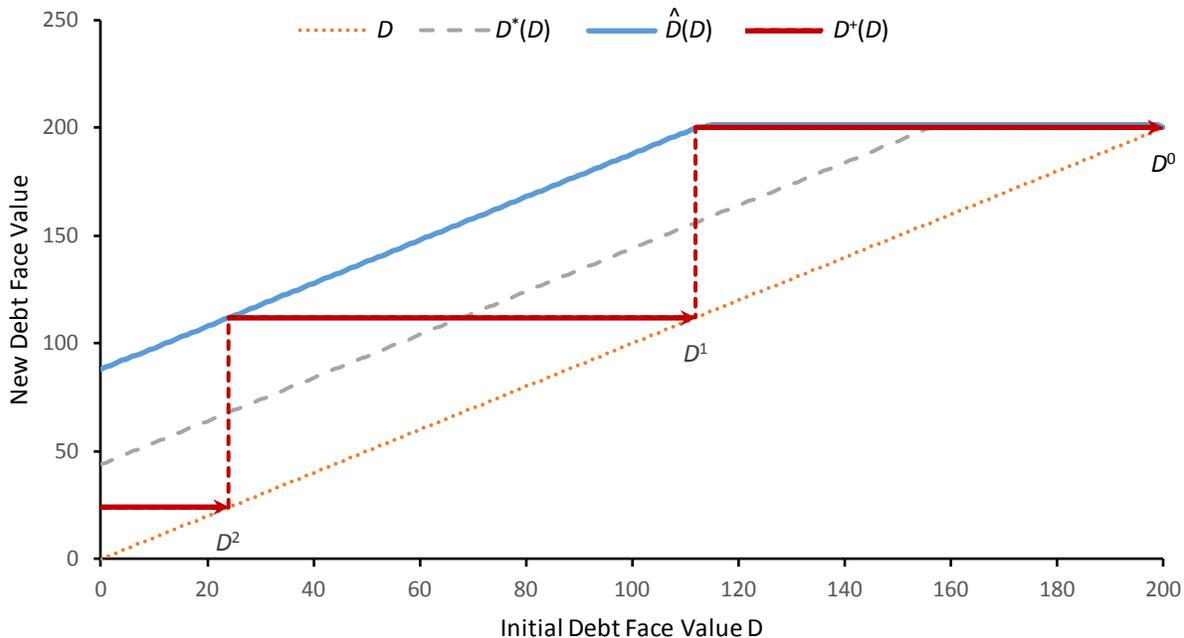
Figure 3: Stable Leverage Equilibrium

When the environment remains fixed, stable leverage levels D^n are determined recursively by (26). There is no incentive to increase debt beyond a stable level because the debt price will drop discontinuously to its value at the next stable level.

The example shows that, because of the leverage ratchet effect, the equilibrium level of firm leverage can be quite different from anything predicted by the static tradeoff approach. Whereas the tradeoff approach predicts a debt level of 44, in this example the lowest equilibrium debt level with the ratchet effect is 24; however, once debt is at 24, a small perturbation might lead shareholders to raise the level all the way to 112. And while the first level is almost 50% lower than that predicted by the tradeoff theory, the second is more than 100% higher. More importantly, the mechanisms determining potential equilibrium leverage are quite different from the simple optimization presumed by the tradeoff approach.

The equilibrium described in Proposition 5 is unnaturally stark. In particular, the result that the firm makes a one-time adjustment to its debt and then debt remains stable from that point onward depends on our assumption that all of the parameters of the firm are constant over time. In reality, firms' cash flows, the likelihood of exit and value in liquidation, as well as macro factors such as interest rates and tax rates, are time varying. In that case, permanently stable debt levels cannot be expected, as we demonstrate next.

3.3. Shocks and Leverage Ratchet Dynamics

Thus far, we have allowed the firm to adjust its leverage in a "steady state" environment without fluctuations. In that case, once a stable leverage level is attained, there is no reason for the firm to adjust leverage further. However, suppose the firm is subjected to shocks to its cash flows,

tax rate, or the intensity of bankruptcy or agency costs. How will the firm's leverage respond to these shocks? And how will debt be priced in anticipation of these shocks and the firm's reaction to them?

Intuitively, starting from any stable debt level prior to the shock, it is unlikely that this debt level will remain stable after the shock. Thus, as in our prior analysis, we would expect the firm to increase leverage to the next stable debt level given the new parameters. If shocks are repeated, then after each shock we will see leverage ratchet upward until the point that the tax shield has been exhausted or the firm defaults.

We formalize this by extending our prior model to allow for the Poisson arrival of a regime shift. We index regimes by $j \in \{1, \ldots, J\}$. In regime j, tax rates, interest rates, debt coupons, and cash flows are given by (t_j, r_j, c_j, y_j), and there is a random arrival of a liquidation event with arrival intensity λ_{j0} and payoff function ϕ_j.[32] In addition, there is an independent random arrival with intensity λ_{jk} of a shock that moves to regime k. The debt is priced and leverage decisions are made in anticipation of these potential shocks. We apply the same logic as in Proposition 5 to establish the following:

Proposition 6 (Leverage Ratchet Dynamics): *There exists a subgame perfect equilibrium of the following form: For each regime j there will be a set of stable debt levels*

$$\bar{D}_j \geq D_j^0 = y_j / c_j > D_j^1 > \ldots > D_j^n > \ldots.$$

If when entering regime j, current debt $D > \bar{D}_j$, shareholders immediately default. Otherwise the firm will increase leverage to the next stable level

$$D_j^+(D) = \begin{cases} \min\{D_j^n : D_j^n \geq D\} & \text{if} \quad D \leq D_j^0 \\ D & \text{if} \quad D \geq D_j^0 \end{cases}. \tag{29}$$

Proof: See Appendix. ∎

We show in the proof of Proposition 6 how to construct the stable points for each regime. As before, starting from D_j^n, we find the next lower debt level such that equity holders would just be indifferent between not issuing additional debt, and issuing up to the point D_j^n. The equity value function and debt price are calculated given this issuance policy. Because debt always ratchets upward, we can solve for these values by induction on the level of debt D.

---

[32] We allow all parameters to vary across regimes for generality – coupon rates, for example, might be (imperfectly) indexed to interest rates, etc. Combined with an arbitrary state space, this model can approximate a broad range of economic settings.

Over time, regime shocks will cause the equilibrium debt level to increase monotonically as the firm ratchets up to the next stable point with each transition to a new regime. Unless a stable point is reached that is shared by *all* regimes, Proposition 6 implies that the debt level of the firm will continue to ratchet up over time. Indeed, we have the following immediate result:

Corollary (Limit Values): *Suppose the regimes are recurrent (i.e., starting from any regime there is a positive probability of ultimately transitioning to any other regime). Then starting from debt level D, the debt level will increase over time until the next universal stable point in the set* $\{D : D_j^+(D) = D \text{ for all } j\}$. *If this maximal debt level exceeds* $\min_j \bar{D}_j$, *the firm may default prior to exit.*

Example 5 (Regime Shocks): Consider a setting as in Example 4 with two regimes that differ in terms of the firm's cash flow stream with $y_1 = 10 > y_2 = 8$. Both regimes share the same $(t, r, c, \lambda, \phi(\cdot))$ as in Figure 3. We show in Figure 4 the stable points when the arrival intensity of a regime shift is $\lambda_{12} = \lambda_{21} = 20\%$ (first panel) or 100% (second panel), corresponding to an average regime length of 5 years or 1 year.

In the first panel, starting with no leverage, the firm's initial debt choice is 19 (if cash flows are low) or 0 (if cash flows are high). Then, with each subsequent change in the level of the firm's cash flow, debt will "ratchet up" to the next stable point shown in the figure. After at most 3 shocks, debt will exceed the first best ($D^*(0) = 44$), and after 8 shocks debt will reach its maximal level of 174; at that level of debt the tax shield is exhausted for the low cash flow firm and this level is a common stable point.[33] (Default would occur in the low cash flow state if D exceeded 180, though this level of debt is not reached in equilibrium.)

In the second panel, with an average regime length of only one year, the distances between adjacent stable points are smaller, and so are the steps by which debt ratchets up. In that case it will take up to 9 shocks for debt to exceed $D^*(0) = 44$, and up to 24 shocks to reach the maximal debt level of 181 (with default in the low state not occurring until debt exceeds 183). As the example suggests, more frequent shocks lead to smaller debt increments, so that if regime changes were to arrive continuously, the ratcheting up of debt will become continuous as well (except for a possible jump prior to the final stable point).[34] ∎

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[33] Because coincidence in the discrete stable debt levels D_j^n is non-generic, we would typically expect, as in the example here, that the maximal debt level will be greater than or equal to D_j^0 for all but one state.

[34] See DeMarzo and He (2016) for the development of the model in a general continuous-time framework, as well as a methodology for characterizing equilibrium.
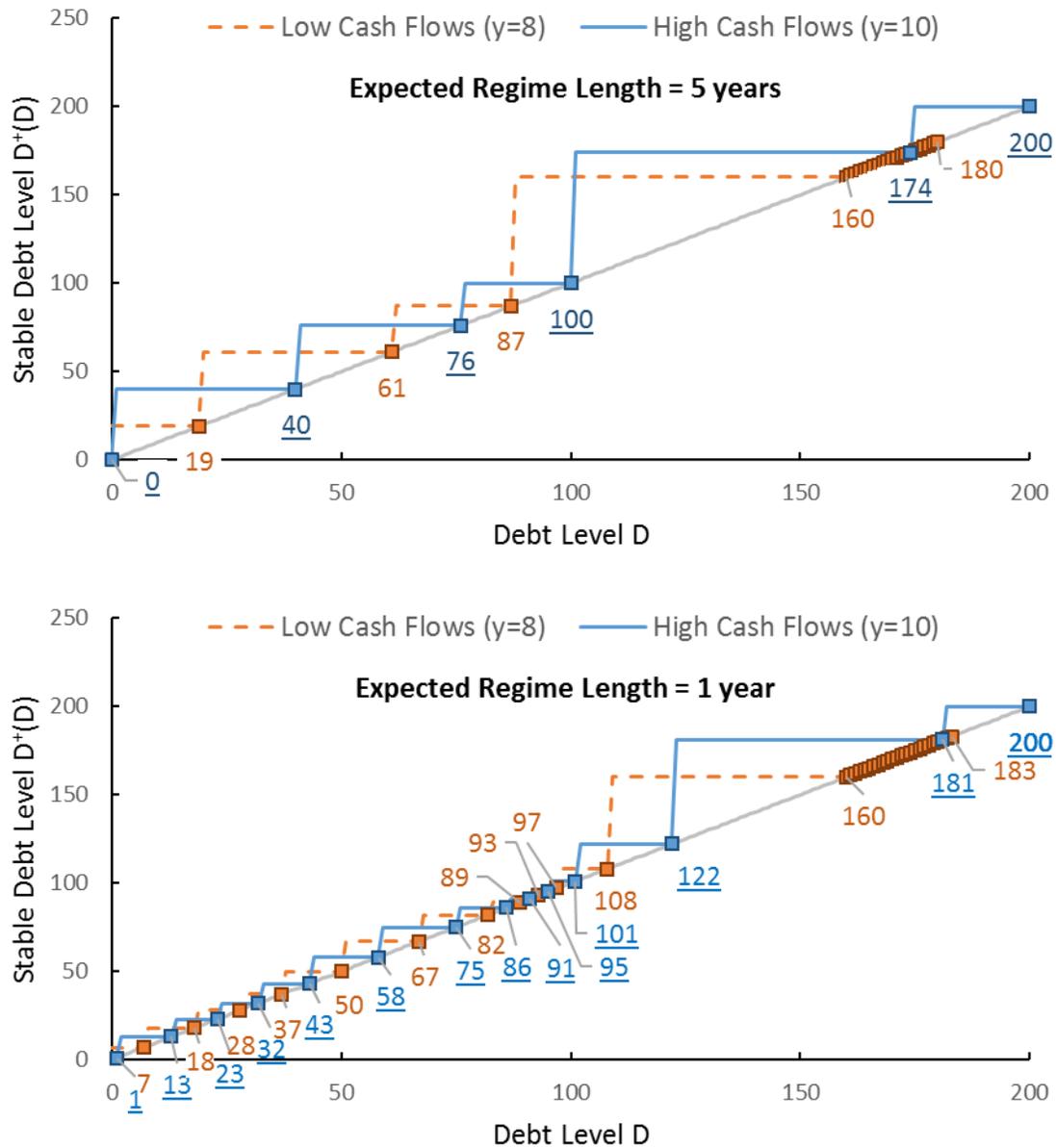
Figure 4: Leverage Ratchet Dynamics with Regime Shocks

In a changing environment, leverage ratchets up to the next stable point with each regime shock. As the arrival rate of shocks increases, the magnitude of the debt increase after each shock declines.

This example confirms the intuition that when there are fluctuations in factors that affect the costs or benefits of leverage, the leverage ratchet effect will induce shareholders to repeatedly "ratchet up" the leverage of the firm. (Similar results are obtained, for example, if the tax rates or default rates fluctuate over time.) Naturally, in anticipation of future ratchets, the equity holders limit the amount of additional leverage they are willing to take on today. Whereas the firm will be initially under-leveraged relative to the value-maximizing debt choice, it will ultimately have excessive leverage, with leverage at any point determined by its cash flow history.

3.4. Countervailing Forces: Covenants, Maturity, and Growth

Our analysis has shown that shareholders of a corporation resist leverage reductions and instead may increase leverage even when doing so reduces the total value of the firm. As the preceding example starkly illustrates, absent countervailing forces, the resulting leverage ratchet effect may lead initial leverage to be low — because the cost of debt reflects the anticipated future inefficiencies due to agency conflicts — but with a gradual transition to ever higher leverage as debt is increased in response to exogenous shocks.

Of course, in practice there are countervailing forces that may prevent leverage from increasing or cause it to decline. First, debt covenants may place some restriction on funding choices by shareholders. Second, finite debt maturity provides an ex ante commitment to retire at least some existing debt. Third, asset growth may raise the value of equity and thereby reduce leverage. We discuss each of these in turn.

Creditors who understand that they may be harmed by the subsequent behavior of shareholders or managers can put in place covenants to try to prevent such behavior. For example, covenants may impose caps or other restrictions on equity payouts or future debt issuance.[35] In the context of the stable leverage equilibria identified in Proposition 5, a cap on the absolute level of debt below the level y/c at which tax shields are exhausted would have the effect of lowering the initial stable debt level D^0, but all subsequent stable debt levels would be computed according to (26) as before. In the simple case without shocks, setting the cap at d^* (derived in Section 3.1) would constrain the firm to the value-maximizing level of debt.

In reality, the optimal debt level changes over time, and covenants are necessarily incomplete and do not cover all possible states of the world. Leaving discretion for shareholders may be useful for allowing the flexibility to respond to opportunities that may benefit all stakeholders. This discretion leaves room for leverage ratchet effects (as well as other agency costs, such as debt overhang or risk shifting). The scope that shareholders have for exploiting their flexibility is especially large if there are many creditors and free-rider problems prevent creditors from being able to respond effectively to such abuses.

Issuing debt with a finite maturity is equivalent to an ex ante commitment by shareholders to repurchase debt at face value and thus reduce leverage at a pre-specified time. In the context of our example in Section 3.3, if some portion of the firm's debt were to mature, leverage would fall and the ratchet would "restart" from a lower debt level. If all of the firm's debt were to mature at the same time, shareholders and managers would again reevaluate leverage from the position of an unleveraged firm.

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[35] Note, however, that the common restriction that requires any new debt to be junior to existing debt is insufficient to prevent the costs associated with the leverage ratchet effect. Because of default costs and agency costs, even the issuance of junior debt can harm existing creditors. Gertner and Scharfstein (1991) also point out that covenant restrictions may be circumvented via exchange offers.

We can see the effect of debt maturity in the context of our current example in a stylized but tractable way by introducing a maturity intensity λ_M that determines the arrival of a maturity "event" at which time the firm is required to repay all of its existing debt (or default).[36] We assume debt is fully prioritized and there are no bankruptcy costs, but there are agency costs in the event of an exit (as in the parameterization of Example 3). We illustrate the resulting equilibrium in Figure 5. In comparison with Figure 4, we can see that while debt maturity leads to periodic reductions in leverage, in equilibrium the firm responds by increasing leverage more aggressively in response to shocks. In this case debt increases to its maximum level of 170 within 6 shocks. Once the debt matures, the firm starts over with new debt of 14 (if cash flows are high) or 41 (if cash flows are low). Intuitively, because creditors anticipate that debt will be periodically reduced through maturity, the impact of the leverage ratchet effect on the price of debt is diminished, which in turn makes issuing debt more attractive to shareholders.[37]
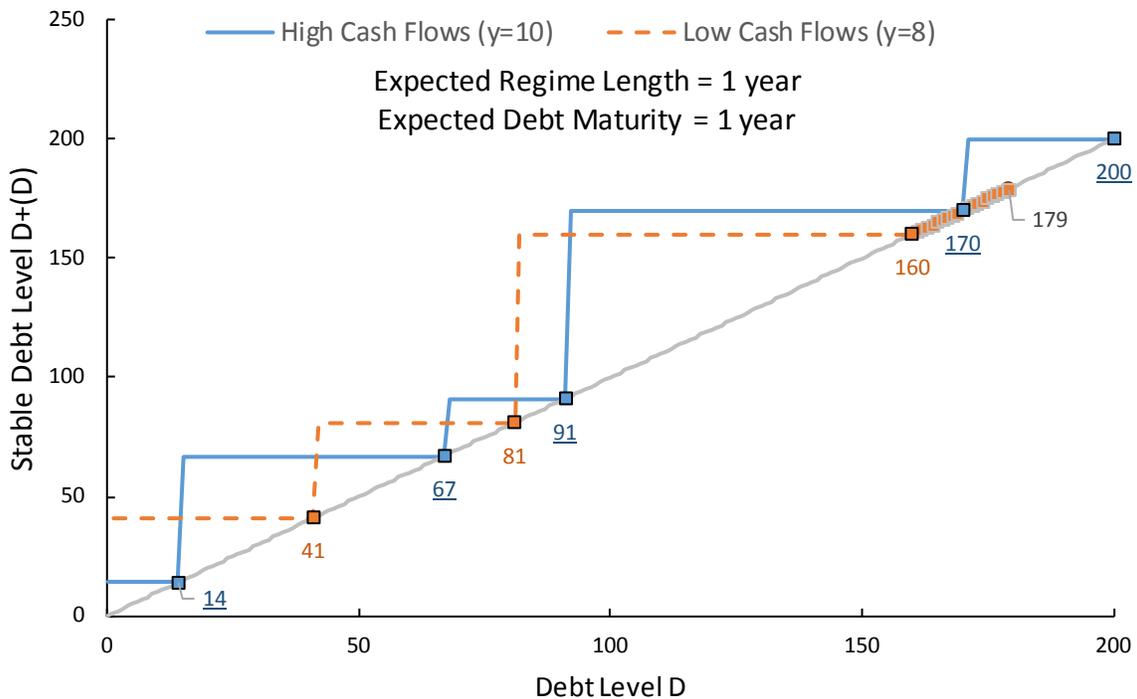
Figure 5: Leverage Ratchet and Debt Maturity
With short-term debt, the firm increases leverage more aggressively prior to maturity.

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[36] Modeling maturity as arriving randomly avoids complicating the analysis with calendar effects. Note that equity holders will default rather than pay the debt if the amount due exceeds the firm's unlevered value $V_j^E(0)$. In that case we assume the value $V_j^E(0)$ is disbursed to creditors according to strict priority.

[37] DeMarzo and He (2016) explore alternative maturity structures in a diffusion model of cash flow growth and demonstrate that shorter debt maturity leads to higher and more rapidly adjusting firm leverage, but that leverage ratchet effects do not disappear even with infinitesimal maturity.

Of course, in this example we have assumed that the debt rollover is perfectly efficient. In practice, relying on short-maturity borrowing, or concentrating debt maturity, exposes the firm to rollover risk or to the risk of a "debt run" which can provoke a costly liquidation of assets or outright default.[38] This risk is particularly high if the firm's assets are illiquid, their value is uncertain, and debt maturities are short. In this case, incomplete information about asset values may induce creditors to run upon the slightest piece of bad news, driving the firm into default and insolvency because the assets can only be liquidated at large discounts, if at all.

Asset growth provides an alternative mechanism by which leverage might decline. If asset values rise, the value of equity grows, which counteracts the ratchet effect. Asset growth can occur simply because the market value of the firm's existing assets increases. Growth may also arise endogenously if new investment opportunities are sufficiently profitable so that shareholders and managers want to exploit them even if they must put up additional equity. (Here, of course, the underinvestment effect of Myers (1977) works in the opposite direction.)

Changes in asset values, however, are not always beneficial. Indeed, the leverage ratchet effect is likely to be most costly in the aftermath of a decline in asset values. In related work, DeMarzo and He (2016) develop a continuous-time model similar to Leland (1998), with finite debt maturity and (potentially endogenous) investment in assets whose cash flows evolve as geometric Brownian motion. They demonstrate that absent commitment, the leverage ratchet effect leads to continuous debt issuance by the firm, at a speed determined by the ratio of the tax benefits to the convexity of the equity value function (analogous to (25)). The countervailing effects of debt maturity and asset growth lead to a mean-reverting leverage policy in which the equilibrium debt level is strongly history dependent and proportional to a weighted-aggregate of the firm's past earnings.[39]

Fundamentally, our results in this paper suggest that standard tests of "tradeoff theory" are likely to fail empirically. Observed levels of leverage will depend on historical choices and their interaction with both covenants and other frictions, with adjustments to leverage driven by the preferences of shareholders. The leverage ratchet effect has the clear implication that firms will respond asymmetrically to shocks that impact leverage choices, such as changes in tax rates. Specifically, increases in the value of the debt tax shield should induce increases in leverage, but reductions in the value of the tax shield would not cause a similar fall in leverage. Such asymmetry has been documented empirically by Heider and Ljungqvist (2015).

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[38] See, e.g. He and Xiong (2012) and He and Milbradt (2014). Diamond and He (2014) also show that short maturity may exacerbate debt overhang.

[39] Specifically, given earnings Y_t, D_t is proportional to $\left(\int_0^t e^{-\rho s} Y_{t-s}^{1/\gamma} ds \right)^\gamma$, where the dependence on past history (determined by ρ) is higher if the debt has longer maturity, volatility is high, or cash flow growth is low. DeMarzo and He (2016) also show that while the debt price is lower, the share price is unaffected by new debt issues, as tax benefits are dissipated via increased agency or default costs.

4. Alternative Ways to Adjust Leverage

In our analysis so far, changes in leverage were accomplished only by pure recapitalizations (buying and selling debt and equity) while the (real) assets of the firm were assumed fixed. In practice, however, changes in leverage are often intertwined with transactions that involve the firm's assets. For example, leverage can be reduced if assets are sold and the proceeds are used to buy back debt or if new equity is issued and the proceeds are used to buy new assets. Conversely, leverage can be increased if the firm issues debt to buy new assets or if it sells assets and makes payouts to shareholders.

This section extends the analysis of Section 2 of shareholder attitudes to *one-time* recapitalizations to include leverage changes that involve asset transactions.[40] In particular, suppose that due to regulations or covenants the firm must reduce leverage by a given amount, or alternatively that the firm's managers, whose interests may not strictly align with shareholders, intend to reduce leverage. We consider which types of transactions to achieve a given leverage change will be most preferred by shareholders. Our results shed light on the effects that existing leverage has on the interaction of investment and funding decisions when decisions involving both assets and liability structure are made to benefit shareholders in the already-indebted firm.

To see how leverage changes can be accomplished in alternative ways, consider a firm that because of covenants or regulations must reduce its ratio of debt to assets from 90% to 80%.[41] Figure 6 shows three different ways in which this can be done. If shareholders are forced to reduce leverage, which mechanism will they prefer?
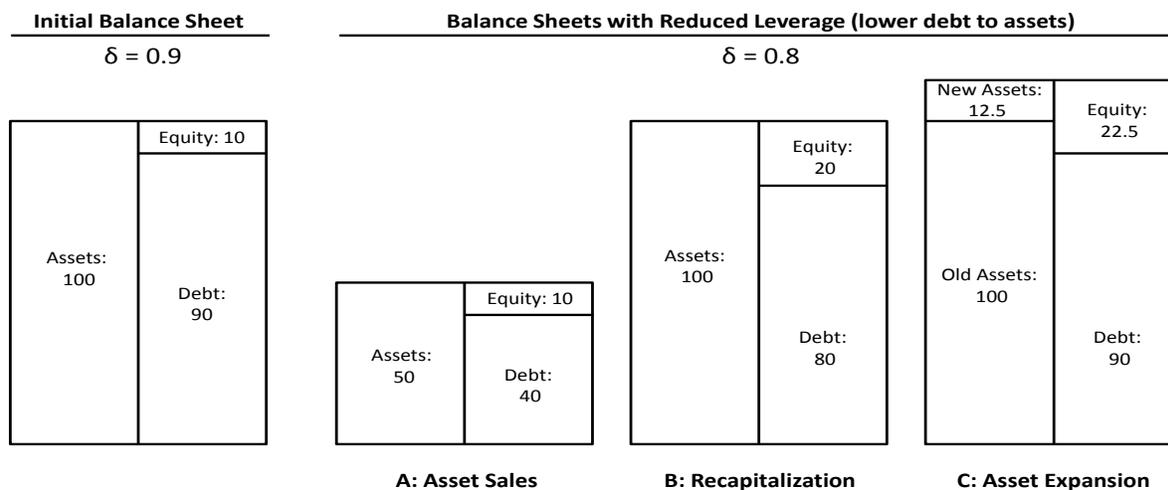
Figure 6: Alternative Methods to Reduce Leverage
If shareholders must reduce leverage to a target debt-to-asset ratio, they may sell assets and repurchase debt (asset sales), issue equity and repurchase debt (recapitalization), or issue equity and purchase assets (asset expansion).

---

[40] Extending this analysis to a fully dynamic equilibrium as in Section 3 is beyond the scope of this paper, though see DeMarzo and He (2016) for a model of the interaction of leverage ratchet and real investment.
[41] Here we are measuring debt in terms of its face or book value, as would be the case in typical covenants.

We begin in Section 4.1 with an equivalence result, showing conditions under which shareholders will be indifferent between all three methods. These conditions will require in particular that (i) the firm has a single class of debt, (ii) asset transactions are zero NPV, and (iii) assets are homogenous. These conditions are clearly highly restrictive, and in the remainder of this section we examine how relaxing these conditions affects shareholder preferences and creates a bias toward asset sales.

4.1. An Equivalence Result

Let A_0 be the current level of assets for the firm (which we earlier normalized to 1) and D_0 be the current face value of debt. The firm's current debt-asset ratio is then $\delta_0 = D_0 / A_0$. Suppose that shareholders are considering changing the debt-asset ratio to δ_1. If the firm can choose any combination of debt and assets (D_1, A_1) satisfying this debt-asset ratio – i.e., such that $D_1 = \delta_1 A_1$ – which combination will shareholders prefer?

If $A_1 \neq A_0$, then assets will be either sold or purchased as part of the leverage reduction. For ease of exposition, we restrict our attention to a compact set C of pairs (D, A) that contains the pure-recapitalization point $(\delta_1 A_0, A_0)$ and assume that, on this set, the technology exhibits constant returns to scale. In particular, we assume that the assets are perfectly homogeneous, so that each unit of the assets today will generate a payoff $\tilde{x}$. We also assume that frictions related to taxes and net bankruptcy costs are proportional to firm size. Specifically, for all (A, D) and $\delta = D / A$,

$$t(xA, D) = t(x, \delta) A \quad \text{and} \quad n(xA, D) = n(x, \delta) A. \tag{30}$$

In addition, we assume that if agency costs exist, they are also proportional to firm size. In particular, letting $V^E(D, A, \theta)$ be the value of equity given assets A, debt D, and actions θ,

$$\theta^* \equiv \arg\max_\theta V^E(D, A, \theta) = \arg\max_\theta V^E(\delta, 1, \theta). \tag{31}$$

Using the expressions for the value of debt and equity in Section 2, we see that when the assets and all frictions are homogeneous, the total value of the firm (equity plus debt) is proportional to its asset holdings and is given by:

$$
\begin{aligned}
V^F(A, D) &= \int_\delta^\infty xA - t(xA, D) \, dF(x, \theta^*) + \int_0^\delta xA - n(xA, D) \, dF(x, \theta^*) \\
&= \left[\int_0^\infty x \, dF(x, \theta^*) - \int_\delta^\infty t(x, \delta) \, dF(x, \theta^*) - \int_0^\delta n(x, \delta) \, dF(x, \theta^*) \right] A \\
&\equiv v(\delta) A.
\end{aligned}
\tag{32}
$$

The homogeneity of the firm's assets also implies that the average price of the firm's debt, which we denote by $q(\delta)$, depends only on the leverage ratio $\delta = D/A$:

$$
\begin{aligned}
q(\delta) &= \frac{V^D(D,A)}{D} \\
&= \int_{D/A}^{\infty} dF(x,\theta^*) + \int_0^{D/A} \frac{xA - n(xA,D)}{D} dF(x,\theta^*) \\
&= \int_{\delta}^{\infty} dF(x,\theta^*) + \frac{1}{\delta} \int_0^{\delta} (x - n(x,\delta)) dF(x,\theta^*).
\end{aligned}
\tag{33}
$$

Recall from Section 2 that if the firm has a single class of debt outstanding, it will be forced to pay the price $q(\delta_1)$ in repurchasing its outstanding debt in the market to lower leverage or will be able to sell additional debt at $q(\delta_1)$ in increasing leverage. The cost of the debt transaction is $q(\delta_1)(D_0 - D_1)$, with the understanding that if this is negative it results in positive proceeds.

Assume that the firm is a price taker in the asset market and the price at which the firm can buy or sell assets is p. It follows that to move from initial balance sheet positions (D_0, A_0) to the new balance sheet positions (D_1, A_1), the value of equity the firm must issue is:

$$
\text{Value of New Equity Issued} = N = p(A_1 - A_0) + q(\delta_1)(D_0 - D_1).
\tag{34}
$$

The total change in the firm's equity value from the transaction is given by:

$$
\text{Change in Total Equity Value} = \nabla V^E = V^E(D_1, A_1) - V^E(D_0, A_0).
\tag{35}
$$

We can therefore determine the effect of the leverage change on existing shareholders by subtracting (34) from (35).

We can now ask whether shareholder losses or gains differ depending on the transactions involved in moving from $D_0/A_0 = \delta_0$ to $D_1/A_1 = \delta_1$. Recall from (13) that

$$
v(\delta_1) = \int_0^{\infty} x\, dF(x,\theta^*) - \int_{\delta_1}^{\infty} t(x,\delta_L)\, dF(x,\theta^*) - \int_0^{\delta_1} n(x,\delta_1)\, dF(x,\theta^*),
\tag{36}
$$

is the expected per-unit payoff of the assets to the firm net of taxes and of (net) default costs at leverage level δ_1. We first consider the case where the market price p at which the firm can buy or sell assets is equal to $v(\delta_1)$ so that asset sales or purchases have zero NPV. In this case, shareholder losses or gains are the same for all transactions that change leverage from δ_0 to δ_1 (conditional on staying in region with constant returns to scale).

Proposition 7 (An Equivalence Result): *Assume that there is only one class of debt, and the firm faces no transactions costs in buying or selling assets or the securities it issues. If $p = v(\delta_1)$, then starting from any initial position (D_0, A_0) with $\delta_0 = D_0 / A_0$, shareholders are indifferent among all transactions that lead to final positions $(D_1, A_1) = (\delta_1 A_1, A_1)$ in the set C on which the technology exhibits constant returns to scale. The change in shareholder wealth is equal to*

$$\left(v(\delta_1) - v(\delta_0)\right) A_0 - \left(q(\delta_1) - q(\delta_0)\right) D_0, \tag{37}$$

and is negative if $\delta_1 < \delta_0$.

Proof: See Appendix. ■

As an immediate corollary, this proposition implies that, under the given conditions, shareholders will resist leverage reductions through asset sales or asset expansion just as they resist leverage reductions through pure recapitalization. In other words, the leverage ratchet effect applies regardless of the mode of leverage adjustment.

The intuition for this result is straightforward. If asset sales or purchases have zero NPV. i.e., $p = v(\delta_1)$, they cannot change the total value of the firm. All changes in shareholder wealth must come from the change in leverage itself, evaluated at the original position (D_0, A_0). The first term represents the efficiency gains from the transaction, determined by the change in v. The second term represents the wealth transfer to creditors, determined by the change in q. The overall effect on shareholders depends then on whether the wealth transfer to the debtholders exceeds the efficiency gains from change in leverage.

While the expression for shareholder losses is natural, it is not immediately obvious that its sign should be negative for leverage reductions. As remarked in Section 2, the standard intuition is that the sign should be positive if the efficiency gains are "large enough." But Proposition 2 guarantees that the former will always dominate, even when the leverage reduction increases firm value ($v(\delta_1) > v(\delta_0)$).

Alternative Asset Prices

If the market price p at which the firm can buy or sell assets is not equal to $v(\delta_1)$, so that the NPV of asset sales or purchases is not equal to zero, the change in shareholder wealth is equal to

$$\nabla V^E - N = \left(v(\delta_1) - v(\delta_0)\right) A_0 - \left(q(\delta_1) - q(\delta_0)\right) D_0 + \left(v(\delta_1) - p\right)\left(A_1 - A_0\right), \tag{38}$$

with an additional term reflecting gains or losses from the NPV of the asset transaction. Indeed, we can interpret (38) as arising from a pure recapitalization, which generates the gains or losses in (37), followed by a pure asset transaction in which equity and debt are issued (or repurchased) proportionally to fund the asset purchase (or disburse the proceeds of an asset sale). Therefore, if

firm changes leverage to δ_1, within the set C on which we have constant returns to scale, shareholders will strictly prefer transactions with higher A_1 if $p < v(\delta_1)$ and with lower A_1 if $p > v(\delta_1)$; in other words, conditional on a given change in leverage, preferences over asset transactions depend solely on NPV.[42] As we will see, if assets or debt are heterogeneous, shareholder preferences will generally be biased and this efficiency result no longer holds.

Feasibility and Limited Liability

If the reduction in leverage is mandated by regulation or covenants, a question that arises is whether the reduction can be achieved without violating the limited liability of shareholders. For the move from (D_0, A_0) to (D_1, A_1) to be compatible with limited liability of existing shareholders, we must have $N \leq V^E(D_1, A_1)$; that is, the amount raised via new equity cannot exceed the market value of the firm's equity after the change – as this value is the maximum value of the claim that can be given to new investors. The following result shows the conditions for which a leverage reduction is feasible via an asset sale or pure recapitalization.

Proposition 8 (Limited Liability and Leverage Reduction): *Assume that there is only one class of debt, and the firm faces no transactions costs in buying or selling assets or the securities it issues. Then a leverage reduction from δ_0 to δ_1 is compatible with limited liability of existing shareholders if and only if $\delta_0 \leq v(\delta_1)/q(\delta_1)$ for a pure recapitalization, or $\delta_0 \leq p/q(\delta_1)$ in the case of an asset sale.*

Proof: See Appendix. ∎

Because $q(\delta_1) \leq 1$, a sufficient condition for the leverage reduction to be feasible is $pA_0 \geq D_0$, which is the conventional condition for assessing the firm to be solvent. Under this condition, a leverage reduction can always be achieved via an asset sale.

4.2. Multiple Classes of Debt

In many settings, the conditions under which Proposition 7 holds are violated, and shareholders have a preference for one mode of leverage change over the others. In this section we show that an important case in which shareholders may have strong preferences about how to reduce leverage is when not all debt in the firm's capital structure has the same priority.

We continue to assume that, over some range C, the asset payoffs and the costs and benefits associated with frictions are perfectly homogenous with firm size. But now suppose the firm has multiple classes of *existing* debt with different levels of priority. In this case, if $D_1 < D_0$, it is

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[42] In these cases, the preferred final position would be outside the set C, presumably at a point, where the marginal asset valuation is equal to p. The finding that the desire to buy or sell assets depends only on the NPV of the transaction remains valid. This benchmark setting differs from Myers (1977) for two reasons. First, we assume the leverage reduction must occur with or without investment. Second we allow the firm to issue debt of equal priority to existing shareholders; we consider the case where new debt must be junior in Section 4.2.

optimal for the firm to repurchase the most junior debt first, as it will be the least expensive. The price at which junior debt can be repurchased depends on the precise capital structure of the firm (as well as any default costs or subsidies). Without going into the details of this dependence, we note that the price q^J at which junior debt can be repurchased must satisfy

$$\Pr(x \geq \delta_1 \mid \theta_1^*) \leq q_1^J < q(\delta_1). \tag{39}$$

where, as above, $\theta_1^* = \arg\max_\theta V^E(\delta_1, 1, \theta)$. The lower bound in (39) reflects the fact that the price of the junior debt should be no less than the probability that the firm does not default, since in that case it will be repaid. The strict inequality for the upper bound follows as long as seniority "matters" in the sense that there exist some states of the world in which junior debt holders have lower recovery rates in default than more senior creditors.

The fact that junior debt is cheaper to repurchase breaks the indifference obtained in Proposition 7. Now, shareholders will be better off the more junior debt that is repurchased. In particular, we have the following important result, which states that with multiple classes of debt, shareholder preferences are biased towards asset sales:

Proposition 9 (Multiple Classes of Existing Debt): *Assume that the firm must reduce leverage from δ_0 to $\delta_1 < \delta_0$, that $p = v(\delta_1)$ and that (39) holds. If the firm can repurchase junior debt, then shareholders find asset sales preferable to a pure recapitalization, which in turn is preferable to an asset expansion.*

Proof: See Appendix. ■

To understand this result, note that the only change relative to Proposition 7 is the price at which the debt is repurchased. There, we assumed debt was reduced from D_0 to D_1 at price $q(\delta_1)$. If instead the debt is repurchased at price $q_1^J < q(\delta_1)$, then relative to (37) shareholders save

$$(q(\delta_1) - q_1^J)(D_0 - D_1). \tag{40}$$

If leverage is reduced via an asset expansion, there is no change in the level of debt ($D_1 = D_0$), and thus the cost of the leverage reduction to shareholders is unchanged from Proposition 7. But with leverage reduced through a recapitalization, the amount of debt is reduced, and so (40) is strictly positive and the cost to shareholders is lower. (We know from Proposition 2, however, that despite this benefit a recapitalization is still costly to shareholders.)

If the firm chooses to reduce leverage using an asset sale, the final debt level will be even lower than in a recapitalization (see Figure 6). As a result, the savings in (40) are even larger, making an asset sale the least costly method for shareholders to reduce leverage.[43] Indeed, we show in the proof of Proposition 9 that *shareholders may actually gain from leverage reduction*

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[43] While this intuition is correct, the proof is complicated by the fact that with multiple classes of debt, the average price at which the junior debt is repurchased may in general be higher in the event of an asset sale. Despite this potential price difference, we show that the net cost to shareholders is always reduced relative to a recapitalization.

achieved through an asset sale if the debt is sufficiently junior and the required reduction in leverage is not too large.

Shareholder bias to deleverage via asset sales can be viewed as another manifestation of the classic underinvestment problem associated with debt overhang identified by Myers (1977). Here we assumed asset sales are zero NPV ($p = v(\delta_1)$), but shareholders would prefer asset sales even if sold at a price somewhat below their fair market value; in other words, shareholders can gain from a "fire sale" even though it reduces total firm value. This result is important in a policy context in which covenants or regulations are designed by creditors or regulators to cap leverage. Setting a target leverage ratio while allowing shareholder discretion regarding which debt securities to repurchase creates a bias towards asset sales, which may be both inefficient for the firm and create additional negative externalities.

Importantly, however, our results demonstrate that this preference arises only if shareholders have the option to repurchase *junior* debt. If covenants or regulations require that senior debt be retired first, the preferences in Proposition 9 would be reversed. More generally, we can consider a full range of transactions in which the firm buys or sells assets and issues or repurchases securities. Letting a be the increase in assets (at price p), and d_i be the increase in debt class i (at price q^i), both of which could be negative if assets are sold or debt is bought back, the gain to equity holders as a result of such a transaction is given by

$$\underbrace{V^E\left(A+a, D+\sum_i d_i\right) - V^E(A,D)}_{\text{change in equity value}} \underbrace{- pa + \sum_i q^i d_i}_{\text{transaction proceeds}}.$$

There are obviously many possible combinations of transactions, and shareholders' preferences among them will depend both on the impact on the redistribution of payoffs among existing and new claimholders and various potential frictions. The following result provides a partial ranking based on transactions involving exchanges of assets or equity for a single class of debt assuming that asset sale and purchases do not by themselves create value.

Proposition 10 (Transaction Ranking): *Assume $p = v(\delta)$, and assets and frictions are homogenous. Consider transactions in which a leveraged firm exchanges assets or equity for debt, where the debt traded is either homogenous (i.e. there is a single class or all classes participate pro rata), or is strictly junior or strictly senior to all other debt. Then, for given small change in leverage δ, shareholder preferences across such transactions are given by* Figure 7.

Proof: See Appendix. ∎

Proposition 9   Indifference (Prop. 7)

Leverage Decreasing δ↓	Sell\Buy:		A\D$_J$	>	E\D$_J$	>	E\D / A\D / E\A	>	E\D$_S$	>	A\D$_S$
	Perfect Market:		+/-		-		-		-		-

Leverage Increasing δ↑	Sell\Buy:	D$_S$\A	>	D$_S$\E	>	D\E / D\A / A\E	>	D$_J$\E	>	D$_J$\A
	Perfect Market:	+		+		+		0		-

Indifference (Prop. 7)   Classic Debt Overhang (Myers 1977)

Figure 7: Shareholder Preferences for Alternative Transactions

Shareholders prefer to reduce leverage by first by selling assets, then by issuing equity to repay junior debt. If debt is homogeneous all transactions are equivalent. Least preferred is selling equity, or worse, assets, to repay senior debt. The ranking is reversed for leverage increasing transactions. Signs indicate whether shareholders gain or lose from these transactions with perfect markets (no taxes, distress, or agency costs); payoffs are not symmetric across reverse transactions due to the holdup problem when repurchasing debt.

Figure 7 shows the relative ranking of shareholder payoffs for a marginal increase or decrease in leverage. Also shown is the sign of the shareholder payoff absent taxes or other frictions; taxes will lower the payoff for leverage decreasing transactions and raise it for leverage increasing ones. Highlighted in the figure are the indifference results of Proposition 7, as well as the results for junior debt in Proposition 9.

With perfect markets, shareholders are indifferent to increasing leverage by issuing junior debt in exchange for equity. Shareholders always lose, however, in the reverse transaction (selling equity and purchasing junior debt). The reason for the asymmetry is the holdup problem shareholders face when repurchasing debt – the price for all claims is the value of the *last* dollar repurchased. A similar asymmetry holds when exchanging assets and junior debt.

We also highlight in Figure 7 the classic result of Myers (1977) that debt overhang implies a cost for shareholders when funding asset purchases with junior securities (equity or junior debt).[44] Note that, absent taxes, this cost is the same whether it is leverage increasing or leverage decreasing. Alternatively, if asset purchases can be financed with senior or pari passu debt, shareholders will gain from the transaction, which may lead to overinvestment.

4.3. Asset Heterogeneity

Proposition 7 treats the firms' assets as homogeneous with returns that are perfectly correlated (i.e. each asset unit has return $\tilde{x}$ so that the total return on all assets is simply $\tilde{x}A$.) In reality, firms have many distinct assets with imperfect correlation and differing risk and return characteristics. In that case, the results of Proposition 7 apply only if any asset sales or purchases

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[44] In Section 2.4, we showed that shareholders gain from issuing junior debt to pay out to equity, which transfers value from senior creditors to shareholders (assuming frictions such as bankruptcy cost). Buying assets, by contrast, protects senior creditors and transfers value from shareholders to senior creditors.

correspond to a "representative portfolio," that is, the firm is selling or buying proportional amounts of each of the firm's current assets.

Of course, given the option, shareholders will generally have preferences with respect to which assets to sell or purchase. If a firm deleverages through asset sales, shareholders prefer to sell relatively safe assets. In contrast, they will prefer to purchase relatively risky assets if the firm expands. This preference is another manifestation of the asset substitution agency problem.

It is natural to expect that if the firm is forced to reduce leverage, covenants or other regulations are likely to limit the firm's ability to make large investments in new assets that significantly increase risk. Firms often have discretion, however, regarding the assets that they sell, as well as the ability to purchase new assets of comparable risk to their existing assets. Even if assets have identical return distributions, as long as they can be traded distinctly, shareholders are again biased toward deleveraging via asset sales.

To illustrate this result, suppose the firm initially holds a portfolio of A_0 distinct assets, where each asset i has return $\tilde{x}_i = \eta + \epsilon_i$, where η is a common factor and ϵ_i are i.i.d. Then the aggregate return of the portfolio is[45]

$$\tilde{x} \equiv \frac{1}{A_0} \sum_{i=1}^{A_0} \tilde{x}_i = \eta + \frac{1}{A_0} \sum_{i=1}^{A_0} \epsilon_i . \tag{41}$$

Although all of the assets are identically distributed, the equivalence result in Proposition 7 applies only when all asset purchases and sales are proportional (i.e. the firm keeps the set of assets fixed and adjusts the quantity held of each by the factor A_1 / A_0). If instead the firm has discretion to trade each asset independently, we have the following result:

Proposition 11 (Heterogeneous Assets): *Suppose the conditions of* Proposition 7 *hold, asset returns are as in* (41) *and the firm can hold up to one unit of each asset. Then if mean-preserving spreads of asset returns increase the value of equity, and if assets can be traded separately, shareholders prefer asset sales to a pure recapitalization, which they in turn prefer to an asset expansion.*

Proof: See Appendix. ∎

The bias toward asset sales arises because selling individual assets reduces diversification and therefore increases overall risk relative to the case when all assets are sold proportionally. Conversely, buying new assets improves diversification compared to a proportional increase in the holdings of the firm's current assets. Reducing diversification via asset sales is preferable to shareholders as a way for shareholders to expose debt holders to more risk.

This bias toward asset sales is only exacerbated if assets differ in terms of their riskiness. In that case, shareholders can gain by selectively sell the safest assets first. As a concrete example, suppose the firm holds a mix of risky assets and safe assets. In particular, suppose it holds quantity

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[45] For ease of notation, we have written have assumed the initial asset quantity A_0 is an integer. Non-integer amounts can be handled by adjusting the quantity of one of the assets.

A_r of risky assets with return $\tilde{x}_r$ and A_s of safe assets with a riskless payoff equal to market price p. Thus the firm has total assets $A = A_r + A_s$ with aggregate return $\tilde{x}$ given by

$$\tilde{x} \equiv \frac{\tilde{x}_r A_r + pA_s}{A}.$$

Suppose the firm considers reducing leverage by selling safe assets and using the proceeds to buyback debt. We then have the following corollary to Proposition 7, showing the equivalence of "selective" asset sales and asset substitution:

Corollary (Asset Sales and Asset Substitution): *Reducing leverage via the sale of safe assets is equivalent, in terms of shareholder payoff, to recapitalizing the firm (to the same leverage ratio) and simultaneously selling safe assets and purchasing risky ones.*

Proof: Suppose the firm first exchanges its holdings a_s of safe assets for risky ones at the market price p, and then sells those risky assets to reduce leverage through an asset sale. This transaction clearly has the same shareholder payoff as simply selling the safe assets directly. But since the firm's assets are homogenous after the asset exchange, by Proposition 7 this has the same shareholder payoff as an asset exchange followed by a pure recapitalization. ∎

4.4. Asymmetric Information

When the firm's managers have private information about the firm's assets and growth opportunities that outside investors do not have, managers will want to sell assets that the market is overvaluing and similarly will want to issue equity if they perceive the market is overpricing the firm's shares. The possibility that managers will make strategic choices based on their private information can account for a significant part of the bid/ask spread for transactions involving the firm's assets and securities, explaining why transactions costs for these are likely to be larger than those associated with debt buybacks.

Asymmetric information that affects the valuation of the firm's assets in an asset sale would also give rise to similar concerns if the firm issues equity directly to recapitalize or expand its assets.[46] If a recapitalization must be done through a new share issuance as opposed to a rights offering, it is not immediately obvious whether it will be more expensive for the firm's shareholders to sell assets and deleverage or to sell equity and recapitalize.

In some circumstances asymmetric information about asset values makes the shareholders indifferent between deleveraging and recapitalizing. Assume that the market undervalues the

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[46] In Myers and Majluf (1984), asymmetric information relates to the value of both assets in place and new investments. A key assumption is that the firm can only raise equity through an offering of common shares and not, for example, through a rights offering. With asymmetric information, the mode of issuance matters. In a sale of new shares, the market's assessment of the firm directly impacts the amount of money raised. In a rights offering, if it succeeds, the market's assessment does not affect the amount of money raised, but only the value of the shares and the rights. The attitude of existing shareholders to a rights offering then depends on whether they are short-term investors, who are interested only in the current share price, or long-term investors à la Myers-Majluf, who care about share prices in the future, when the market will have learned about the underlying values.

firm's assets in the following sense: while managers know that the realized value on the firm's assets will be $\tilde{x}(1+\omega)A$ for $\omega > 0$, the market assumes that the realized value of the assets will only be $\tilde{x}A$. Essentially this means that for each asset unit that the market perceives, the firm effectively has $1+\omega$ units and this difference is perceived by the firm's managers who make decisions about how the firm should change its leverage.

Proposition 12 (Equivalence with Asymmetric Information): *Assume that there is only one class of debt, the firm has homogeneous assets and faces no transactions costs in buying or selling assets or the securities it issues other than that implied by the market's undervaluation of its assets and the firm must decrease its leverage (as measured by outsiders) from δ_H to δ_L. Then for all $\omega \geq 0$, assets sales and pure recapitalization through a common share offering are equally undesirable for shareholders based on the managers' information.*

Proof: See Appendix. ∎

Asymmetric information imposes costs on the current shareholders in both the asset sale and pure recapitalization cases because the firm is selling assets or equity at prices below their values. Although a greater dollar amount of assets must be sold in the asset sales approach than the dollar amount of equity that needs to be issued to effect a recapitalization, the underpricing of equity is larger in percentage terms because of leverage, and this difference is just sufficient to make the loss due to underpricing the same.

Of course, the homogeneous asset condition of the theorem is a knife edge case. Once we allow for asset heterogeneity, shareholders would again benefit from asset sales rather than a recapitalization. Underpricing due to asymmetric information is likely to be lowest for the least risky assets. Even if all assets have equivalent risk as in (41), if there is heterogeneity in the degree of mispricing across assets, individual asset sales are preferable to a pooled sale via equity (see e.g. DeMarzo (2005), Theorem 1). The firm can sell more of its lowest quality assets to reduce the costs of mispricing (i.e. there is no lemon's cost if you are selling lemons). Furthermore, as shown in Section 4.3, assets sales additionally benefit by reducing shareholders by reducing diversification. Thus both mispricing and risk-shifting concerns combine to push shareholders toward deleveraging via asset sales.

Finally, note that one way to reduce leverage that avoids asymmetric information costs is for the firm to retain earnings and build equity "internally." Adverse selection costs can also be eliminated by raising equity through a rights offering. Shareholders resistance to these methods of reducing leverage is entirely due to conflicts of interest related to the leverage ratchet.

5. Concluding Remarks

Our paper's most important message is that the observed funding mixes of corporations are unlikely to be explained by an optimal initial contracting decision followed by a sequence of decisions that maximize the total value of the firm at each date. If shareholders cannot completely commit to subsequent capital structure choices, the evolution of funding over time will depend on

how their incentives play out given contractual constraints and the realization of random events, such as unexpected changes in profits, investment opportunities, and financial frictions.

The evolution of incentives and tradeoffs is itself critically affected by past funding choices, particularly by prior debt issuances. Once debt is in place, shareholders have an aversion to reducing debt and incentives to increase it. These two considerations generate the *leverage ratchet effect*, with funding dynamics that are skewed toward gradual increases in leverage.

If creditors anticipate the potential harm to their interests created by the leverage ratchet effect, they will require higher interest rates ex ante. In consequence, equilibrium leverage may initially be lower than a simple tradeoff analysis with commitment would predict. Over time, the leverage ratchet effect may lead to much higher leverage than a simple tradeoff analysis would predict for the same exogenous conditions.

Our paper presents a major challenge for empirical work. Whereas a static tradeoff analysis proceeds by specifying a tradeoff and explaining observed funding patterns in terms of the parameters of this tradeoff, such an analysis fails to take account of hysteresis effects that make today's tradeoffs depend on past funding decisions, which in turn reflect the joint evolution of asset values and financial frictions that affect the debt cost of funding relative to that associated with equity. A static approach also fails to take account of the behavior of investors who anticipate how shareholders' incentives shape the firm's future funding choices and account for the implications of these choices for their own return prospects.

Our analysis of equilibrium funding dynamics in this paper is rudimentary rather than comprehensive. Whereas we give an encompassing account of the pervasiveness of leverage ratchet incentives, we only present some examples showing the equilibrium dynamics that may emerge. More systematic analyses of the equilibrium dynamics when imperfect commitment affects the interplay of shocks and funding outcomes would generate stronger empirical predictions about the time series and cross-sections of funding choices that we can observe. (DeMarzo and He (2016) provide initial developments in this direction.) Nonetheless, our analysis yields a number of empirical predictions:

- Firms will react more strongly to shocks that make funding by debt more attractive (for firm value) than they react to shocks that reduce the attractiveness of debt (for firm value). In the context of tax rate changes, Heider and Ljungqvist (2015) provide evidence of such asymmetric responses.
- Resistance to reducing leverage is particularly strong for firms in distress, even though for such firms leverage reductions may carry the greatest benefits due to the avoidance of costly default. Once in distress, shareholders resist investing in a recapitalization and prefer to make payouts to themselves ahead of bankruptcy.
- The leverage ratchet effect is particularly strong if debtholders are unable or unwilling to impose and enforce covenants limiting subsequent debt issues that might counter the effect. This condition is particularly relevant if debt is held by many small investors so that the free-rider problems in imposing and enforcing covenants are large, which helps explain why banks – funded by small and dispersed depositors – tend to have much

higher leverage than non-financial firms.[47] The incentives to impose and enforce covenants are also weak when debtholders are protected by explicit or implicit government guarantees, or when selected debtholders are protected by collateral and exempted from bankruptcy process.

- If a corporation is forced to reduce leverage, by covenants or by regulation, shareholders will prefer the form of leverage reduction that provides the most scope to have incumbent senior debtholders bear some of the burden. We show in almost all cases, shareholders are biased toward asset sales, repurchasing subordinated debt funded by sales of relatively safe assets, even at distress (or "fire sale") prices. This prediction is consistent with the behavior of banks and other financial institutions during and since the financial crisis.

Our paper also has important implications for welfare and policy analysis. Whereas in static models starting with an initially unleveraged firm, funding choices that maximize shareholder value also maximize firm value and therefore can be deemed to be constrained-efficient, in dynamic models without full commitment, shareholder value maximization and firm value maximization no longer coincide. Once debt is in place, the external effects that additional funding choices have on incumbent debtholders introduce a wedge between shareholder and firm value maximization. The leverage ratchet effect makes shareholders resist leverage reductions even when they would raise firm value – and, from an ex ante perspective, shareholders would have benefited if they had entered into commitments that would force them to reduce leverage ex post.

Given these inefficiencies arising from imperfect commitment, the question is whether suitable policy measures might improve on outcomes under *laissez-faire* either by strengthening commitment devices or by providing substitutes. For example, minimum equity requirements for banks can be interpreted as an antidote to the leverage ratchet effect in an industry where the effect is particularly strong. When leverage reductions are imposed via covenants or regulation, mandating that they be met via new equity issues may prevent costly fire sales. Additionally, the use of short-term debt commits the firm to reduce leverage via maturity, but may entail other costs (e.g. rollover risk or financial runs). Finally, our paper also highlights the harmful effect of the corporate tax preference of debt funding over equity, which creates incentives to put in place and subsequently increase leverage to levels that are both privately and socially inefficient and excessive.

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[47] Becht et al (2011) note that creditors discipline is particularly weak in banking.

Appendix: Remaining Proofs

Proof of Proposition 3: Note that the expectation in (8) is with respect to the information z. Then, using the same argument as in Proposition 1, holding the policy functions fixed,

$$
V^E(D-d,\theta,a) - V^E(D,\theta,a) =
$$

$$
E_z\left[d \times \left(1 - F\left(\frac{D-d}{1+a(z)}\bigg| z, \theta(z)\right)\right) + \int_{(D-d)/(1+a(z))}^{D/(1+a(z))} \left(x(1+a(z)) - D\right) dF\left(x|z,\theta(z)\right)\right.
$$

$$
- \int_{(D-d)/(1+a(z))}^{\infty} t\left(x(1+a(z)), D-d\right) dF\left(x|z,\theta(z)\right)
$$

$$
\left. + \int_{D/(1+a(z))}^{\infty} t\left(x(1+a(z)), D\right) dF\left(x|z,\theta(z)\right)\right]
\tag{42}
$$

$$
< E_z\left[d \times \left(1 - F\left(\frac{D-d}{1+a(z)}\bigg| z, \theta(z)\right)\right)\right]
$$

$$
= d \times \Pr\left[\tilde{x}(1+a(z)) > D-d \mid \theta^*(z)\right]
$$

As in Proposition 1, the inequality follows because shareholders forfeit their default option for final asset values between $D-d$ and D, and have a higher expected tax burden. The last equality states that the increase in the value of equity per dollar of debt repurchased is less than the ex-ante probability of no default at the lower level of leverage.

The proof then follows using exactly the same argument as in (10). Let θ^* and a^* be the optimal risk and investment policy functions for equity holders given debt $D-d$:

$$
V^E(D-d) = \max_{\theta,a} V^E(D-d,\theta,a) = V^E(D-d,\theta^*,a^*)
\tag{43}
$$

Then,

$$
V^E(D-d) - V^E(D) = V^E\left(D-d,\theta^*,a^*\right) - \max_{\theta,a} V^E(D,\theta,a)
$$

$$
\leq V^E\left(D-d,\theta^*,a^*\right) - V^E\left(D,\theta^*,a^*\right)
$$

$$
< d \times \Pr\left[\tilde{x}_{\theta^*(z)}(1+a^*(z)) > D-d\right]
\tag{44}
$$

$$
\leq d \times q^D(D-d)
$$

The first inequality follows since we have fixed the investment policy functions at a level that may not be optimal with higher leverage (due to agency costs), the second follows from above, and the third follows since the repurchase price of the debt will be at least the no default probability (and will be strictly higher if the debt has a non-zero recovery rate in any default states). ∎

Proof of Proposition 4: Note that $q^J(0,D) = q(D)$, and therefore $Dq^J(0,D) = V^D(D)$. That is, proceeds from issuing debt are equal to the total value of the firm's debt. Hence,

$$
G(0,D) = V^E(D) - V^E(0) + Dq^J(0,D) = V^E(D) + V^D(D) - V^E(0)
\tag{45}
$$

Thus, D maximizes $G(0,D)$ if and only if it maximizes total firm value.

For the second result, note that our earlier results already establish that shareholders lose if the firm reduces debt ($D' < D$) regardless of the seniority of the debt that is repurchased. Therefore, it is enough to establish that the marginal benefit of an increase in leverage from its current level is positive. Specifically, we need to show the right-hand derivative of G at $D' = D$,

$$\left.\frac{\partial G(D,D')}{\partial D'}\right|_{D'=D_+} = \left.\frac{\partial\left(V^E(D')+(D'-D)q^J(D,D')\right)}{\partial D'}\right|_{D'=D_+}, \tag{46}$$

is positive. Let θ^* be the optimal risk choice with debt level D. From the definition of V^E, and using the fact that holding the risk choice fixed at θ^* only reduces the gain to equity holders, we have

$$\left.\frac{\partial V^E(D')}{\partial D'}\right|_{D'=D_+} \geq \left.\frac{\partial}{\partial D'}\left[\int_{D'}^{\infty}\left(x-t(x,D')-D'\right)\,\mathrm{d}F\left(x|\theta^*\right)\right]\right|_{D'=D_+}$$

$$= -\int_{D_+}^{\infty}\mathrm{d}F\left(x|\theta^*\right) - \int_{D_+}^{\infty}t_D\left(x,D\right)\mathrm{d}F\left(x|\theta^*\right)$$

$$> -\int_{D_+}^{\infty}\mathrm{d}F\left(x|\theta^*\right) = -\Pr\left(\tilde{x}>D|\theta^*\right)$$

where the final inequality follows from the assumption that tax benefits are positive.

Next, for $D'\geq D$, define $\pi(D')$ to be the proceeds raised from the new debt:

$$\pi(D') \equiv (D'-D)q^J(D,D') = (D'-D)\int_{D'}^{\infty}\mathrm{d}F\left(x|\theta'\right) + \int_0^{D'}\left(x-n(x,D')-D\right)^+\,\mathrm{d}F\left(x|\theta'\right).$$

Then let $\hat{\pi}(D') \equiv (D'-D)\int_{D'}^{\infty}\mathrm{d}F\left(x|\theta'\right)$. Because $\pi(D)=\hat{\pi}(D)=0$ and $\pi(D')\geq\hat{\pi}(D')$ for $D'>D$, we have

$$\pi'(D_+) \geq \hat{\pi}'(D_+) = \lim_{\varepsilon\downarrow 0}\frac{\hat{\pi}(D+\varepsilon)-\hat{\pi}(D)}{\varepsilon} = \lim_{D'\downarrow D}\int_{D'}^{\infty}\mathrm{d}F\left(x|\theta'\right) = \lim_{D'\downarrow D}\Pr\left(\tilde{x}>D'|\theta'\right).$$

That is, the marginal price per dollar of junior debt is at least the probability of no default (and could be higher in the presence of default subsidies). Thus we have shown

$$\left.\frac{\partial G(D,D')}{\partial D'}\right|_{D'=D_+} > \lim_{D'\downarrow D}\Pr\left(\tilde{x}>D'|\theta'\right) - \Pr\left(\tilde{x}>D|\theta^*\right) = 0 \tag{47}$$

where for the final equality we use the fact that the probability of default is continuous at D. ∎

Proof of Lemma: For the pure bankruptcy cost model, define the distribution for X such that

$$\Pr(X\leq D) = 1+\phi'(D) = 1-\Pr(X_{\theta(D)}>D) = Pr(X_{\theta(D)}\leq D).$$

Then we have $\phi^{BC}{}'(D) = -\Pr(X>D) = -Pr(X_{\theta(D)}\leq D) = \phi'(D)$. Because the derivatives match and they share the same limit, we have $\phi^{BC} = \phi$.

For the pure moral hazard model, define $\theta(D) = -\phi'(D)$. Next define g as $g(\theta(D)) \equiv D+\dfrac{\phi(D)}{\theta(D)}$ on the range of $\theta(D)$ and zero elsewhere. Then we can rewrite the shareholders' optimization as

$$\phi^{AC}(D) = \max_{\theta}\theta(g(\theta)-D)^+ = \max_{\hat{D}}\theta(\hat{D})(g(\theta(\hat{D}))-D)^+$$

Now

$$\theta(\hat{D})(g(\theta(\hat{D}))-D) = \theta(\hat{D})(\hat{D}-D)+\phi(\hat{D}) = \phi(\hat{D})+\phi'(\hat{D})(D-\hat{D}) \leq \phi(D)$$

where the last inequality follows from the convexity of ϕ. Hence we have $\phi^{AC} = \phi$. ∎

Proof of Proposition 5: To verify an equilibrium, note first that given the equilibrium leverage strategy, the debt pricing is rational for creditors since the firm is expected to maintain leverage permanently at $D^n = D^+(D)$. Next note that if $D < D^n = D^+(D)$, then $G(D, D^n) > 0$ from (26). Thus, shareholders gain from increasing debt to D^n. Moreover, it is suboptimal to delay this increase in debt, as it would delay earning the gain $G(D, D^n)$.

Finally, we need to establish that shareholders would not prefer some alternative debt choice or sequence of choices. From the prior argument, it is sufficient to consider only changes to some other stable point $D^m \neq D^n$. Note that for $D^m < D$, $G(D, D^m) < 0$ since shareholders both lose tax benefits and bear (via the debt price) incremental agency or bankruptcy costs when buying back debt. For $D \leq D^n < D^m$, note that because $q(D^m) \leq q(D^n)$,

$$G(D, D^m) \leq G(D, D^n) + G(D^n, D^m) \leq G(D, D^n),$$

where the last inequality follows since $G(D^n, D^{n-1}) \leq 0$ by (26). ∎

Proof of Proposition 6: Let $V_j^E(D)$ and $q_j(D)$ be the payoff to equity and the price of debt if the firm has stable debt D until the next shock arrives, and let λ_j be the total arrival rate $\sum_k \lambda_{jk}$. If the firm enters regime k with debt D, let $D_k^+(D)$ denote the next stable debt level in regime k, as in (29). Then upon entering regime k, the value of equity and the debt price will be

$$\hat{V}_k^E(D) = V_k^E\left(D_k^+(D)\right) + \left(D_k^+(D) - D\right) q_k\left(D_k^+(D)\right), \text{ and } \hat{q}_k(D) = q_k\left(D_k^+(D)\right) \tag{48}$$

where

$$V_j^E(D) = \left(\frac{(y_j - c_j D) - t_j(y_j - c_j D)^+ + \lambda_{j0}\phi_j(D) + \sum_{k>0}\lambda_{jk}\hat{V}_k^E(D)}{r_j + \lambda_j}\right)^+, \text{ and} \tag{49}$$

$$q_j(D) = \frac{c_j - \lambda_{j0}\phi_j{'}(D) + \sum_{k>0}\lambda_{jk}\hat{q}_k(D)}{r_j + \lambda_j} \times 1_{D \leq \bar{D}^j}.$$

Note that in (49), we account for equity's option to default when $D > \bar{D}_j$, where $\bar{D}_j = \sup\{D : V_j^E(D) > 0\}$. By the identical logic as Proposition 5, the stable points are defined by

$$D_j^{n+1} = \max\{D < D_j^n : G_j(D, D_j^n) \leq 0\}, \tag{50}$$

where $$G_j(D, D') \equiv V_j^E(D') - V_j^E(D) + q_j(D')(D' - D). \tag{51}$$

Note that we can calculate the equilibrium value function and stable points via backward induction on the debt level D, beginning from debt level $D^0 \equiv \max_j D_j^0$. Once $D = D^0$, there will be no further increases in debt, and the system (48) and (49) can be solved using standard methods. ∎

Proof of Proposition 7: After the change, the total value of equity will be:

$$V^E\left(A_1, D_1\right) = v(\delta_1) A_1 - q(\delta_1) D_1. \tag{52}$$

Therefore,

$$\nabla V^E = \left(v(\delta_1) A_1 - q(\delta_1) D_1\right) - \left(v(\delta_0) A_0 - q(\delta_0) D_0\right). \tag{53}$$

Thus, the total change in value for existing shareholders is

$$\nabla V^E - N = \left(v(\delta_1)A_1 - q(\delta_1)D_1\right) - \left(v(\delta_0)A_0 - q(\delta_0)D_0\right)$$
$$- p(A_1 - A_0) - q(\delta_1)(D_0 - D_1)$$
$$= \left(v(\delta_1) - p\right)A_1 - \left(q(\delta_1) - q(\delta_0)\right)D_0 - \left(v(\delta_0) - p\right)A_0 \tag{54}$$
$$= -\left(q(\delta_1) - q(\delta_0)\right)D_0 - \left(v(\delta_0) - v(\delta_1)\right)A_0.$$

Since this does not depend on either A_1 or D_1, it is the same for all changes that lead to a given reduction in the leverage ratio. We know already from our results in Section 2 that shareholder losses are positive for a pure recapitalization, so they must be positive for any method, proving the result. ∎

Proof of Proposition 8: Compatibility with limited liability of shareholders requires that

$$V^E(A_1, D_1) = v(\delta_1)A_1 - q(\delta_1)D_1 \geq N = p(A_1 - A_0) + q(\delta_1)(D_0 - D_1),$$

which is equivalent to $v(\delta_1)(A_0 + A_1 - A_0) \geq p(A_1 - A_0) + q(\delta_1)D_0$, or,

$$v(\delta_1) \geq q(\delta_1)\delta_0 + \left(p - v(\delta_1)\right)\left(\frac{A_1 - A_0}{A_0}\right) \tag{55}$$

which leads to the condition $v(\delta_1) \geq q(\delta_1)\delta_0$ when $p = v(\delta_1)$. If $p \neq v(\delta_1)$, then (55) is relaxed given shareholder's preferred choice of A_1. For a pure asset sale, we also need to check that the firm can deleverage to new level $\delta_1 < \delta_0$ without needing to raise new equity; that is, there exists $A_1 \in [0, A_0]$ such that $p \times (A_0 - A_1) = q(\delta_1)(D_0 - \delta_1 A_1)$. Solving for A_1 we have

$$A_1 = \frac{\left(p - q(\delta_1)\delta_0\right)}{\left(p - q(\delta_1)\delta_1\right)} A_0, \tag{56}$$

which is in the range $[0, A_0]$ if and only if $p = v(\delta_1) \geq q(\delta_1)\delta_0$. ∎

Proof of Proposition 9: As before we have

$$\nabla V^E = \left(v(\delta_1)A_1 - q(\delta_1)D_1\right) - \left(v(\delta_0)A_0 - q(\delta_0)D_0\right),$$

but given the lower cost q_1^J of repurchasing the junior debt, the total value of equity issued is

$$N^J = p \times (A_1 - A_0) + q_1^J \times (D_0 - D_1) = N - \left(q(\delta_1) - q_1^J\right)(D_0 - D_1),$$

and therefore the change in value for existing shareholders is:

$$\nabla V^E - N^J = \nabla V^E - N + \left(q(\delta_1) - q_1^J\right)(D_0 - D_1)$$
$$= -\left(q(\delta_1) - q(\delta_0)\right)D_0 - \left(v(\delta_0) - v(\delta_1)\right)A_0 + \left(q(\delta_1) - q_1^J\right)(D_0 - D_1) \tag{57}$$

For a pure asset expansion, we have $D_0 = D_1$, and thus the loss to shareholders in (57) is identical to that in the case of a single debt class. However, this loss is reduced with a recapitalization or asset sale, since then $\left(q(\delta_1) - q_1^J\right)(D_0 - D_1) > 0$. But while shareholders' losses are smaller in a recapitalization, we know from Proposition 2 that shareholders still lose even if they can repurchase the junior debt at the minimal price in (39).

Next we show that asset sales are preferable to a recapitalization. Let D_{1R} be the debt remaining after a recapitalization and D_{1A} be the debt remaining after an asset sale, and note that $D_0 > D_{1R} > D_{1A}$. Let q_{1R}^J be the average repurchase price of the junior debt $(D_0 - D_{1R})$ in a recapitalization. Therefore, relative to an asset expansion, shareholders gain

$$\left(q(\delta_1) - q_{1R}^J\right)\left(D_0 - D_{1R}\right)$$

from a recapitalization. For the asset sale, let q_{1A}^{J1} be the average repurchase price of the most junior $(D_0 - D_{1R})$ of debt, and q_{1A}^{J2} be the average repurchase price of the remaining $(D_{1R} - D_{1A})$ of "mezzanine" debt. Then the gain relative to an asset expansion for an asset sale can be written as

$$\left(q(\delta_1) - q_{1A}^{J1}\right)\left(D_0 - D_{1R}\right) + \left(q(\delta_1) - q_{1A}^{J2}\right)\left(D_{1R} - D_{1A}\right).$$

Comparing the two, we see that the gain for an asset sale is larger because $q_{1A}^{J2} \le q(\delta_1)$ (mezzanine debt has weakly lower priority to the remaining senior debt) and $q_{1A}^{J1} \le q_{1R}^J$ (since the most junior tranche has even lower priority relative to the remaining senior debt in an asset sale), and moreover at least one of the inequalities must be strict given at least two classes of debt.

Finally, we show that shareholders *may* gain with asset sales if they can repurchase junior debt – it is sufficient to consider the case with no frictions. In that case, given assets A and debt $D(A)$, the value of equity is

$$V^E = \int_{D(A)/A}^{\infty} \left(xA - D(A)\right) dF(x)$$

With perfect markets, assets can be sold for $p = E[x]$, with the proceeds used to purchase debt with face value p/q^J. Therefore, $D'(A) = p/q^J$ and so,

$$\frac{d}{dA}V^E = \int_{\delta}^{\infty}\left(x - D'\right)dF(x) = \int_{\delta}^{\infty} x\,dF(x) - D'\int_{\delta}^{\infty} dF(x) = \left(E[x \mid x > \delta] - \frac{E[x]}{q^J}\right)\Pr(x > \delta)$$

Equity holders gain from asset sales if the above expression is negative, or equivalently,

$$q^J E[x \mid x > \delta] < E[x]. \tag{58}$$

Now, the value of the junior debt can be written

$$q^J = \int_{\delta}^{\infty} dF(x) + \alpha\int_{0}^{\delta} x/\delta\, dF(x)$$

where $\alpha \in [0,1]$ is the expected recovery rate of the junior debt relative to average recovery rate of the firm's debt (which is strictly positive given our assumption that F has full support). If the debt is fully prioritized so that all debt repurchased is junior to any debt retained, then $\alpha = 0$, whereas if the debt is all of one class, then $\alpha = 1$ and $q^J = q$. Substituting this value for q^J in (58) we get

$$\left(\int_{\delta}^{\infty} dF(x) + \alpha\int_{0}^{\delta} x/\delta\, dF(x)\right)E[x \mid x > \delta] = \int_{\delta}^{\infty} x\,dF(x) + \alpha\int_{0}^{\delta} x\,dF(x)\frac{E[x \mid x > \delta]}{\delta} < \int_{0}^{\infty} x\,dF(x) \tag{59}$$

Simplifying, we see that (59) holds and shareholders can gain from an asset sale if the debt repurchased is sufficiently junior so that its relative recovery rate satisfies $\alpha < \delta / E[x \mid x > \delta]$. ∎

Proof of Proposition 10: For the case of leverage reductions, the only case left to prove is when debt repurchased must be senior, in which case the total value of equity issued is

$$N^S = p \times (A_1 - A_0) + q_1^S \times (D_0 - D_1) = N - \left(q(\delta_1) - q_1^S\right)(D_0 - D_1),$$

where $q_1^S > q(\delta_1)$ is the price of senior debt. The change in value for existing shareholders is:

$$\nabla V^E - N^S = \nabla V^E - N + \left(q(\delta_1) - q_1^S\right)(D_0 - D_1)$$
$$= -\left(q(\delta_1) - q(\delta_0)\right)D_0 - \left(v(\delta_0) - v(\delta_1)\right)A_0 + \left(q(\delta_1) - q_1^S\right)(D_0 - D_1). \tag{60}$$

For a recapitalization or asset sale, $D_0 > D_1$, and so relative to an asset expansion (in which debt is unchanged), shareholder losses increase by $\left(q_1^S - q(\delta_1)\right)(D_0 - D_1) > 0$. By a similar argument as in the proof of Proposition 9, because the amount of debt repurchased is larger in an asset sale, the total loss from an asset sale exceeds that from a recapitalization.

The ranking for leverage increases follows by a symmetric argument, with signs reversed. When selling junior debt rather than repurchasing it, however, note that the firm will receive the average value of the new debt issued rather than repurchasing it at the marginal value of the last dollar repurchased. As a result, with perfect markets, selling strictly junior debt and repurchasing equity has zero value to shareholders, while the reverse transaction (selling equity and purchasing junior debt) entails a strict loss. ∎

Proof of Proposition 11: Any portfolio with between 0 and 1 unit of each asset can be constructed as a convex combination of portfolios with only 0 or 1 unit of each asset. Because asset returns are exchangeable, Jensen's inequality implies that portfolios with only 0 or 1 unit of each asset maximize risk for any level of holdings. Thus, given any A_1, it is optimal for shareholders to hold one unit each of A_1 assets. Comparing the case in which the firm holds one unit each of A_0 assets with the case in which it holds one unit each of A_1 assets, if $A_1 > (<)A_0$, the new portfolio is less (more) risky than if the holdings in the initial assets had be rescaled proportionally instead, as assumed in Proposition 7. Thus, rather than be indifferent, shareholders will be worse off with an asset purchase and better off with an asset sale. ∎

Proof of Proposition 12: Let $q(\delta)$ be the market value of a unit of debt (face value is equal to 1) when the perceived leverage is δ and let $e(\delta)pA$ be the *total* market value of equity when the market value of assets is equal pA and the (perceived) leverage is δ. In a recapitalization the firm must issue equity sufficient to buy back Δ_D units of debt so that

$$\frac{D - \Delta_D}{pA} = \delta_L, \text{ or } D - \Delta_D = p\delta_L A \tag{61}$$

The true value of current equity holders' claim after recapitalization will be:

$$\left(1 - \frac{q(\delta_L)\Delta_D}{pA - q(\delta_L)(D - \Delta_D)}\right)e\left(\frac{\delta_L}{1+\omega}\right)p(1+\omega)A \tag{62}$$

The *total* value of equity after the leverage reduction (from the perspective of the informed insiders) is $e\left(\delta^{True}\right)p(1+\omega)A$ where $\delta^{True} = \delta_L/(1+\omega)$ and $p(1+\omega)A$ is the managers' assessment of the value of the assets. Note that true leverage as perceived by the managers is less than the market perceived leverage since the market is undervaluing the assets. The fraction of the total equity claim retained by current shareholders is based on the amount that must be raised through issuing equity to buy back the debt, i.e., $q(\delta_L)\Delta_D$, and the market's valuation of equity after the recapitalization, i.e., $pA - q(\delta_L)(D - \Delta_D)$.

Substituting (61) into (62), we have

$$\left(1 - \frac{q(\delta_L)\Delta_D}{pA - q(\delta_L)(D - \Delta_D)}\right)e\left(\frac{\delta_L}{1+\omega}\right)p(1+\omega)A = \left(\frac{pA - q(\delta_L)D}{pA - q(\delta_L)(D - \Delta_D)}\right)e\left(\frac{\delta_L}{1+\omega}\right)p(1+\omega)A$$
$$= \left(\frac{pA - q(\delta_L)D}{pA - q(\delta_L)pA\delta_L}\right)e\left(\frac{\delta_L}{1+\omega}\right)p(1+\omega)A = \left(\frac{pA - q(\delta_L)D}{1 - q(\delta_L)\delta_L}\right)e\left(\frac{\delta_L}{1+\omega}\right)(1+\omega) \tag{63}$$

In reducing leverage through assets sales the amount of debt bought back must solve:

$$\frac{D-\Delta_D}{pA-q(\delta_L)\Delta_D} = \delta_L \quad \text{or} \quad \Delta_D = \frac{D-pA\delta_L}{1-q(\delta_L)\delta_L} \tag{64}$$

Since $A - q(\delta_L)\Delta_D / p$ will be the new level of assets after the deleveraging is completed, the value of the equity claim after the asset sales is:

$$e\left(\frac{\delta_L}{1+\omega}\right)p(1+\omega)\left(A - \frac{q(\delta_L)\Delta_D}{p}\right) \tag{65}$$

Using (64), we find that the new level of assets will be:

$$\left(A - \frac{q(\delta_L)}{p}\left(\frac{D-pA\delta_L}{1-q(\delta_L)\delta_L}\right)\right) = \left(\frac{pA-pAq(\delta_L)\delta_L-q(\delta_L)D+pAq(\delta_L)\delta_L}{p-pq(\delta_L)\delta_L}\right) = \left(\frac{pA-q(\delta_L)D}{p-pq(\delta_L)\delta_L}\right) \tag{66}$$

This means that (65) becomes

$$e\left(\frac{\delta_L}{1+\omega}\right)(1+\omega)\left(\frac{pA-q(\delta_L)D}{1-q(\delta_L)\delta_L}\right) \tag{67}$$

Since this is precisely equal to (63), the shareholders are indifferent between recapitalization and asset sales.

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