Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?

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Discussion by
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INTRODUCTION

- Paper asks if models with incomplete markets help explaining the behavior of exchange rates, and specifically
  - Low volatility of exchange rates relative to other asset prices
  - Deviations from uncovered interest rate parity
  - Low correlation between exchange rates and economic "fundamentals"
- The contribution is to develop an approach to address this question
  - Take stochastic properties of SDF as given
  - Incomplete markets modeled as a "wedge"
  - Characterize restrictions on the wedge due to trading in risk-free bonds
- Paper finds that the wedge, *per se*, cannot do much

Overview of discussion

Very useful and clean exercise. It should be thought to PhD's students

1. Overview of the paper
2. Two main comments
   - Incomplete markets and the SDF
   - A more formal test?

Complete Markets and the Exchange Rate Puzzles

Under complete markets, and no trading restrictions across countries:

$$M^*_{t+1} = M_{t+1}\frac{S_{t+1}}{S_t}$$

Then, we have:

- $\mathrm{var}_t(\Delta s_{t+1}) = \mathrm{var}_t(m^*_{t+1}) + \mathrm{var}_t(m_{t+1}) - 2\mathrm{cov}_t(m^*_{t+1}, m_{t+1})$

  Volatility puzzle: For plausible values of sdf variances, we have that $\mathrm{var}_t(\Delta s_{t+1}) >> \text{data}$, unless m_{t+1} and m^*_{t+1} highly correlated

- $r^*_t - \left(r_t - \Delta s_{t+1}\right) = \frac{1}{2}\left[\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^*_{t+1})\right]$

  UIP puzzle: Hard to generate sizable deviations from UIP

- $\frac{\mathrm{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})}{\mathrm{var}_t(\Delta s_{t+1})} = 1$

  Backus-Smith puzzle: Δs_{t+1} orthogonal to $\Delta c^*_{t+1} - \Delta c_{t+1}$

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The Incomplete Markets Wedge

With incomplete markets we have:

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TRADING IN RISK-FREE BONDS

Assuming that risk-free bonds are freely traded across countries, we have

$$\mathbb{E}_t[M^*_{t+1}R^\star_t] = \mathbb{E}_t\left[M_{t+1}\frac{S_{t+1}}{S_t}R^\star_t\right] = 1 \quad \mathbb{E}_t[M_{t+1}R_t] = \mathbb{E}_t\left[M^\star_{t+1}\frac{S_t}{S_{t+1}}R_t\right] = 1$$

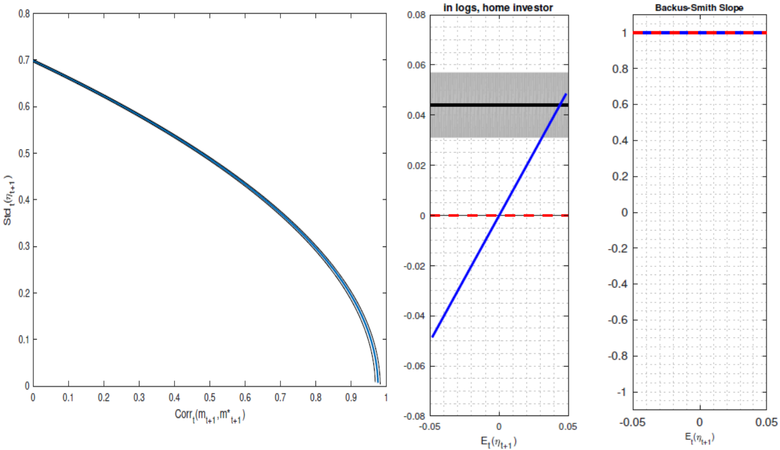
This generates restrictions on $\{\eta_{t+1}\}$. Specifically, we must have

$$\text{cov}_t(m^*_{t+1} - m_{t+1}, \eta_{t+1}) = -\text{var}_t(\eta_t),$$

implying

- $\text{var}_t(\Delta s_{t+1}) = \text{var}_t(m^*_{t+1} - m_{t+1}) - \text{var}_t(\eta_{t+1})$
- $r^*_t - \left(r_t - \Delta s_{t+1}\right) = \frac{1}{2}\left[\text{var}_t(m_{t+1}) - \text{var}_t(m^*_{t+1})\right] + \mathbb{E}_t[\eta_{t+1}]$
- $\frac{\text{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})}{\text{var}_t(\Delta s_{t+1})} = 1$

Incomplete Markets and the Puzzles



Main result: incomplete spanning might help in addressing volatility and UIP puzzle, but it does not help with Backus-Smith puzzle

Comment 1: Incomplete markets and the SDF

- The paper takes $\{m_{t+1}, m^\star_{t+1}\}$ as given in the exercise. But market incompleteness modifies the properties of $\{m_{t+1}, m^\star_{t+1}\}$

- In models with segmented and incomplete markets, stochastic discount factors are typically functions of the leverage of "experts"

- A growing literature documents that leverage-based SDF outperforms consumption-based SDF in explaining asset prices (Adrian, Etula and Muir, 2016; Bocola, 2016)

- Would be interesting to know if this holds true for currencies. For example, how does the Backus-Smith slope looks in the data when using a leverage-based pricing kernel?

Comment 2: A more formal test?

- Paper suggests that incomplete markets do not help much fitting the behavior of exchange rates
  - Backus-Smith puzzle
  - Need $\mathbb{E}_t[\eta_{t+1}] > 0$ to deal with UIP puzzle. But this introduces predictability in exchange rates changes

- It would be nice to have a more formal test. For example, one can use

$$\Delta s_{t+1} = m^{\star}_{t+1} - m_{t+1} + \eta_{t+1}$$

  as a measurement equation and compare marginal data densities of the incomplete market model and the model with $\eta_{t+1} = 0$

- This would give a more precise answer to the authors' question

CONCLUSION

- Very nice paper
- Two comments/questions
  - How to think about the implications of incomplete markets for $\{m^*_{t+1}, m_{t+1}\}$?
  - A more formal test of the hypothesis?