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Exchange Rate Policies at the Zero Lower Bound

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We study the problem of a monetary authority pursuing an exchange rate policy that is inconsistent with interest rate parity because of a binding zero lower bound constraint. The resulting violation in interest rate parity generates an inflow of capital that the monetary authority needs to absorb by accumulating foreign reserves. We show that these interventions by the monetary authority are costly, and we derive a simple measure of these costs: they are proportional to deviations from the covered interest parity (CIP) condition and the amount of accumulated foreign reserves. Our framework can account for the recent experiences of "safe-haven" currencies and the sign of their observed deviations from CIP.

Key words: Capital flows, CIP deviations, Currency pegs, Foreign exchange interventions, International reserves.

JEL Codes: F31, F32, F41

1. INTRODUCTION

Many central banks often manage, implicitly or explicitly, their exchange rate. In a financially integrated world, the path for the exchange rate determines, together with nominal interest rates, the relative desirability of assets denominated in domestic and foreign currency. A long tradition, which dates back at least to Krugman (1979), has focused on how inconsistent fiscal and monetary policies can make domestic assets less attractive than foreign ones and lead to episodes of capital outflows, depletion of foreign reserves, and currency devaluations.

Since the global financial crisis, however, several countries have experienced opposite dynamics, that is, capital inflows, accumulation of foreign reserves, and currency appreciations. The case of Switzerland is emblematic in this respect. Over the period 2010–7, despite a zero or

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negative nominal interest rate, Switzerland experienced a large increase in private capital inflows that was accompanied by an equally large increase in holdings of foreign reserves by the Swiss National Bank (SNB), which was attempting to prevent an appreciation of the Swiss franc.

In this article, we argue that episodes of this sort can arise because of a conflict between an exchange rate policy and the zero lower bound constraint on nominal interest rates. To understand our argument, consider a situation in which a monetary authority is pegging the exchange rate, but there are future states of the world in which it would abandon the peg and appreciate. If nominal interest rates are at zero at home and abroad, local currency assets will be attractive to foreigners because the expected future appreciation is not offset by lower domestic interest rates. We show that this force induces capital inflows that *need* to be absorbed by the monetary authority through foreign exchange interventions, and that such unconventional interventions are *costly*. We provide a measure of these costs and show that they can be substantial. For the Swiss franc, the monthly costs of the exchange rate policies carried out by the SNB peaked at about 0.6% of monthly gross domestic product (GDP). Moreover, our framework can help to rationalize the recent emergence of deviations from covered interest parity (CIP) for economies with nominal interest rates close to zero.

We formalize this argument in a canonical small open economy model with two main ingredients. First, we assume that foreign financial intermediaries that trade with the domestic economy face potentially binding financial constraints, a feature implying that arbitrage in international financial markets might fail. That is, risk-adjusted returns on domestic currency assets could be higher than those on foreign ones, and only a *finite* amount of capital would flow into the country. Second, we introduce money in the model, which leads to a potentially binding zero lower bound on nominal interest rates, as is standard in monetary models. In such a framework, we study the problem of a benevolent monetary authority that uses its balance sheet to implement a given state-contingent path for its exchange rate.

Let's start from the (risk-augmented) interest rate parity condition,

$$(1+i_t)=\frac{(1+i_t^*)}{\mathbb{E}_t[e_t/e_{t+1}]}-\mathbb{C}\text{ov}_t\left(\Lambda_{t+1},(1+i_t)\frac{e_t}{e_{t+1}}\right), \tag{IP}$$

where i_t and i_t^* are, respectively, the nominal interest rates on risk-free bonds at home and abroad, e_t is the exchange rate (the price of foreign currency in terms of domestic currency), and Λ_{t+1} is the financial intermediaries' stochastic discount factor. This equation defines the level of i_t that makes intermediaries indifferent between holding domestic currency or holding foreign currency bonds given the foreign interest rate and the exchange rate policy.[1]

The central bank's exchange rate policy (e_t, e_{t+1}) *does not conflict* with the zero lower bound if equation (IP) holds for some non-negative i_t, given i_t^* and Λ_{t+1}. In such a scenario, the monetary authority can always implement the desired exchange rate policy by choosing a level of i_t that makes intermediaries indifferent between investing in the small open economy or not. We show that, in this case, it is optimal for the monetary authority to choose this particular nominal rate. As a result, interest parity holds, and capital flows between the small open economy and the rest of the world arise only to absorb the desired excess domestic net savings of the private sector.

This implementation, however, is not feasible when the exchange rate policy *conflicts with the zero lower bound*, that is, when there is no non-negative i_t that is consistent with equation (IP). The zero lower bound then implies that in any equilibrium that implements the exchange rate policy (e_t, e_{t+1}), interest rate parity will be violated. In this regime, foreign intermediaries

1. The deterministic log-linearized version reduces to $i_t=i_t^*+\ln(e_{t+1})-\ln(e_t)$, which is the usual condition for nominal exchange rate determination in workhorse open economy models (see Engel, 2014, for a recent survey).

have incentives to purchase domestic currency assets, generating a potentially large inflow of capital toward the small open economy. We show that in this situation, the private sector does not have incentives to absorb this inflow of capital, and the monetary authority is forced to issue domestic liabilities and accumulate foreign assets. By issuing high-yielding domestic assets and purchasing low-yielding foreign ones, the trades of the monetary authority induce a resource cost for the small open economy. To implement its desired exchange rate policy, it is optimal for the monetary authority to set interest rates at zero, so as to minimize these costs, while accumulating foreign reserves.

Equation (IP) clarifies the conditions under which a given exchange rate policy might conflict with the zero lower bound on nominal interest rates. The conflict is more likely to arise when (1) the foreign nominal interest rate is low, (2) there is an expected future appreciation of the domestic currency, or (3) the currency of the small open economy is perceived to be a "safe haven," that is, when future appreciations coincide with "bad" times for intermediaries (generating a high covariance between Λ_{t+1} and the exchange rate).

In our view, these three circumstances describe well the environment faced by the Swiss National Bank (SNB) after the global financial crisis. In an effort to dampen the appreciation pressures on the Swiss franc, the SNB established a currency floor vis-à-vis the euro in 2011 and announced that it would not tolerate an exchange rate beyond 1.2 Swiss francs per euro. Such policy was implemented during a period in which interest rates were at zero in all major advanced economies, and the policy itself was not perfectly credible, as financial markets attached a positive probability that the SNB would abandon the floor and let the franc appreciate (Jermann, 2017). Moreover, there is evidence that the Swiss franc was expected to appreciate during adverse worldwide economic conditions.[2] Consistent with our reading, the Swiss franc was characterized throughout this period by deviations from CIP that made Swiss-denominated assets attractive, and the foreign reserves of the SNB jumped from roughly 10% of GDP in 2010 to more than 100% in 2016. In our theory, both observations are symptoms of a conflict between an exchange rate policy and the zero lower bound.

We use the experience of the Swiss franc as a laboratory to measure the costs of an exchange rate policy. Specifically, we show that these resource costs can be approximated by combining central bank balance sheet data and CIP deviations. CIP deviations of the franc with respect to the euro reached a monthly average of 85 basis points in January 2015, at a time when the foreign reserves of the SNB reached almost 80% of GDP; these two observations imply an estimate of substantial losses—on the order of 0.6% of monthly GDP at that time.

While offering a prototypical example of a conflict between exchange rate policies and the zero lower bound, the Swiss experience is not an isolated one, and our framework is useful for interpreting the behaviour of other advanced economies. As documented in a recent paper by Du *et al.* (2018), systematic failures from CIP have occurred for several currencies after 2008, and these deviations are associated with low interest rates. We complement their findings, showing that CIP deviations are large only when interest rates are close to zero. Also we document that CIP deviations are associated with large increases in official holdings of foreign reserves, suggesting that these deviations are associated with monetary authorities actively pursuing exchange rate policies. Finally, we examine two notable cases of countries that pursued an explicit exchange rate policy while at the zero lower bound, that is, Switzerland in the late 1970s and the Czech Republic in the late 2010s. We show that in both cases, these policies are associated with large foreign reserve accumulation and large CIP deviations.

2. For example, following the intensification of the European debt crisis in May 2012, there was a massive increase in the demand for Swiss francs by international investors. At that stage, speculations that the SNB would abandon the currency floor intensified; see Alice Ross, "Swiss franc strength tests SNB," *Financial Times*, 24 May 2012, for instance.

Our article contributes to the literature on exchange rate determination in segmented capital markets. Backus and Kehoe (1989) derive general conditions under which sterilized official purchases of foreign assets do not affect equilibrium allocations and therefore are irrelevant for the determination of the nominal exchange rate—a result in the spirit of the irrelevance of standard open-market operations by Wallace (1981) and Sargent and Smith (1987). A key assumption underlying this irrelevance result is the absence of financial constraints and asset market segmentation

We follow the contributions by Alvarez *et al.* (2009) and Gabaix and Maggiori (2015) in relaxing these assumptions, and we study foreign exchange interventions in the presence of limited international arbitrage. Our work is also connected to Cavallino (2019) and Fanelli and Straub (2017). Cavallino (2019) studies the role of foreign exchange interventions for the management of the terms of trade (as in Costinot *et al.*, 2014) in the presence of exogenous shocks to capital flows. Fanelli and Straub (2017) consider a real deterministic model with limited international arbitrage in which the government uses foreign exchange interventions to mitigate the distributional consequences of exchange rate fluctuations. In such a framework, they show how foreign exchange interventions generate a resource cost proportional to the difference between the domestic and foreign real interest rates while also analysing credibility and international coordination issues.[3] We complement these papers by studying a monetary environment with uncertainty, and examine the optimal implementation of a policy for nominal exchange rates with an explicit zero lower bound constraint for the nominal interest rate. The presence of uncertainty allows us to address the question of whether one should use deviations from covered or uncovered interest rate parity when measuring the intervention costs in the data. In addition, we show how a model of limited international arbitrage can provide a consistent narrative of some of the unusual behaviour observed in major currencies post-financial crisis.

In relation to the intervention costs, Calvo (1991) first raised the warning about the potential costs of sterilized foreign exchange interventions. A mostly empirical literature has subsequently discussed and estimated the "quasi-fiscal" costs of these operations and similarly identified them as a loss in the budget constraint of the government, proportional to the interest parity deviations and the size of the accumulated reserves (see Kletzer and Spiegel, 2004; Devereux and Yetman, 2014; Liu and Spiegel, 2015, and references therein). The common practice in this literature, prominent also in policy discussions about the merits of sterilized interventions, is to use deviations from the uncovered interest parity (UIP) condition when computing these costs.[4] Our article clarifies that this practice might lead to biases: as will become clear from our analysis, using deviations from UIP in these calculations is equivalent to computing the ex-ante net costs from foreign exchange interventions without appropriately discounting them.

The failure of CIP since 2008 has been documented in detail by Du *et al.* (2018). They provide evidence that such deviations and the resulting failure of arbitrage were due to balance sheet constraints on financial intermediaries, likely induced by tighter banking regulations following the financial crises. They also uncover a negative cross-country relation between nominal interest rates and deviations from CIP, meaning that currencies that were most attractive were also characterized by lower interest rates. To the best of our knowledge, our article is the first to provide a formal framework for interpreting these findings and investigating their welfare implications. Specifically, we provide a theory in which failures from CIP arise from the binding balance sheet constraints of financial intermediaries, which explains why positive CIP deviations may appear

3. Jeanne (2012) studies foreign reserve accumulation as a tool to manage the real exchange rate, but in the context of a real model with a closed capital account for the private sector.

4. See, for example, Adler and Mano (2016) and Sarno and Taylor (2001) for reviews of the literature.

for some currencies and not for others, and we explain their connections to official holdings of foreign reserves and low interest rates.

Finally, our work is related to the literature that studies unconventional policies when monetary policy is constrained, either by a zero lower bound or by a fixed exchange rate regime. Correia *et al.* (2013), Adao *et al.* (2009), and Farhi *et al.* (2014) emphasize how various schemes of taxes and subsidies can achieve the same outcomes that would prevail in the absence of constraints to monetary policy. Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2012) study capital controls as second-best policy instruments to deal with capital flows under a fixed exchange rate regime. In contrast to these studies, we investigate foreign exchange interventions as a tool to implement a given exchange rate policy at the zero lower bound.[5] Both limitations and benefits are associated with these different policies, and more research is needed to tease out the appropriate policy mix.

The structure of the article is as follows. Section 2 introduces the model, while Section 3 characterizes the monetary equilibria for a given exchange rate policy. In Section 4, we introduce the problem of the monetary authority, characterize the optimal balance sheet policy, conduct a comparative statics analysis, and discuss alternative policies. Section 5 shows how to measure the costs of foreign exchange interventions, and Section 6 presents empirical evidence. Section 7 concludes. Throughout the article, we assume that the monetary authority wishes to implement an exogenous exchange rate target. In the Supplementary Appendix, we endogenize this target in a model with sticky wages and show that our implementation results continue to hold in this environment.

2. THE MODEL

We consider a small open economy that lasts for two periods, indexed by $t \in \{1,2\}$. There is an uncertain state $s \in \{s_1,\ldots,s_N\} \equiv S$ that is realized at $t=2$, and we denote by $\pi(s) \in (0,1]$ the probability that state s occurs. There is only one good and no production.

The small open economy is inhabited by a representative household and a monetary authority. The rest of the world is populated by a mass of financial intermediaries which trade on behalf of passive foreign households.

The domestic household receives an endowment of the consumption good, $(y_1,\{y_2(s)\})$, and decides on a consumption allocation, $(c_1,\{c_2(s)\})$.[6] In addition, the household also receives a lump-sum transfer (or a tax, if negative) of $\{T_2(s)\}$ from the monetary authority in the second period.[7]

There is an international financial market with a full set of Arrow–Debreu securities, indexed in foreign currency. The price level in the international financial market is normalized to one, so

5. For other work exploring the open economy dimension of the zero lower bound, see Krugman (1998), Cook and Devereux (2013), Svensson (2003), Benigno and Romei (2014), Acharya and Bengui (2018), Fornaro (2018), Caballero *et al.* (2015), Eggertsson *et al.* (2016), and Corsetti *et al.* (2017). For the interaction between the zero lower bound and safe-haven currencies, see Gourinchas and Rey (2016).

6. We use the following notation: a vector of the form $(x_1,\{x_2(s)\})$ denotes an x_1 value at $t=1$ and a value of $x_2(s)$ at $t=2$ conditional on the state s.

7. Here, we are consolidating the fiscal authority and the central bank into a single decision-making unit. We could separate the fiscal authority from the Central Bank. If transfers between these two authorities were allowed, and they were to share the same objective (*i.e.* maximize household welfare), the resulting model would be equivalent to the present setup. However, this equivalence may no longer hold if transfers between these two authorities are constrained by political economy or institutional considerations. See Amador *et al.* (2016) for an example in which a cap on the amount of transfers that the central bank can receive from the fiscal authority imposes constraints on the exchange rate policies that can be implemented.

that foreign prices are effectively quoted in units of the consumption good. Let $q(s)$ be the price, in foreign currency as of period 1, of the Arrow–Debreu security that pays one unit of foreign currency in state s in period 2 and zero in all others. The price $q(s)$ is exogenous and taken as given by all agents.

The small open economy has its own currency in circulation, as well as a full set of Arrow–Debreu securities denominated in domestic currency. We denote by $p(s)$ the domestic currency price in period 1 of the domestic Arrow–Debreu security that pays one unit of domestic currency in state s in period 2, and zero otherwise. There is a nominal exchange rate in periods 1 and 2, $(e_1,\{e_2(s)\})$, which denotes the amount of domestic currency necessary to purchase a unit of foreign currency at any period and state. Goods trade is costless, and as a result, the law of one price holds: the domestic price level at any state is equal to the exchange rate.

2.1. *The domestic households*

The budget constraint of the domestic household in the initial period is

$$y_1=c_1+\sum_{s\in S}\left[q(s)f(s)+p(s)\frac{a(s)}{e_1}\right]+\frac{m}{e_1}, \tag{2.1}$$

where $f(s)$ and $a(s)$ denote the purchases of domestic and foreign Arrow–Debreu securities, m are money holdings, and where we have assumed that all initial asset positions of the households are zero.

In period 2 at state s, the budget constraint of the household becomes

$$y_2(s)+T_2(s)+f(s)+\frac{a(s)+m}{e_2(s)}=c_2(s)\ \text{ for all } s\in S. \tag{2.2}$$

Domestic households can purchase and sell any amount of domestic securities. They can also purchase an unrestricted non-negative amount of foreign assets. However, we assume that the household cannot short-sell foreign securities:

$$f(s)\geq 0, \text{ for all } s\in S. \tag{2.3}$$

This assumption guarantees that the financial constraints of the financial intermediaries will matter for the equilibrium allocation. The zero in the above equation, however, is not important, as all our results would survive if domestic households had a strictly positive borrowing limit in foreign currency.

The household's problem is to choose $(c_1,\{c_2(s)\},m,\{f(s),a(s)\})$, subject to the budget constraints, to maximize the following utility function:

$$u(c_1)+h\left(\frac{m}{e_1}\right)+\beta\sum_{s\in S}\pi(s)u(c_2(s)), \tag{2.4}$$

where $u(c)$ is a standard strictly increasing, strictly concave, and differentiable utility function, and h is an increasing, differentiable, and concave function, with a satiation level $\overline{x}$ (*i.e.* $h(x)=h(\overline{x})$ for all $x\geq\overline{x}$).

2.2. *The foreign intermediaries*

There is a mass one of foreign financial intermediaries, which trade on behalf of foreign households. They start the period with some amount of capital, $\overline{w}>0$, which they use to purchase domestic assets, including money, issued by the small open economy and foreign financial assets. They choose their portfolio $(m^*,\{a^*(s),f^*(s)\})$ and dividend stream $(d_1^*,\{d_2^*(s)\})$ to maximize the expected discounted present value of dividends:

$$d_1^*+\sum_{s\in S}\pi(s)\Lambda(s)d_2^*(s), \tag{2.5}$$

where $\Lambda(s)$ represents the stochastic discount factor of the foreign households. We assume that this stochastic discount factor, $\Lambda(s)$, also prices each of the *foreign* Arrow–Debreu securities, and thus $\pi(s)\Lambda(s)=q(s)$.

In the initial period, their budget constraint is

$$\overline{w}=\frac{m^*}{e_1}+\sum_{s\in S}\left[\frac{p(s)a^*(s)}{e_1}+q(s)f^*(s)\right]+d_1^*. \tag{2.6}$$

In period 2 at state s, their budget constraint is

$$d_2^*(s)=\frac{m^*+a^*(s)}{e_2(s)}+f^*(s). \tag{2.7}$$

These intermediaries cannot issue negative dividends in the first period and have limited ability to borrow in both domestic and foreign financial markets:

$$d_1^*\geq 0, f^*(s)\geq 0, \text{ and } a^*(s)\geq 0 \quad \text{for all } s\in S. \tag{2.8}$$

As was the case for the household, the zero in these constraints is not critical for our results, and its only role is to make certain expressions in the article less cumbersome. The important assumption here is that there are some limits on the ability of the intermediaries to issue equity or borrow.

2.3. *The monetary authority*

We impose that the monetary authority has a given nominal exchange rate objective, which we denote by $(e_1,\{e_2(s)\})$. In general, an exchange rate objective would arise from the desire to achieve a particular inflation or output target. In the Supplementary Appendix, we study optimal exchange rate policies in a model with wage rigidities. For the moment, however, we simply assume that the monetary authority follows this objective, and we define an equilibrium *given* $(e_1,\{e_2(s)\})$. This allows us to transparently illustrate the role of the balance sheet of the monetary authority in determining the nominal exchange rate.

To achieve its exchange rate objective, the monetary authority issues money and an uncontingent bond denominated in domestic currency, (M,A). We denote by $\overline{p}$ the price of the risk-free domestic bond. It also purchases foreign reserves in the form of an uncontingent bond denominated in foreign currency, F, at price $\overline{q}$. As with the households, we restrict $F\geq 0$.

In the second period, the monetary authority withdraws the money from circulation and redistributes the returns of its portfolio holdings to the domestic household. The associated budget constraints are

$$\frac{\overline{p}A+M}{e_1}=\overline{q}F, \tag{2.9}$$

$$T_2(s)=F-\frac{A+M}{e_2(s)} \quad \text{for all } s\in S \tag{2.10}$$

for periods 1 and 2, respectively.

The prices of the domestic and foreign uncontingent bond, which can be replicated from the set of domestic and foreign Arrow–Debreu securities, respectively, are

$$\bar{p} = \sum_{s \in S} p(s) \equiv \frac{1}{1+i} \qquad \bar{q} = \sum_{s \in S} q(s) \equiv \frac{1}{1+i^*}, \tag{2.11}$$

where we have defined the domestic and international risk-free interest rate as i and i^*.

2.4. *Monetary equilibrium*

An *equilibrium given an exchange rate policy* $(e_1, \{e_2(s)\})$ is a household's consumption profile, $(c_1, \{c_2(s)\})$, and its asset positions, $(m, \{a(s), f(s)\})$; intermediaries' dividends policy, $(d_1^*, \{d_2^*(s)\}$, and its asset positions, $(m^*, \{a^*(s), f^*(s)\})$; the monetary authority's transfer to the households, $\{T_2(s)\}$, and its asset positions, (M, F, A); and domestic asset prices $\{p(s)\}$, such that

1. The domestic household chooses consumption and portfolio positions to maximize utility, (2.4), subject to the budget constraints, (2.1) and (2.2), as well as the no-borrowing constraints, (2.3), while taking prices $\{q(s), p(s)\}$ and transfers $\{T_2(s)\}$ as given.
2. Intermediaries choose the dividend policy and portfolio positions to maximize their objective, (2.5), subject to their budget constraints, (2.6) and (2.7), as well as the non-negativity restriction on their asset holdings and first-period dividends, (2.8), while taking prices $\{q(s), p(s)\}$ as given.
3. The purchases of assets by the monetary authority and its transfers to the households satisfy its budget constraints, (2.9) and (2.10) for all $s \in S$, together with (2.11) and the non-negativity restriction on foreign reserves, $F \geq 0$.
4. Domestic asset markets clear:

$$a(s) + a^*(s) = A \quad \text{for all } s \in S, \tag{2.12}$$

$$m + m^* = M. \tag{2.13}$$

The above definition does not specify an objective function for the monetary authority. For a given exchange rate policy $(e_1, \{e_2(s)\})$, there are potentially many possible monetary equilibria, indexed by particular balance sheet positions for the monetary authority. Our objective is to study how a benevolent monetary authority that maximizes household welfare sets its balance sheet optimally in order to implement $(e_1, \{e_2(s)\})$. Before studying this problem, though, it is useful to first characterize some useful properties of monetary equilibria.

3. CHARACTERIZING MONETARY EQUILIBRIA

This section characterizes monetary equilibria. We start by defining a "first-best" consumption allocation, which will be a useful benchmark for the optimal policy of the monetary authority.[8] We then move to describe some key equilibrium conditions and present a characterization of the monetary equilibria.

8. The definition of this first-best consumption allocation does not incorporate real money balances. The separability between money balances and consumption in utility makes such a definition helpful in characterizing the best monetary equilibrium, a feature we exploit in Proposition 1.

3.1. *First best in a real economy*

We define the *first-best* consumption allocation as the allocation $(c_1^{fb}, \{c_2^{fb}(s)\})$ that solves

$$\max_{c_1,\{c_2(s)\}} \left\{ u(c_1) + \beta \sum_{s\in S} \pi(s) u(c_2(s)) \right\} \tag{3.1}$$

subject to

$$y_1 - c_1 + \sum_{s\in S} q(s)(y_2(s) - c_2(s)) \geq 0. \tag{3.2}$$

In what follows, we impose an assumption that guarantees that this consumption allocation could be implemented as a monetary equilibrium, absent the zero lower bound constraint:

Assumption 1 *Intermediary capital is such that*

$$\sum_{s\in S} q(s) \max\left\{ y_2(s) - c_2^{fb}(s), 0 \right\} \leq \overline{w}.$$

This condition guarantees that the intermediaries have enough capital to cover the external gross liability/inflow position of the economy generated by the first-best allocation.[9]

The first-best allocation equalizes the ratio of marginal utility in the first period to marginal utility in the second period across states, adjusted by prices and probabilities. That is,

$$\frac{\beta\pi(s)}{q(s)} \frac{u'(c_2^{fb}(s))}{u'(c_1^{fb})} = 1$$

for all $s \in S$.

The property of equalizing this ratio, but not necessarily to one, is shared by consumption allocations of a different type, which, under certain conditions, will be part of any monetary equilibrium. We define them as "equal gaps" consumption allocations.

Definition 1 *We say that a consumption allocation features equal gaps if it satisfies*

$$\frac{\beta\pi(s)}{q(s)} \frac{u'(c_2(s))}{u'(c_1)} = \frac{\beta\pi(s')}{q(s')} \frac{u'(c_2(s'))}{u'(c_1)}, \tag{3.3}$$

for all $s, s' \in S$.

These consumption allocations feature no intratemporal distortions in the second period, just as in the first best, but may feature an intertemporal distortion. An alternative way of interpreting

9. From the budget constraints of the households and the monetary authority, we have that $y_2(s) - c_2^{fb}(s) + f(s) + F = x^*(s)$, where $x^*(s) \geq 0$ represents the payoff to intermediaries on their domestic investments in state s. Given that $f(s) \geq 0$ and $F \geq 0$, and $x^*(s) \geq 0$, it follows that $x(s) \geq \max\{y_2(s) - c_2^{fb}(s), 0\}$. In the first-best allocation, domestic state prices would be equalized with foreign ones, and thus summing over across states, using the state price $q(s)$, we get that the total domestic investments made by the intermediaries must be $\sum_{s\in S} q(s)x^*(s) \geq \sum_{s\in S} q(s)\max\{y_2(s) - c_2^{fb}(s), 0\}$. But the total domestic investments of the intermediaries cannot be bigger than $\overline{w}$ as of time 1, and so $\sum_{s\in S} q(s)x^*(s) \leq \overline{w}$, generating the condition in Assumption 1.

these allocations is that the second-period consumption allocation is the solution to the following static planning problem, indexed by C_2:

$$U(C_2) \equiv \max_{\{c_2(s)\}} \left\{ \sum_{s \in S} \pi(s) u(c_2(s)) \text{ subject to } \overline{q} C_2 = \sum_{s \in S} q(s) c_2(s) \right\}, \tag{SP}$$

where C_2 are the second-period expenditures necessary to purchase the consumption bundle $\{c_2(s)\}$. If an equilibrium features equal gaps, we only need to determine initial consumption and the second-period aggregate C_2. Along with the prices of foreign securities, this is sufficient to characterize the second-period consumption in every state. It is also useful to define an "average" of the second-period endowment, Y_2:

$$Y_2 \equiv \frac{\sum_{s \in S} q(s) y_2(s)}{\overline{q}}. \tag{3.4}$$

3.2. *Equilibrium conditions*

We now discuss the key equilibrium conditions of the model, starting with the optimality conditions for the household.

3.2.1. Household optimality and domestic prices. The household solves a standard consumption-saving problem, with multiple assets (domestic and foreign securities) and potentially binding borrowing constraints. Recall that these constraints apply only when the household borrows in foreign currency. Because of that, the first-order condition of the household with respect to domestic securities holds with equality and implies that their price is given by

$$p(s) \frac{e_2(s)}{e_1} = \frac{\beta \pi(s) u'(c_2(s))}{u'(c_1)} \tag{3.5}$$

for all $s \in S$.

Their optimality condition with respect to foreign asset s might instead hold with inequality because of the borrowing constraint,

$$q(s) \geq \frac{\beta \pi(s) u'(c_2(s))}{u'(c_1)}, \tag{3.6}$$

for all $s \in S$. When the above condition holds with strict inequality for some s, the household chooses not to invest in the associated foreign security, that is, $f(s) = 0$, because this security is strictly dominated by the domestic one.

3.2.2. The zero lower bound on the nominal interest rate. The household also chooses its money holdings. The household's optimality condition with respect to money holdings can then be written as

$$h'\left(\frac{m}{e_1}\right) = u'(c_1)\left(1 - \sum_{s \in S} p(s)\right) = u'(c_1) \frac{i}{1+i}, \tag{3.7}$$

where we have used the definition of the risk-free rate on a nominal bond in (2.11).

Note that equation (3.7) implies that domestic nominal interest rates cannot be negative. Because $h' \geq 0$ and $u' \geq 0$, we must have that $i \geq 0$ in any monetary equilibrium.

3.2.3. Intermediary's optimality and profits. The intermediary chooses investment in foreign and domestic securities, including money. Let us denote by Π their period 1 profits, that is, the difference between the expected discounted present value of their dividends and their initial capital. Because they share the same stochastic discount factor that prices the foreign securities, investing in foreign assets yields no profits. However, investing in domestic ones may, depending on the equilibrium prices. In particular, their profits Π are

$$\Pi = \frac{m^*}{e_1}\left[\sum_{s\in S}\frac{q(s)e_1}{e_2(s)}-1\right]+\sum_{s\in S}\frac{p(s)a^*(s)}{e_1}\left[\frac{q(s)e_1}{p(s)e_2(s)}-1\right], \tag{3.8}$$

where m^* and a^* are non-negative and such that $m^*/e_1+\sum_s p(s)a^*(s)/e_2(s)\leq \overline{w}$.

The terms in square brackets are return differentials. The first is the return differential of holding money and the foreign nominal risk-free bond.[10] The second is the return differential between domestic and foreign Arrow–Debreu securities.

Given the linearity of their objective function, the optimal portfolio decision of intermediaries is to channel all of their wealth into the domestic security that yields the largest differential return.

3.2.4. The intertemporal resource constraint. We can obtain an intertemporal resource constraint in this economy by consolidating the household and the monetary authority budget constraints. Specifically, solving for $f(s)$ using the household's budget constraint in the second period and plugging it back into the household's first-period budget constraint, we obtain

$$y_1 = c_1+\sum_{s\in S}\left[q(s)\left(c_2(s)-y_2(s)-T_2(s)-\frac{a(s)+m}{e_2(s)}\right)+p(s)\frac{a(s)}{e_1}\right]+\frac{m}{e_1}.$$

Using the budget constraints of the monetary authority, we have that the transfer in the second period can be expressed as

$$T_2(s)=\frac{1}{\overline{q}}\left[\frac{\overline{p}A+M}{e_1}\right]-\frac{A+M}{e_2(s)}.$$

Substituting this expression in the previous equation and collecting terms, we obtain

$$y_1=c_1+\sum_{s\in S}\left[q(s)\left(c_2(s)-y_2(s)+\frac{A-a(s)+M-m}{e_2(s)}\right)+p(s)\frac{A-a(s)}{e_1}\right]+\frac{M-m}{e_1}.$$

Market clearing implies that $A(s)-a(s)=a^*(s)$ and $M-m=m^*$, and thus we obtain the following condition that must hold in any equilibrium:

$$y_1-c_1+\sum_{s\in S}q(s)(y_2(s)-c_2(s))-\Pi=0. \tag{3.9}$$

This equation is similar to the first-best intertemporal resource constraint, equation (3.2), but is adjusted to incorporate a potential loss for the small open economy, Π. When foreign

10. To see this, we can use $q(s)=\pi(s)\Lambda(s)$ to obtain

$$\sum_{s\in S}\frac{q(s)e_1}{e_2(s)}-1=\mathbb{E}\left[\Lambda(s)\left(\frac{e_1}{e_2(s)}-(1+i^*)\right)\right].$$

intermediaries make profits by purchasing domestic assets, someone in the small open economy is taking the opposite side and incurring a loss. This loss is always non-negative because the intermediaries can always choose a portfolio yielding zero profits. That is, in equilibrium, $\Pi \geq 0$. We highlight that this loss is the equivalent (in our environment) to the losses obtained by Fanelli and Straub (2017) in a deterministic environment (and also featured in Cavallino, 2019). As we show below, our model with uncertainty implies that risk premia may now play a key role.

3.2.5. Gross capital flows and trade balance. Using the household budget constraint in the first period, as well as the monetary authority budget constraints, we obtain the following equality, linking the trade deficit to the evolution of the net foreign asset position:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{m^* + \sum_s p(s)a^*(s)}{e_1}}_{\text{foreign liabilities}} - \underbrace{\left[\sum_s q(s)f(s) + F\right]}_{\text{foreign assets}}. \tag{3.10}$$

3.3. *Monetary equilibria featuring equal gaps*

Under certain conditions, equal gaps allocations are the only possible equilibrium outcome. We proceed to show this next. Toward this end, we make the following assumption:

Assumption 2 *One of the following holds:*

(a) $q(s)/\pi(s)$ is a constant for all $s \in S$ and

$$g_0 \leq \overline{w},$$

(b) u is CRRA, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 0$, and

$$g_0 + \left(g_1^{1/\sigma} - 1\right)(y_1 + \bar{q}Y_2) \leq \overline{w},$$

(c) u is CARA, $u(c) = -\exp(-\sigma c)$ with $\sigma > 0$, and

$$g_0 + \frac{\bar{q}}{\sigma}\log(g_1) \leq \overline{w},$$

where $g_0 \equiv \bar{q} \times \max_{s_1,s_2}\{y_2(s_1) - y_2(s_2)\}$, $g_1 \equiv \max_{s_1,s_2}\left(\frac{\pi(s_1)q(s_2)}{q(s_1)\pi(s_2)}\right)$, and Y_2 is defined in (3.4).

Part (a) tells us that the assumption is satisfied for any utility function when $q(s)/\pi(s)$ is constant, and the cross-state variation in the second-period endowment is not large when compared to the intermediary capital. For example, if the second-period endowment is constant, then the assumption is satisfied for any positive intermediary capital level. Parts (b) and (c) consider cases in which $q(s)/\pi(s)$ is not constant. For them, the assumption holds when the cross-state variation in the second-period endowment and the variation in $q(s)/\pi(s)$ (which determines the variation in consumption in the second period) are not large, or when the capital of foreign intermediaries is sufficiently large.

This assumption guarantees that, in any monetary equilibrium, there is sufficient intermediary capital so that the private agents equalize the ratio of marginal utilities adjusted by the foreign prices and probabilities. We state this in the following lemma:

Lemma 1 Suppose that Assumption 2 holds. Then the consumption allocation of any monetary equilibrium features equal gaps.

When a consumption allocation features equal gaps, the intermediary's problem simplifies. Using condition (3.3), we must have that excess returns on all domestic securities are equalized:

$$0 \leq \frac{q(s')e_1}{p(s')e_2(s')} - 1 = \sum_{s\in S}\left[\frac{p(s)}{\overline{p}} \times \left(\frac{q(s)e_1}{p(s)e_2(s)}\right)\right] - 1 = \sum_{s\in S}\frac{q(s)e_1}{e_2(s)}(1+i) - 1 \tag{3.11}$$

for any $s' \in S$. The first inequality follows from the household's optimality conditions, (3.5) and (3.6), which require that $p(s) \leq q(s)e_1/e_2(s)$. The first equality follows from the definition of equal gaps and that $p(s)/\overline{p}$ sums to one (by definition of $\overline{p}$). The second equality follows from the definition of i.

Let us define $\Delta(i)$ to be the right-hand term of (3.11):

$$\Delta(i) \equiv \sum_{s\in S}\frac{q(s)e_1}{e_2(s)}(1+i) - 1. \tag{3.12}$$

In an equal gaps allocation, $\Delta(i)$ captures the profits per unit of capital. When $\Delta(i) > 0$, intermediaries optimally invest all of their wealth in domestic securities. When $\Delta(i) = 0$, intermediaries make zero profits. Thus, we can write their profits as

$$\Pi = \Delta(i) \times \overline{w}.$$

This expression also captures the losses for the small open economy in the resource constraint (3.9).

The value of $\Delta(i)$ has another interpretation. Consider a simpler problem in which the intermediaries decide between two assets. An intermediary can invest in the domestic risk-free nominal bond with return i or in the foreign currency risk-free bond with return i^*. The difference in payoffs between these two assets, from the perspective of an intermediary, is

$$\mathbb{E}\left[\Lambda(s)\left(\frac{e_1}{e_2(s)}(1+i) - (1+i^*)\right)\right] = \left[\sum_{s\in S} q(s)\frac{e_1}{e_2(s)}\right](1+i) - 1 = \Delta(i), \tag{3.13}$$

thus, $\Delta(i)$ is the "risk-adjusted" difference between the domestic and foreign risk-free bond returns. When $\Delta(i) = 0$, we say that *interest rate parity* holds. However, in our model it could be that $\Delta(i) > 0$. Such a violation of interest rate parity can arise because intermediaries and households face potentially binding borrowing constraints.

Monetary equilibria featuring an equal gaps allocation are particularly tractable because they can be described by just three values: initial consumption, c_1, the second-period consumption expenditures, C_2, and money balances, m.

Lemma 2 (Characterization of equilibrium) Under Assumptions 1 and 2, a consumption allocation $(c_1, \{c_2(s)\})$ and money holdings m are part of an equilibrium given the exchange rate policy $(e_1, \{e_2(s)\})$ if and only if there exists an i such that

$$y_1 - c_1 + \overline{q}(Y_2 - C_2) = \Delta(i)\overline{w}, \tag{3.14}$$

$$\frac{\overline{q}u'(c_1)}{\beta U'(C_2)} = 1 + \Delta(i) \geq 1, \tag{3.15}$$

$$h'\left(\frac{m}{e_1}\right) = u'(c_1)\frac{i}{1+i}, \tag{3.16}$$

and $\{c_2(s)\}$ solves the static planning problem (SP) given C_2; and where Y_2 and U are defined in (SP) and (3.4). Household welfare in this equilibrium is

$$u(c_1) + h(m/e_1) + \beta U(C_2). \tag{3.17}$$

Equation (3.15), the novel addition in this lemma, represents the household's Euler equation for foreign assets. Here, we have used the envelope condition for the static planning problem, (SP), with the equal gaps condition, (3.11). Recall that (3.16) implicitly imposes the zero lower bound.

Note that equations (3.14) and (3.15) have a solution only if $\Delta(i)\overline{w} < y_1 + \overline{q}Y_2$. Intuitively, the losses need to be lower than the present value of the country's endowment in order to have positive consumption. If $\Delta(i)\overline{w} > y_1 + \overline{q}Y_2$, then the exchange rate policy is infeasible. Moreover, first-period consumption c_1 is below the first best, and it is decreasing in $\Delta(i)$ and $\overline{w}$. An increase in $\overline{w}$ when $\Delta(i) > 0$ induces a negative income effect that pushes households to consume less today. An increase in $\Delta(i)$ generates a similar negative income effect, but also a negative substitution effect which further reduces first-period consumption. As this result is useful for the analysis to follow, we summarize it below.

Corollary 1 Suppose $\Delta(i)\overline{w} < y_1 + \overline{q}Y_2$. There is a unique pair (c_1, C_2) that solves (3.14) and (3.15). When $\Delta(i) = 0$, c_1 coincides with the first-best consumption. In addition, c_1 strictly decreases with $\Delta(i)$ and strictly decreases in $\overline{w}$ for $\Delta(i) > 0$.

In the analysis that follows, we will maintain Assumption 2, which guarantees that we can restrict attention to monetary equilibria featuring equal gaps allocations. This is useful for a number of reasons. First, as shown in Lemma 2, Assumption 2 makes the characterization of equilibria simple, which allows for a clear exposition of our main results. Second, we will see that, under equal gaps allocations, there are easily measurable empirical counterparts to $\Delta(i)$, something that we exploit in our application. Note, however, that if we were to remove Assumption 2, or allow the government to purchase a richer set of foreign securities, equilibrium allocations may feature unequal gaps. We explore these cases in Amador *et al.* (2018).

4. THE PROBLEM OF THE MONETARY AUTHORITY

We now study the problem of the monetary authority. Section 4.1 characterizes the monetary equilibrium that maximizes the welfare of domestic households, which we refer to as the "best equilibrium." Section 4.2 describes the balance sheet policy that allows the monetary authority to implement the best equilibrium. We conclude the section with a graphical illustration of the main results and with a discussion of comparative statics and alternative policies.

4.1. *Best equilibrium*

The objective of the monetary authority is to choose an equilibrium, given an exchange rate policy $(e_1, \{e_2(s)\})$, that maximizes the domestic household's welfare. Given Lemma 2, the problem of the monetary authority can be formulated as follows:

$$\max_{c_1, C_2, m, i} \left\{ u(c_1) + h(m/e_1) + \beta U(C_2) \right\} \tag{MP}$$

subject to (3.14), (3.15), and (3.16).

We refer to the solution for (MP) as a "best equilibrium." Note that even though the monetary authority's problem seems deterministic, uncertainty and risk play a role, as they determine the shape of $\Delta(i)$, thus affecting the intertemporal resource constraint (3.14).

The solution to (MP) can be characterized by two cases depending on the exchange rate policy and its effect on $\Delta(0)$.

First, consider the case in which the exchange rate policy is such that $\Delta(0) \leq 0$. Then, there exists a non-negative domestic nominal interest rate, $\tilde{i}$, such that $\Delta(\tilde{i}) = 0$. We can show that in such a scenario, the monetary authority sets $i = \tilde{i}$ and implements the first-best allocation.

Proposition 1 Suppose Assumptions 1 and 2 hold. If $\Delta(0) \leq 0$, then the best equilibrium features $(c_1^{fb}, C_2^{fb}, m, i)$ where

$$C_2^{fb} = \sum_{s \in S} q(s) c_2^{fb} / \overline{q},$$

$$i \geq 0 \text{ and such that } \Delta(i) = 0,$$

$$m \text{ such that } h'(m/e_1) = u'(c_1^{fb}) \frac{i}{1+i}.$$

When $\Delta(0) < 0$, the best equilibrium necessarily requires a deviation from the Friedman rule, that is, $i > 0$. The reason is that, given an exchange rate policy such that $\Delta(0) < 0$, an interest rate equal to zero requires negative profits per unit of capital, which cannot be part of an equilibrium (see equation (3.13)). When $\Delta(0) = 0$, in the best equilibrium, the Friedman rule holds and $i = 0$.

Importantly, the above solution cannot be an equilibrium if $\Delta(0) > 0$: in this case, there is no non-negative nominal interest rate consistent with interest rate parity. The following proposition describes the optimal solution in this case, which is our main result.

Proposition 2 Suppose Assumptions 1 and 2 hold. If $\Delta(0) > 0$ and $\Delta(0)\overline{w} < y_1 + \overline{q} Y_2$, then the best equilibrium features (c_1, C_2, m, i) such that

$$i = 0, \ \frac{m}{e_1} \geq \overline{x}, \text{ and}$$

$$(c_1, C_2) \text{ are the unique solutions to (3.14) and (3.15),}$$

and where $\overline{x}$ is the satiation level of real money balances.

That is, the best equilibrium features zero nominal interest rates, a failure of interest rate parity, and a consumption allocation distorted away from the first best. In this case, the monetary authority is trying to implement an exchange rate policy that makes domestic assets attractive even if nominal interest rates were set to zero, $\Delta(0)>0$. As $\Delta(i)$ increases with i, any equilibrium necessarily features a deviation from interest rate parity. Intermediary capital will flow into the country, generating the losses captured by $\Delta(i)\overline{w}$. By setting the lowest possible domestic interest rate, $i=0$, and thus selecting the lowest possible $\Delta(i)$, the monetary authority alleviates the costs associated with this capital inflow.

Before turning to study the implementation analysis, it is useful to discuss the conditions under which $\Delta(0)>0$ is more likely to emerge. For this purpose, we can write $\Delta(0)$ as follows:

$$\begin{aligned}\Delta(0)&=\mathbb{E}\left[\Lambda(s)\left(\frac{e_1}{e_2(s)}-(1+i^*)\right)\right]\\&=\frac{\mathbb{E}[e_1/e_2(s)]}{1+i^*}-1+\mathbb{C}\text{ov}\left(\Lambda(s),\frac{e_1}{e_2(s)}\right).\end{aligned} \tag{4.1}$$

Three main forces determine whether $\Delta(0)>0$: the rate of appreciation of the domestic currency, $\mathbb{E}[e_1/e_2(s)]$, the foreign interest rate, i^*, and the covariance of the appreciation rate, $e_1/e_2(s)$ with the stochastic discount factor of the foreign markets, $\Lambda(s)$. Note that these three components are exogenous in our model, as the interest rate and the stochastic discount factor of the foreign markets are given to the small open economy, and the exchange rate policy is taken as given by the monetary authority in our analysis.

Thus, the zero lower bound is more likely to be a problem for the monetary authority when the expected appreciation is high, the foreign interest is low, and the covariance term is positive. These results are intuitive. A high expected appreciation of the currency or a low foreign interest rate makes the domestic asset more attractive for a given nominal rate. The same occurs if the domestic currency tends to appreciate in bad states of the world for the foreigners, a property referred to as a "safe haven" in the literature.

The above can also help us to understand how external factors beyond i^*, in particular the international price of risk, affect $\Delta(0)$ and the domestic equilibrium. In our environment, it is possible for changes in $\Lambda(s)$ to affect $\Delta(0)$ even when neither the exchange rate policy nor the foreign interest rate changes. To see this, consider starting from a situation in which there is a strictly positive correlation between the exchange rate and $\Lambda(s)$. Suppose then that the variance of $\Lambda(s)$ increases, while its expected value and its correlation with the exchange rate do not change. Such an increase in variance leads to an increase in $\Delta(0)$ even though $i^\star$ does not change.[11] Monetary authorities of safe-haven currencies are thus more likely to face a conflict between their exchange rate policy and the zero lower bound constraint when the international price of risk increases (*i.e.* when the variance of $\Lambda(s)$ increases).[12]

4.2. *Implementation*

We now study the role of the monetary authority's balance sheet for the implementation of the best equilibrium, that is, we characterize the positions F, M, and A underlying the best equilibrium of the previous section. It turns out that we only need to characterize F: the value of M is, in fact, pinned down by the households' demand for money, while the total amount bought in domestic securities A follows from the budget constraint of the monetary authority.

11. These external effects that operate beyond short-term interest rate levels have been prominently discussed by Rey (2013), who highlights the role of VIX in the global financial cycle.

12. A related interesting point made by Hassan *et al.* (2016) is that a central bank that induces a real appreciation in bad times lowers its risk premium in international markets and increases capital accumulation.

We first consider the case discussed in Proposition 1, where the monetary authority optimally chooses an allocation that maintains interest parity and operates away from the zero lower bound.

Corollary 2 (Implementation away from the zero lower bound) Suppose Assumptions 1 and 2 hold. If $\Delta(0) \leq 0$, the monetary authority implements the best equilibrium with any $F \in [0, (y_1 - c_1^{fb} + \overline{w})/\overline{q}]$.

In this first scenario, accumulating reserves is not necessary to implement the exchange rate policy. Moreover, interest parity holds and the accumulation of foreign reserves does not affect the equilibrium outcomes (locally), thus mirroring the classic irrelevance result of Backus and Kehoe (1989). The reason for this irrelevance is that, as long as the intervention is not too large, there is sufficient intermediary capital for private agents to undo the interventions of the monetary authority.

We next consider the case discussed in Proposition 2, where the zero lower bound binds, and the monetary authority chooses an allocation that violates interest rate parity. In this case, it is necessary for the monetary authority to engage in foreign reserve accumulation. It optimally does so by selecting the minimum amount of reserves necessary to sustain its exchange rate policy. We summarize it in the following corollary.

Corollary 3 (Implementation at the zero lower bound) Suppose Assumptions 1 and 2 hold. If $\Delta(0) > 0$, the monetary authority implements the best equilibrium with $F = (y_1 - c_1 + \overline{w})/\overline{q} > 0$, where c_1 is the best equilibrium first-period consumption.

Why does the monetary authority need to accumulate foreign reserves? In the best equilibrium when $\Delta(0) > 0$, domestic assets strictly dominate foreign ones. As a result, the capital of foreign intermediaries flows to the small open economy. This capital must be absorbed by either a trade deficit or capital outflows. That is, from equation (3.10),

$$\underbrace{\sum q(s) f(s) + \overline{q} F}_{\text{capital outflow}} + \underbrace{(c_1 - y_1)}_{\text{trade deficit}} = \underbrace{\overline{w}}_{\text{capital inflow}}. \tag{4.2}$$

From Lemma 1, we know that the trade deficit is lower in the best equilibrium than it is in the first best, as $c_1 < c_1^{fb}$. Because capital inflows are higher in the best equilibrium relative to the first best, they must be absorbed by an outflow of resources. Domestic households have no incentives to purchase foreign assets because, under the best equilibrium, those assets are dominated by domestic ones. So, they set $f(s) = 0$ for all s. It follows that the best equilibrium *must* feature an accumulation of foreign reserves by the monetary authority, $F > 0$.

An important observation is that the necessity of foreign reserve accumulation by the monetary authority is independent of the sign of the trade balance in the resulting equilibrium. *Both a trade deficit and a trade surplus are possible outcomes.*

4.3. *A simple illustration*

We now provide a graphical illustration of the key results of this section. To this end, we leverage the results of Lemma 2 and describe the consumption allocation that arises in the best equilibria using a simple diagram in the (c_1, C_2) space, where C_2 represents the value of the second-period consumption allocation $\{c_2(s)\}$.

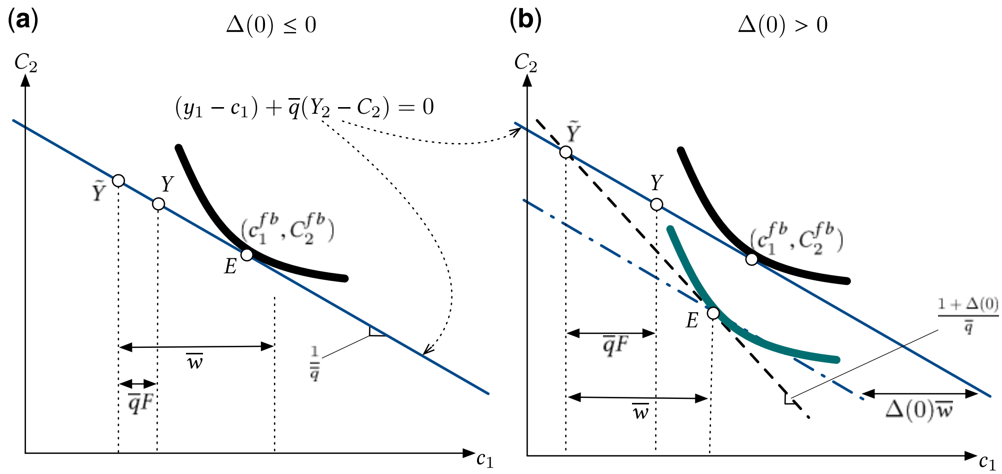

FIGURE 1

Reserves (F) and the best equilibrium

In both panels of Figure 1, the thick solid lines represent indifference curves, that is, combinations of (c_1, C_2) delivering the same level of welfare,

$$u(c_1)+\beta U(C_2),$$

with $U(C_2)$ defined in (SP). The thin solid lines delimit the set of feasible allocations to the small open economy in the first-best problem, that is, those that satisfy (3.2). The tangency between the indifference curves and this feasibility line represents the first-best consumption allocation, denoted by (c_1^{fb}, C_2^{fb}). In both panels, we denote the endowment point (y_1, Y_2) by Y and the consumption allocation in the best equilibrium by E.

Figure 1(a) describes the case in which $\Delta(0) \leq 0$. As discussed in Proposition 1, the best equilibrium features the first-best consumption allocation and the nominal interest rate that guarantees $\Delta(i)=0$. The graph is also useful in understanding why changes in foreign reserves are locally irrelevant, as we discussed in Corollary 2. Specifically, foreign reserves holdings F by the monetary authority shift the endowment point from point Y to point $\tilde{Y}=(y_1-F, Y_2+F/\overline{q})$. When F is small (*i.e.* $F<y_1+\overline{w}-c_1^{fb}$), these interventions have no effects on the equilibrium consumption allocation because the private sector undoes the external position taken by the monetary authority by borrowing more from foreigners.[13]

Figure 1(b), instead, describes the case in which $\Delta(0)>0$. As discussed in Proposition 2, the best equilibrium features a nominal interest rate equal to 0 and deviations from interest rate parity given by $\Delta(0)$. The dash-dotted line represents the constraint (3.14) evaluated at $i=0$. This line is parallel to the first-best feasibility constraint, but reduced by a magnitude $\Delta(0)\overline{w}$, which captures the profits of foreign intermediaries and the losses for the small open economy. The best equilibrium is the point on this line where the slope of the indifference curve satisfies (3.15) with $i=0$. This slope is $(1+\Delta(0))/\overline{q}$ and is represented in the figure by the dashed line. This dashed line is also useful for understanding the role of reserves. In particular, its intersection with the first-best feasibility constraint, denoted by $\tilde{Y}$, determines the magnitude of the foreign

13. Note that, as mentioned previously, if $\Delta(0)<0$, the domestic interest rate is strictly positive, and the economy will be away from the Friedman rule.

reserve accumulation that is necessary to implement the best equilibrium. The figure shows that it is useful to decompose the welfare reduction that arises as a consequence of the exchange rate policy into two channels: a resource loss, captured by the parallel shift in the thin solid line, and the intertemporal distortion, captured by the steeper dashed line.

In this section, we assumed that the monetary authority takes as given the exchange rate policy to focus on the optimal implementation. Clearly, there are reasons why the monetary authority might choose these exchange rate policies in the first place, and one may worry that, in a more general model where the exchange rate is endogenous, the monetary authority might choose an implementation that is not the best implementation characterized above. In the Supplementary Appendix, however, we show that this concern is not valid in our setup. That is, even though the monetary authority optimally chooses an exchange rate policy, it will carry it out using the best implementation described in this section.

4.4. *Comparative statics*

Let us briefly discuss two comparative statics of the model by zooming in on the two terms that determine the losses: $\overline{w}$ and $\Delta(i)$.[14]

Consider first an increase in intermediary capital, $\overline{w}$, in a situation in which $\Delta(0)>0$ and the monetary authority sets $i=0$ to implement the best equilibrium. As can be seen from equation (3.14), an increase in $\overline{w}$ increases the losses because intermediaries are able to obtain higher profits. Because of the higher losses and the fact that there are no changes in the intertemporal distortion, equation (3.15), households are unambiguously worse off. We can also see from Figure 1(b), that an increase in intermediary capital induces a higher reserve accumulation by the monetary authority. If intermediaries are better capitalized, the interventions done by the monetary authority to reverse the capital inflows need to be larger.[15] Note that when intermediary capital is larger than $(y_1+\overline{q}Y_2)/\Delta(i)$, the exchange rate policy is infeasible, as the losses to intermediaries are larger than the present value of the country's endowment.

The second comparative statics refer to the role of the exchange rate policy, the foreign interest rate, and the role of risk, all captured in $\Delta(0)$. From equation (4.1), $\Delta(0)$ increases when (1) there is a larger expected appreciation of the domestic currency, (2) the covariance of the appreciation rate with the stochastic discount factor of the intermediaries is larger, and (3) i^* is lower. For a given $\overline{w}$, the increase in $\Delta(0)$ has two effects on the best equilibrium. It increases the magnitude of the losses in (3.14) and increases the intertemporal distortion as compared to the first-best allocation, as seen in (3.15).[16] As a result, the domestic households are unambiguously worse off. Similar to the discussion above, a larger $\Delta(0)$ also requires a larger reserve accumulation by the monetary authority.

This discussion highlights that if a country is better integrated with the international financial markets (*i.e.* financial intermediaries can invest more resources in it) or if its currency has better hedging properties (*i.e.* it is a safe haven), then the larger are the interventions required to implement the exchange rate policy under a binding zero lower bound, and the larger are the associated costs.

14. For more detail on the arguments, we refer the reader to an earlier version of this article (Amador *et al.*, 2017).

15. It is important to highlight that an increase in intermediary capital is not beneficial in part because there is already enough capital to finance the first-best consumption (Assumption 2).

16. A reduction in i^*, which is equivalent to an increase in $\overline{q}$, also affects the resource constraint, equation (3.14). One may have conjectured that whether such a reduction is beneficial would depend on whether the economy is a net external lender or a borrower. However, we can show that with a binding zero lower bound, a reduction in i^* unambiguously reduces welfare, even for a net external borrower. The key is that with a binding zero lower bound, a net external borrower effectively borrows at a rate higher than i^*, and the monetary authority ends up saving at a lower interest rate.

4.5. *Alternative policies*

The comparative statics with respect to $\overline{w}$ underscores that restricting the quantity of foreign inflows is beneficial for an economy that pursues an exchange rate policy under a binding zero lower bound constraint. By restricting foreign inflows, the government reduces the resource losses generated by the resulting the deviation from interest rate parity. In an earlier version of this article, Amador *et al.* (2017), we analyse capital controls, in the form of either taxes on foreign inflows of capital or quantity restrictions. There, we show that both of these tools allow the government to implement any exchange rate policy for any nominal interest rate without the need to engage in *costly* foreign exchange interventions.[17]

In practice, however, governments face both implementation hurdles and potential costs of introducing capital controls. First, capital controls often require some form of coordination between the monetary authority and the fiscal authority, whereas foreign exchange interventions can usually be carried out directly by the central bank. Second, capital controls are subject to well-known evasion problems. With nominal interest rates close to zero, it becomes especially challenging to implement capital controls, as money or near-money financial instruments need to be taxed for capital controls to be effective; if not, capital inflows would redirect toward these financial instruments. Finally, other costs that we do not model, for example, reputational considerations, might arise.

Rather than imposing capital controls or engaging in costly foreign exchange interventions, the central bank could alter its exchange rate policy. In our benchmark analysis so far, the exchange rate policy is given. However, in the Supplementary Appendix, we consider a model with nominal rigidities in which the exchange rate policy is a government's choice. With nominal rigidities, the level of the exchange rate matters. In particular, we consider a situation in which, in the absence of the zero lower bound, the optimal policy (which achieves perfect stabilization) involves a depreciated current exchange rate and an appreciated future one. When facing the zero lower bound, choosing such a policy may violate interest rate parity, requiring interventions in foreign exchange markets with their associated costs. If the cost of interventions is high (*e.g.* if $\overline{w}$ is large), the government will depreciate the future exchange rate, deviating from perfect stabilization in the future. This can be interpreted as a form of forward guidance. When the cost of interventions is low (*e.g.* if $\overline{w}$ is low), the government resorts to foreign exchange interventions, as these reduce the deviation from perfect stabilization in the future. In other words, foreign exchange rate interventions and forward guidance are substitutes, and, depending on the fundamentals that affect the cost of foreign exchange interventions, the government may rely more heavily on one policy or the other.

5. MEASURING THE COSTS OF FOREIGN EXCHANGE INTERVENTIONS

In the previous sections, we have shown that certain exchange rate policies require the monetary authority to actively intervene in foreign exchange markets and that these interventions are costly for the small open economy. We have identified two distinct welfare costs associated with these interventions: an intertemporal distortion in the consumption allocation and a resource cost. This latter in our stylized model is the product of two objects: the deviations from interest rate parity, $\Delta(i)$, and the amount of capital that foreign intermediaries devote to the small open economy, $\overline{w}$. In this section, we show how to use available data to measure this second cost.

17. Taxes on foreign inflows, depending on the direction of capital flows, may be superior to quantity restrictions. For more details, see the cited paper.

5.1. *Measuring* $\Delta(i)$

In the literature, measuring return differentials on bonds denominated in different currencies can be done in two ways: the UIP condition and the CIP condition. An important question is which of these two conditions should be used as a proxy for $\Delta(i)$ when measuring the costs of foreign exchange interventions. A standard practice in the literature is to use deviations from the UIP condition; see, for example, Adler and Tovar Mora (2011) and Adler and Mano (2016). In what follows, we show that UIP deviations are, in general, not the right empirical counterpart to $\Delta(i)$. We next show that, under reasonable assumptions, CIP deviations should be used instead to proxy for $\Delta(i)$.

We start by rewriting the resource loss per unit of capital inflow, $\Delta(i)$ in equation (3.13), as follows:

$$\Delta(i)=\underbrace{\left\{\frac{1+i}{1+i^{*}}\mathbb{E}\left[\frac{e_1}{e_2(s)}\right]-1\right\}}_{\text{UIP deviation}}+\underbrace{\mathbb{C}\text{ov}\left[\frac{q(s)}{\pi(s)},\frac{e_1}{e_2(s)}\right]}_{\text{risk premium}}.$$

From the above equation, we can immediately see that deviations from UIP would be an imperfect measure of $\Delta(i)$ as long as the risk premium component is different from zero.

A simple example might be useful in explaining why UIP should not be used to measure the costs of foreign exchange interventions. Consider a situation in which $\Delta(i)=0$ but the deviations from UIP are negative. In our model, this occurs when the currency of the small open economy has good hedging properties (when it appreciates in bad times for foreign financial intermediaries). Assume also that the monetary authority in period 1 accumulates foreign reserves and finances this accumulation by issuing a domestic currency risk-free bond. The returns from this strategy per unit of foreign bond purchased are

$$r_2(s)=1-\frac{1+i}{1+i^{*}}\frac{e_1}{e_2(s)}.$$

It is clear from this example that the monetary authority makes profits on average from this strategy because it is shorting assets with low yields and purchasing high-yielding ones. That is, $\mathbb{E}[r_2(s)]>0$. However, it should also be clear that these profits, when appropriately discounted, equal zero from an ex ante perspective. Indeed, using equal gaps, we have

$$\mathbb{E}\left[\frac{\beta u'(c_2(s))}{u'(c_1)}r_2(s)\right]=\mathbb{E}\left[\frac{\Lambda(s)}{1+\Delta(i)}r_2(s)\right]=\frac{\Delta(i)}{1+\Delta(i)}.$$

Thus, if $\Delta(i)=0$, from the perspective of the households, the economy is not gaining or losing anything from the monetary authority's strategy: the monetary authority is purchasing a riskier asset than the one it is shorting, and the profits it receives in expectation exclusively reflect a fair compensation for undertaking such risk.[18]

If UIP deviations are not the right measure, could we use deviations from CIP to proxy for $\Delta(i)$? To examine CIP deviations within our model, we need to open a forward exchange rate

18. When $\Delta(i)=0$, this result is related to Backus and Kehoe (1989). Backus and Kehoe (1989) showed that changes in the currency composition of the government's balance sheet that do not alter the ex post payout from the balance sheet in any state are irrelevant. Our result is stronger, as we do not impose that the payout from changes in the government balance sheet remains the same. This is due to the presence of lump-sum transfers in our environment.

market. Given that we have complete markets in each of the two regions (domestic and foreign asset markets), we could open such a forward market in either of the two.

Let us consider, then, the price of a forward exchange rate contract in the international financial markets.[19] The idea is to consider the following trade. A foreign household has a claim to a unit of domestic currency in period 2. She would like to exchange it for a claim to a constant amount of foreign currency in period 2. Let $\hat{e}$ denote the price of this contract (*i.e.* the forward exchange rate). The value $\hat{e}$ must satisfy the following condition:

$$\sum_{s\in S} q(s)\left[\frac{1}{e_2(s)} - \frac{1}{\hat{e}}\right] = 0, \tag{5.1}$$

which implies that the forward exchange rate equals

$$\hat{e} = \frac{\sum_{s\in S} q(s)}{\sum_{s\in S} \frac{q(s)}{e_2(s)}}.$$

From the definition of $\Delta(i)$, we have that

$$\Delta(i) = \left[\sum_{s\in S} \frac{q(s)e_1}{e_2(s)}\right](1+i) - 1 = \underbrace{\frac{1+i}{1+i^*}\frac{e_1}{\hat{e}} - 1}_{\text{CIP deviation}}.$$

Hence, direct observation of a CIP deviation provides a correct estimate of a loss per unit of capital inflow, $\Delta(i)$.

This distinction between UIP and CIP is an important one. Going back to our previous example, a safe-haven currency might experience negative deviations from UIP and, at the same time, observe positive deviations from CIP. If we were to use deviations from the UIP condition, we would incorrectly conclude that the small open economy is gaining from foreign exchange interventions while, in reality, these interventions are costly.

As we discuss in our empirical application, this situation is indeed relevant when studying the experience of the SNB. The literature also discusses an alternative interpretation to the safe haven. Consider the case in which a *safety premium* arises not from the risk properties of domestic assets vis-à-vis foreign ones, but rather from foreigners having a strict preference for holding the asset perceived to be safe. As a result, they are willing to hold this asset even when its risk-adjusted rate of return lies strictly below that of foreign ones. In this case, for example, the SNB can, by creating monetary liabilities and accumulating U.S. assets, generate ex ante discounted profits as long as this safety premium on its monetary liabilities vis-à-vis U.S. dollar assets is strictly positive.[20] However, under this interpretation with perfect arbitrage, the CIP deviation between the Swiss franc and the U.S. dollar would be negative: foreigners should be indifferent between holding a Swiss asset at a lower rate of return and holding an equivalent U.S. security and selling forward its dollar return back into Swiss francs. As we show next, there is no evidence of a

19. Under Assumption 2, that is, under equal gaps, it does not matter in which market the forward contracts are traded, as they both deliver the same prices. The reason is that under equal gaps, foreign markets and households share the same ratio of marginal utilities across states in period 2 (*i.e.* the only distortion is intertemporal). A forward contract is a trade across states in period 2, and thus, the forward price should be the same in both markets.

20. This argument is similar to how seigniorage generates revenue for the government in monetary models.

negative CIP deviation between Swiss francs and U.S. dollars. Note that this does not contradict the argument that safe asset demand and supply considerations played an important role during and after the financial crisis of 2008—a point argued strongly in a recent literature, summarized in Caballero *et al.* (2017). Rather, our point is that the measured difference in rates of return does not justify the view that the safety premium on Swiss francs was particularly higher *than for other safe assets* (*i.e.* U.S. securities) during this period.

It is worth noting that CIP deviations are the right measure of the costs of intervention for *any* nominal interest rate. While the best equilibrium in the model features deviations from interest rate parity only when the government is at the zero lower bound, our measure of losses still applies for strictly positive rates. For example, there could be cases in which the central bank has an exchange rate policy but is reluctant to cut interest rates all the way to zero, either because of financial stability considerations or because of other macroeconomic objectives.[21] In these cases, one would observe central bank interventions away from the zero lower bound, and the measure of the costs of intervention would still be captured by deviations from CIP. Emerging markets are an especially relevant case of interventions away from the zero lower bound, as their central banks often purchase reserves even when the domestic nominal rate is high.[22]

5.2. *Measuring $\overline{w}$*

In order to measure the costs of foreign exchange interventions, we need to measure the amount of capital that foreign intermediaries can invest in the small open economy, $\overline{w}$. Unfortunately, this object cannot be directly measured. However, we show that we can use additional equilibrium relations of the model in order to approximate the resource costs using the foreign reserves accumulated by the monetary authority.

Specifically, as we show in Appendix B, we can rewrite the intertemporal resource constraint of the small open economy as follows:

$$y_1 - c_1 + \frac{\overline{q}}{1+\Delta(i)}(Y_2 - C_2) = \underbrace{\frac{\Delta(i)}{1+\Delta(i)}\overline{q}F}_{\text{alternative measure}}, \tag{5.2}$$

which corresponds to the dashed line in Figure 1(b) when $i=0$. Thus, we can approximate the resource loss using the reserves accumulated by the monetary authority and multiplying the amount of foreign reserves by the CIP deviation.

5.3. *Infinite horizon and balance sheet composition*

Two final aspects remain to be addressed regarding our measurement of the costs. First, so far, we have studied a two-period model. The lack of a multiperiod framework makes it difficult to uncover, for example, whether it is the flows or the stocks of reserves that matter in measuring the costs. Second, while in our analysis we have restricted the monetary authority to issue or purchase

21. An example in which the central bank wants to keep a high interest rate in the face of capital inflows is in Corsetti *et al.* (2018). They consider an economy with incomplete markets where interest rate policies can improve international risk sharing.

22. One challenge to measure the losses for emerging markets, however, is the need to obtain interest rates that are free of default risk. In addition, we should note that governments in emerging markets often accumulate reserves for other concerns that go beyond explicit exchange rate management, notably insurance against sudden stops (see *e.g.* Bianchi *et al.*, 2018).

risk-free domestic and foreign bonds, in practice the balance sheet of central banks contains several types of assets and liabilities that differ, for example, by currency of denomination and maturity. A relevant question is whether and how we should account for these different financial assets when computing the costs of interventions.

We tackle these two issues by extending our setting to an infinite horizon economy. Let s^t now index the history of state realizations up to time t. Let $F(s_{t+1}, s^t)$ denote the realized value of the portfolio of foreign reserves in the subsequent state (s_{t+1}, s^t). This value $F(s_{t+1}, s^t)$ is allowed to be state dependent to account for all the potentially different maturities, currencies of denomination, or risk properties of the underlying assets held by the monetary authority. However, independently of the underlying securities that make up the portfolio, the value of the foreign reserve portfolio at the end of period t is

$$\sum_{s_{t+1} \in S} q(s_{t+1}, s^t) F(s_{t+1}, s^t).$$

In Appendix C, we show that under allocations satisfying equal gaps, and taking as given future policies, we can write the resource losses for the small open economy between periods t and $t+1$ in a way that is analogous to equation (5.2):

$$\tilde{y}(s^t) - c(s^t) + \frac{\bar{q}(s^t)(\tilde{Y}_2(s^t) - C_2(s^t))}{1+\Delta(s^t)} = \frac{\Delta(s^t)}{1+\Delta(s^t)} \sum_{s_{t+1} \in S} F(s_{t+1}, s^t) q(s_{t+1}, s^t), \tag{5.3}$$

where $\tilde{y}(s^t)$ and $\tilde{Y}_2(s^t)$ are the "effective endowments" in period t and $t+1$, respectively (see equations (C.8) and (C.9) in Appendix C). The former is constructed by summing the initial net foreign asset position to the period t endowment, while the latter also consolidates the value of the next-period endowment with the next-period savings policy.

We wish to emphasize two main points. First, our measure of resource costs can be interpreted more generally as the *one-period-ahead* costs incurred by the monetary authority taking as given future policies. Second, to approximate the losses at any period, we just need to compute the one-period deviations from CIP and the end-of-period value of the *stock* of *total* reserves. The composition of the monetary authority's balance sheet is irrelevant in equation (5.3) because arbitrage returns are equalized across all securities under an equal gaps allocation. Thus, the market value of total reserves is enough to compute the losses.

6. EMPIRICAL ANALYSIS

In this section, we first use our theoretical results to quantify the resource cost of foreign exchange interventions in the case of Switzerland over the period 2010–7. We argue below that Switzerland during this period is a good example of the economic circumstances analysed in this article: an interest rate close to or at its lower bound, an explicit exchange rate policy, a large accumulation of reserves by the SNB, and persistent and significant CIP deviations.

We then discuss how our framework is useful for understanding the patterns of CIP deviations, interest rates, and foreign reserve accumulation by central banks observed after the financial crisis for major international currencies. We finally show that similar patterns were also observed in another (rare) early episode of interest rates at their lower bound, that is, Switzerland in the late 1970s.

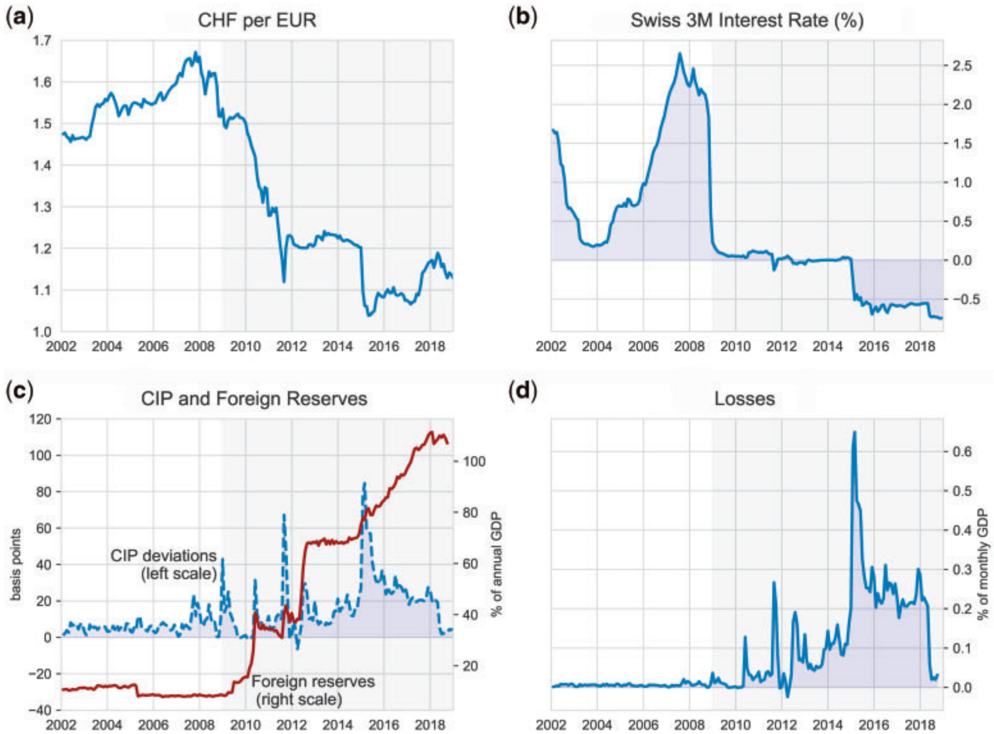

FIGURE 2

Exchange rate, interest rate, foreign reserves, CIP deviations, and losses

Notes: The shaded areas represent the months in which the Swiss interest rate was below 0.5%. The CIP deviation between the Swiss franc and the euro is annualized . Panels (a), (b), and (c) plot monthly averages of daily observations of the exchange rate, the interest rate, and the CIP deviation, respectively. Foreign reserves and losses are shown at the monthly level.

6.1. *The case of the Swiss franc: 2010–2018*

Following the global financial crisis, as shown in Figure 2(a), the Swiss franc appreciated by roughly 25% against the euro. The SNB perceived this appreciation to be damaging for the Swiss economy, and to counteract it, it established a currency floor of 1.2 Swiss francs per euro in 2011.[23] The SNB kept this floor until January 2015, when the floor was abandoned and the Swiss franc appreciated by 15% vis-à-vis the euro.

As Figure 2(b) shows, throughout the 2010–8 period, nominal interest rates in Switzerland were at or below zero. Moreover, all throughout this period there were times in which financial markets assigned a non-trivial probability of franc appreciations (Jermann, 2017), and these expectations of appreciation were correlated with bad economic conditions in Europe and worldwide.[24] The SNB experience during the 2010–7 period is well described by our simple model: a central bank with an interest rate at the zero bound, pursuing an exchange policy that makes its own domestic currency assets attractive relative to a reference foreign currency. Our

23. The 2011 Q3 SNB Quarterly Bulletin stated that with the 1.20 floor, “the SNB is taking a stand against the acute threat to the Swiss economy and the risk of deflationary development that spring from massive overvaluation of the Swiss franc.”

24. As the European crisis deepened following the Greek elections of May 2012, there was increased speculation that the SNB could impose capital controls or abandon the currency floor. See, for example, Alice Ross and Haig Simonian, “Swiss eye capital controls if Greece goes,” *Financial Times*, 27 May 2012, and the article mentioned in footnote 2.

theoretical analysis predicts that, under these circumstances, we should observe foreign reserve accumulation by the SNB, concurrent with strictly positive CIP deviations for the Swiss franc. Figure 2(c) shows that this is indeed the case.

Figure 2(c) reports the (annualized) three-month CIP deviations between the Swiss franc and the euro, along with a monthly series for the stock of foreign reserves held by the SNB as a fraction of annual (trend) Swiss GDP.[25] The panel shows that CIP deviations were virtually absent before the 2008 financial crisis and that these deviations spiked during the crisis.[26] More interestingly for our purpose, the panel shows that, starting in 2010, there is a tight connection between large *positive* CIP deviations and increases in SNB holdings of foreign reserves. First, post-crisis spikes in the CIP deviations correspond to large increases in reserve accumulation (which brought Swiss foreign reserves from 10% to 80% of GDP). Also, over the 2016–7 period, historically sizable CIP deviations (between 20 and 40 basis points) have corresponded with additional reserve accumulation, bringing SNB reserves to 110% of GDP.

In view of our discussion in Section 5, we can use these series to measure the resource costs associated with these foreign exchange interventions. Specifically, we let a period be a month, and, based on equation (5.3), we calculate the losses in period t using

$$\text{Losses}_t = \frac{\Delta_t}{1+\Delta_t} \times F_t, \tag{6.1}$$

where F_t represents the market value of the stock of reserves held in period t, and Δ_t the corresponding one-period CIP deviation.

We let F_t equal the value of the stock of foreign reserves held by the SNB (in current Swiss francs). We approximate Δ_t using the CIP deviations observed for three-month-ahead assets denominated in Swiss francs and euros. Specifically, we let

$$\Delta_t = \left[\frac{\left(1 + \frac{i_t^{\text{CHF,3M}}}{100}\frac{1}{4}\right)}{\left(1 + \frac{i_t^{\text{EUR,3M}}}{100}\frac{1}{4}\right)} \frac{e_t}{\hat{e}_t^{\text{3M}}} \right]^{1/3} - 1, \tag{6.2}$$

where $i_t^{\text{CHF,3M}}$ is the quoted Swiss franc denominated overnight index swap (OIS) rate; $i_t^{\text{EUR,3M}}$ is the quoted OIS rate on euro denominated swaps; e_t denote the spot exchange rate between the Swiss franc and the euro; and $\hat{e}_t^{\text{3M}}$ is the three-month forward exchange rate between the Swiss franc and the euro. All these values are taken at their average between the bids and asks. Note that the resulting Δ_t is a monthly rate. We report the losses as a fraction of monthly (trend) GDP.[27]

The loss is reported in Figure 2(d). As can be seen, the costs of foreign exchange interventions after 2010 were significant, reaching around 0.6% of monthly GDP around January 2015, the month when the SNB decided to abandon the currency floor vis-à-vis the euro.

25. For this analysis, we use the CIP deviations with respect to the euro, as this was the currency used for the floor on the Swiss franc. The deviations with respect to the U.S. dollar are similar in behaviour but larger in magnitude.

26. See, for example, Baba and Packer (2009) for a discussion of how tightening financial constraints might explain the deviations during the financial crisis.

27. The OIS, spot, and forward rates (both bids and asks) are at a daily frequency, obtained from Bloomberg, and averaged over their respective months. The data on foreign reserves and GDP are from the IMF International Financial Statistics and OECD Quarterly National accounts, respectively. Foreign reserves are in Swiss francs at a monthly frequency, while the GDP series is current Swiss francs at a quarterly frequency. To obtain monthly trend GDP, we HP-filtered the quarterly GDP series (with a smoothing parameter of 1,600) and imputed a monthly value from its trend. We choose to use the three-month CIP deviation rather than the one-month CIP deviation because there is less high-frequency variation in the former. The results are not significantly affected by this choice.

Recall from the discussion of Figure 1 that the welfare reduction can be decomposed in two channels. The first channel is the resource loss, which we have computed above. A second channel arises from the intertemporal distortion generated by the wedge in the Euler equation, $\Delta(0)>0$. In terms of first-period consumption, a second-order approximation to this intertemporal distortion implies that the loss is $\frac{1}{\sigma}\Delta(0)^2$, where $1/\sigma$ is the intertemporal elasticity of substitution. Using $\Delta(0)\approx 50$ basis points and $\sigma=2$, we get that this loss is around 0.000625% of first-period consumption, a tiny number when compared to the first channel. This reflects that the intertemporal distortion generates a second-order loss (a triangle), while the resource loss is first order (a rectangle).

6.2. *CIP deviations, foreign reserves, and interest rates across countries*

While the recent Swiss experience provides a clear example of the economic forces studied in this article, we now argue that our results are also useful for interpreting other experiences. In a recent paper, Du *et al.* (2018) have documented that well after the financial crisis of 2008, substantial deviations from CIP have been persistently observed for several advanced economies. In this section, we use the same set of countries studied by these authors and document that (1) positive deviations from CIP are concentrated in countries/periods where the nominal interest rate is close to zero and (2) deviations from CIP are positively related to foreign reserves accumulated by the monetary authority. These results support the idea that some of the CIP deviations observed after the financial crises are due to a conflict between exchange rate policies and the zero lower bound on nominal interest rates.

We collect data on exchange rates (both spot and forward rates) against the U.S. dollar, and on the nominal interest rate (OIS) for the Japanese yen, Danish krone, Swedish krona, Canadian dollar, British pound, Australian dollar, New Zealand dollar, and Swiss franc over the 2010–8 period. We also collect data on total foreign exchange reserves held by monetary authorities in these countries.[28]

Figure 3(a) plots the monthly average of the CIP deviations for each of these currencies (with respect to the U.S. dollar) against their corresponding nominal interest rates. The panel shows that CIP deviations are positive for countries and time periods characterized by very low nominal interest rates, whereas they tend to be small when nominal interest rates are positive. A negative relation between CIP gaps and nominal interest rates has also been documented by Du *et al.* (2018). This graph highlights the non-linearity of the relations: CIP deviations are large only when interest rates are close to zero. This finding lends support to our result that CIP deviations are only part of an optimal equilibrium when the zero lower bound constraint on the nominal interest rate binds.

28. CIP deviations against the U.S. dollar are computed using three-month OIS interest rates and three-month forward exchange rates as in equation (6.2) but using the OIS rate on dollar-denominated swaps as the foreign rate. We correct the local interest rate for differences in the market-day-count conventions where appropriate. The set of countries is the same as in Du *et al.* (2018) with the exception of Norway, which we exclude from our sample because it has no OIS rate. Prior to this period, and with the exception of the 2008–9 financial crisis, CIP deviations were essentially zero for all of these currencies. We exclude from our analysis the very volatile period of the financial crisis. The dollar exchange rate and OIS data were collected at a daily frequency from Bloomberg and were averaged over their respective months. Data sources and methodology for computing the reserve to GDP ratio are the same used for the Swiss series and are detailed in footnote 27. We again choose the three-month CIP deviation over the one-month for the calculations. The results in Figure 3 and Table 1 are not significantly affected by this choice. We have also experimented with including Norway in the sample, by constructing CIP deviations for Norway, using the Norwegian central bank rate as a proxy for OIS. Again, the results in Figure 3 and Table 1 are not affected.

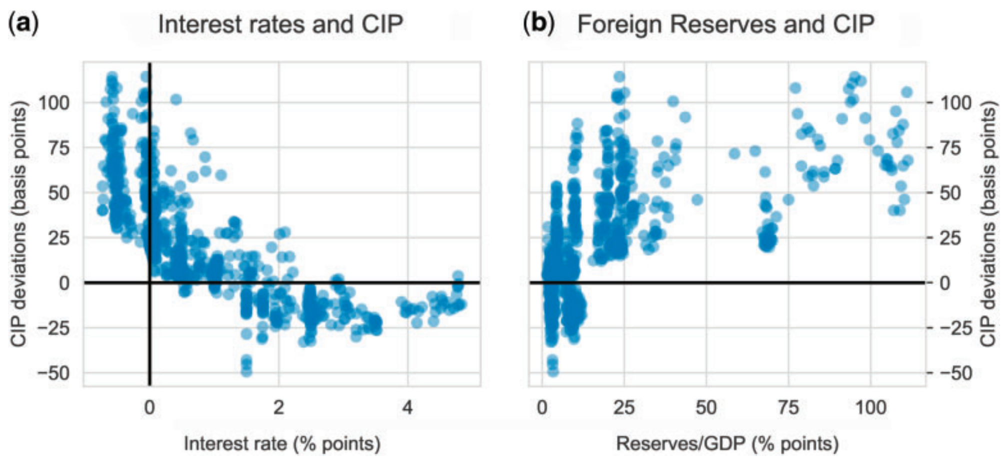

FIGURE 3

Relation between CIP gaps, reserves, and interest rates

Notes: Interest rates and CIP deviations are monthly averages of their respective daily observations. Darker dots represent overlapping observations.

Figure 3(b) plots these monthly CIP deviations against the corresponding level of foreign reserves (normalized by trend GDP). The figure shows a positive relationship between the level of foreign reserves held by the monetary authority and the deviations from CIP. This empirical finding, which to the best of our knowledge has not been previously noted in the literature, is consistent with the mechanism at the heart of our model, whereby the monetary authority is able to sustain a positive CIP deviation by accumulating a sufficiently large position in foreign assets.

We complement Figure 3 with Table 1, which shows the results of regressing the monthly CIP deviations on measures of foreign reserves. The purpose of these regressions is to highlight the correlation between the variables, and no direction of causality should be inferred from them. The table shows that the positive association between CIP deviations and foreign reserves is robust. Specifically, this association holds whether we include country and time fixed effects, whether we drop the Swiss franc from the sample, and whether we do the analysis with the level or the first difference in foreign reserves.[29]

6.3. *Other episodes of exchange rate policies at the zero lower bound*

This last section provides two additional examples of countries that followed explicit exchange rate policies at the zero lower bound.

6.3.1. Switzerland in the 1970s. Figure 4(a) shows the monthly time series for the Swiss franc against the U.S. dollar for the period 1977–9, and it shows that the Swiss franc had been steadily appreciating against the U.S. dollar, just as it did in the aftermath of the 2007–9 crisis.[30] In an effort to prevent further appreciations, the SNB initially reduced the domestic rate, which by the end of 1978 reached levels close to zero (see the shaded area in Figure 4(b)). At this point,

29. We use both the level and the changes in foreign reserves to account for potentially different dynamics in the supply of intermediary capital that could imply different levels of intervention.

30. See Claire Jones, "Swiss tried to put ceiling on franc before", *Financial Times*, 6 September 2011, for a description of the macroeconomic environment in Switzerland at the time.

TABLE 1
CIP deviations and foreign reserves

	(1) No fixed effects	(2) With fixed effects	(3) Excluding Switzerland	(4) No fixed effects	(5) With fixed effects	(6) Excluding Switzerland
$(F/Y)_t$	79.4***	38.2**	222.6***			
	(18.3)	(11.2)	(31.0)			
$\Delta(F/Y)_t$				470.9***	195.7***	187.9*
				(94.3)	(8.0)	(77.5)
Country/time FE	No	Yes	Yes	No	Yes	Yes
N	831	831	726	831	831	726
Adjusted R^2	0.37	0.84	0.84	0.03	0.83	0.82

Clustered (at country level) standard errors in parentheses. * $p<0.10$ ** $p<0.05$, *** $p<0.01$.

Notes: The dependent variable is the (annualized) CIP deviation against the U.S. dollar in basis points, computed using three-month OIS and forward rates. F/Y is the value of foreign reserves divided by (annualized) trend GDP computed as in footnote 27. $\Delta(F/Y)$ is the monthly difference in F/Y with respect to the previous month. The sample includes monthly (average) observations from January 2010 to September 2018 for the Swiss franc, Japanese yen, Danish krone, Swedish krona, Canadian dollar, British pound, Australian dollar, and New Zealand dollar. The regressions without fixed effects include a constant term.

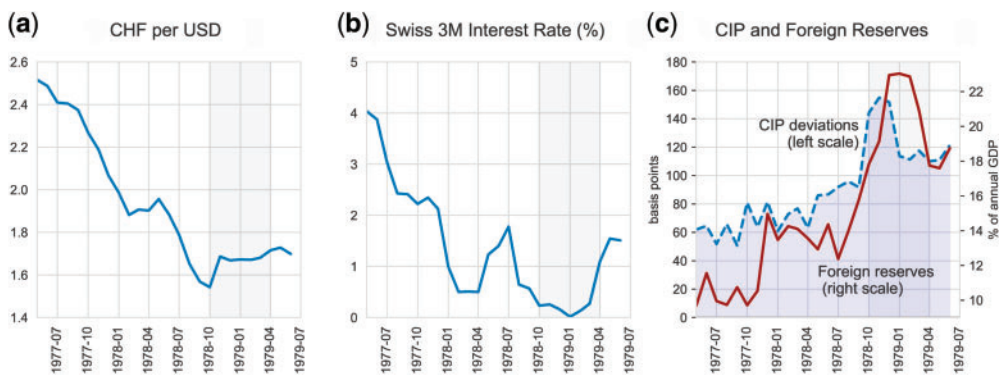

FIGURE 4

Foreign reserves, interest rates, and CIP deviations: Switzerland, 1977–9

Notes: The shaded areas represent the months in which the Swiss interest rate was below 0.5%.

just as it did in 2011, the SNB announced a temporary floor between the Swiss franc and the deutsche mark, and, to maintain the floor, it engaged in large foreign exchange interventions. Figure 4(c) shows the monthly time series of foreign reserves (excluding gold, as a fraction of trend GDP), together with CIP deviations between the Swiss franc and the U.S. dollar, calculated similarly to the previous section.[31]

Figure 4(c) shows that the ratio of foreign reserves to GDP increased by over 10% of GDP, and around the same time, the deviations from CIP increased by over 50 basis points. By mid-1979, the international macroeconomic conditions changed substantially, and the SNB was able to avoid

31. The data source is different, since Bloomberg data are not available for this early period. Three-month nominal interest rates are interbank rates from the OECD Main Economic Indicators, and daily spot and three-month forward rates are provided by the SNB.

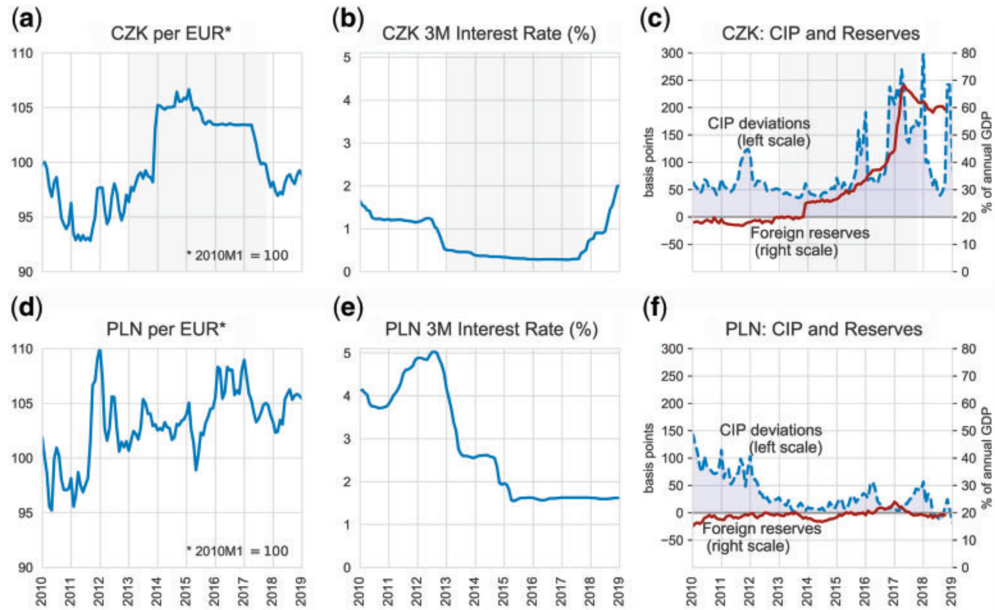

FIGURE 5

Foreign reserves, interest rates, and CIP deviations: Czech Republic and Poland, 2010–8

Notes: The shaded areas represent the months in which the Czech interest rate was below 0.5%. The deviations of CIP are annualized and computed between the local currency and the U.S. dollar using three-month interbank lending rates and forward exchange rates. The panels show monthly averages of daily observations of the exchange rate, the interest rate, and the CIP deviation, respectively.

appreciation of the currency while maintaining a positive interest rate. As a consequence, both the level of foreign reserves and the deviations from CIP abated.

6.3.2. Czech Republic in the 2010s. A more recent example of an economy pursuing an exchange rate policy at the zero lower bound is the Czech Republic.[32] Following a period of poor economic performance in the aftermath of the 2008 crisis, the Czech National Bank depreciated the exchange rate in 2014 and imposed a floor against the euro (see Figure 5(a)). This was done just as the Czech nominal interest rate reached a value very close to zero (Figure 5(b)). Unlike the Swiss experience, however, the Czech currency floor came with an expiration date, as in February 2016 the central bank announced that it would abandon the floor in the second quarter of 2017, creating the expectation of an appreciation in the Czech currency. Figure 5(c) shows that as the abandonment date grew closer, sizable deviations from CIP (over 150 basis points, with the sign predicted by our theory) started to emerge, and at the same time the central bank accumulated a large foreign reserve position, reaching almost 70% of GDP by March 2017.

As a comparison, in the bottom panels of Figure 5, we also look at data from Poland, another central European economy commercially integrated with the Eurozone, and with its own currency. Figure 5(d) plots the exchange rate of the Polish currency versus the euro, showing no evidence of an explicit peg of the Polish currency against the euro. Figure 5(e) plots the Polish interest rate, showing that it never got close to the zero lower bound. Finally, Figure 5(f) shows that

32. We are grateful to an anonymous referee for bringing this case to our attention, as well as to Jarda Borovicka and Matteo Maggiori for helpful discussions. See Franta *et al.* (2014) and "Czech central bank removes currency cap," *Financial Times*, 6 April 2017, for overviews of the Czech episode.

in Poland there were no significant deviations from CIP, nor any significant accumulations of reserves. Overall, comparing Poland and the Czech Republic provides a stark example of how exchange rate policies at the zero lower bound are an important factor in understanding foreign reserve accumulation and deviations from CIP.

7. CONCLUSIONS

This article studied the problem of a monetary authority pursuing an exchange rate policy that is inconsistent with interest rate parity because of a binding zero lower bound constraint. We have shown that even if monetary policy is constrained, it can still achieve an independent exchange rate objective by using foreign exchange interventions that result in observable deviations from arbitrage in capital markets. These interventions, however, are costly from the point of view of the domestic economy. We show how these costs can be measured and document that they were substantial in the recent experience of the SNB. Moreover, the main predictions of our theory are consistent with the behaviour of foreign reserves, nominal interest rates, and deviations from the CIP condition for a panel of advanced economies.

The analysis could be extended in several directions. One question is how the optimal intervention policy and the resulting losses need to be adjusted when domestic agents face uninsurable aggregate or idiosyncratic risk. Another question relates to reserve management: in this article, we have shown that reserve accumulation is a necessary tool for conducting an exchange rate policy at the zero lower bound but have restricted the monetary authority to purchase only foreign risk-less bonds. In Amador *et al.* (2018), we allow central banks to choose a portfolio of foreign reserve assets and characterize its optimal composition.

A. OMITTED PROOFS

A.1. *Proof of Lemma 1*

Suppose that Assumptions 1 and 2 hold. Then, the consumption allocation of any monetary equilibrium features equal gaps.

Proof. Toward a contradiction, suppose that Assumption 2 holds and there exists an equilibrium allocation that features *unequal* gaps. Let us define $\kappa(s)$ to be

$$\kappa(s) \equiv \frac{q(s)e_1}{p(s)e_2(s)} - 1 = \frac{q(s)}{\beta\pi(s)}\frac{u'(c_1)}{u'(c_2(s))} - 1.$$

Note that this implies that

$$u'(c_2(s)) = \left(\frac{(1+\kappa(s))\beta\pi(s)}{q(s)}\right)^{-1} u'(c_1).$$

or alternatively:

$$c_2(s) = \mathcal{C}\left(\left(\frac{(1+\kappa(s))\beta\pi(s)}{q(s)}\right)^{-1} u'(c_1)\right), \tag{A.1}$$

where $\mathcal{C}$ is the inverse of u', that is, $\mathcal{C}(u'(c)) = c$.

Let $\bar{\kappa} \equiv \max_{s\in S}\{\kappa(s)\}$. Given that $\kappa(s) \geq 0$ for all s, it follows that $\bar{\kappa} > 0$ (or else the gaps are all equalized). Let $\overline{S} \equiv \{s|\kappa(s) = \bar{\kappa}\}$. Let $S_0 \equiv \{s|\kappa(s) = 0\}$. And let $\underline{S}$ be their complement, $\underline{S} \equiv S/(\overline{S} \cup S_0)$. We allow for either $\underline{S}$ or S_0 to be empty (but not both).

The intermediaries' problem implies that $a^*(s) = 0$ for all $s \in S_0 \cup \underline{S}$, that is, $\kappa(s) < \bar{\kappa}$. In addition, $m^* = 0$, and $\sum_{s\in\overline{S}} p(s)a^*(s)/e_1 = \overline{w}$.

From the households' problem, $f^*(s) = 0$ for $s \in \overline{S} \cup \underline{S}$, that is, $\kappa(s) > 0$. From the trade balance equation, (3.10),

$$c_1 - y_1 = \overline{w} - \left(\sum_{s\in S_0} q(s)f(s) + \bar{q}F\right).$$

Using the budget constraints, market clearing, and $m^*=0$, we obtain that

$$c_2(s)=\begin{cases} y_2(s)+F & s\in \underline{S} \\ y_2(s)+F-a^*(s)/e_2(s) & s\in \overline{S} \\ y_2(s)+F+f(s) & s\in S_0. \end{cases} \tag{A.2}$$

We will use this second-period consumption allocation, together with the intertemporal optimality conditions, to obtain a contradiction of Assumption 2.

From $\sum_{s\in\overline{S}}p(s)a^*(s)/e_1=\overline{w}$, we have that

$$\overline{w}=\sum_{s\in\overline{S}}p(s)a^*(s)/e_1=\frac{1}{1+\bar{\kappa}}\sum_{s\in\overline{S}}q(s)\frac{a^*(s)}{e_2(s)}, \tag{A.3}$$

where the second equality follows from the definition of $\kappa(s)$ for $s\in\overline{S}$. From $c_2(s)$ above, we have that $a^\star(s)/e_2(s)=c_2(s)-y_2(s)-F$ for $s\in\overline{S}$. So substituting this expression into (A.3) and rearranging, it follows that

$$\sum_{s\in\overline{S}}q(s)c_2(s)=\sum_{s\in\overline{S}}q(s)y_2(s)+\sum_{s\in\overline{S}}q(s)F-(1+\bar{\kappa})\overline{w}.$$

Using (A.1) to substitute for $c_2(s)$ as a function of $\kappa(s)$ and c_1, we obtain

$$\sum_{s\in\overline{S}}q(s)\mathcal{C}\left(\left(\frac{(1+\bar{\kappa})\beta\pi(s)}{q(s)}\right)^{-1}u'(c_1)\right)=\sum_{s\in\overline{S}}q(s)y_2(s)+\sum_{s\in\overline{S}}q(s)F-(1+\bar{\kappa})\overline{w},$$

which implies that

$$F=\frac{\sum_{s\in\overline{S}}q(s)\mathcal{C}\left(\left(\frac{(1+\bar{\kappa})\beta\pi(s)}{q(s)}\right)^{-1}u'(c_1)\right)}{\sum_{s\in\overline{S}}q(s)}-\frac{\sum_{s\in\overline{S}}q(s)y_2(s)}{\sum_{s\in\overline{S}}q(s)}+\frac{(1+\bar{\kappa})\overline{w}}{\sum_{s\in\overline{S}}q(s)}. \tag{A.4}$$

For any $s_0\notin\overline{S}$, $c_2(s)\geq y_2(s)+F$ from equation (A.2) and $f(s)\geq 0$. Using (A.1) we get

$$\mathcal{C}\left(\left(\frac{(1+\kappa(s_0))\beta\pi(s_0)}{q(s_0)}\right)^{-1}u'(c_1)\right)\geq y_2(s_0)+F.$$

Using (A.4) to substitute F, and rearranging, it follows that

$$\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\kappa(s_0))\beta}\right)-\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]$$
$$+\sum_{s\in\overline{S}}q(s)y_2(s)-\left(\sum_{s\in\overline{S}}q(s)\right)y_2(s_0)\geq(1+\bar{\kappa})\overline{w}.$$

Using that $\bar{\kappa}>\kappa(s_0)\geq 0$, and that $\mathcal{C}$ is strictly decreasing, the above implies that

$$\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)-\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]$$
$$+\sum_{s\in\overline{S}}q(s)y_2(s)-\left(\sum_{s\in\overline{S}}q(s)\right)y_2(s_0)>\overline{w}$$

Using that the last term on the left-hand side above is bounded above by g_0, we have that[33]

$$\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)-\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]+g_0>\overline{w}. \tag{A.5}$$

We now show that (A.5) contradicts Assumption 2. We proceed in steps according to the different parts of this assumption.

33. Note that $\sum_{s\in\overline{S}}q(s)y_2(s)-\left(\sum_{s\in\overline{S}}q(s)\right)y_2(s_0)=\sum_{s\in\overline{S}}q(s)(y_2(s)-y_2(s_0))\leq\overline{q}\max_{s_1,s_2}(y_2(s_1)-y_2(s_2))=g_0$.

Assumption 2, part (a). In this case, $q(s)/\pi(s)$ is a constant, and as a result, the first two terms of inequality (A.5) cancel out. It then follows that

$$g_0 > \overline{w},$$

contradicting the condition under Assumption 2, part (a).

For the next two parts, let us first note that

$$\frac{\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]}{\sum_{s\in\overline{S}}q(s)} \geq \mathcal{C}\left(\max_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right), \tag{A.6}$$

as the left-hand side is a weighted average, and $\mathcal{C}$ is decreasing. Using the latter also implies that

$$\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right) \leq \mathcal{C}\left(\min_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right). \tag{A.7}$$

Assumption 2, part (b). In this case, $\mathcal{C}(x) = x^{-1/\sigma}$. We can rewrite inequality (A.5) as

$$\left[\frac{\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)}{\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]}-1\right]\left(\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]\right)+g_0 > \overline{w}.$$

Using (A.6) and (A.7), it follows that

$$\frac{\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)}{\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]} \leq \frac{\mathcal{C}\left(\min_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)}{\mathcal{C}\left(\max_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)} = g_1^{1/\sigma}.$$

Hence, inequality (A.5) implies that

$$g_0 + \left(g_1^{1/\sigma}-1\right)\left(\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]\right) > \overline{w}$$

but note that

$$\left(\sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]\right) \leq y_1 + \overline{q}Y_2$$

from the resource constraint and that $c_2(s) = \mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)$ for $s\in\overline{S}$. Using that $g_1^{1/\sigma}-1 \geq 0$, we then obtain

$$g_0 + \left(g_1^{1/\sigma}-1\right)(y_1+\overline{q}Y_2) > \overline{w},$$

a contradiction of the condition in Assumption 2, part (b).

Assumption 2, part (c). In this case, $\mathcal{C}(x) = -\frac{1}{\sigma}\log(x/\sigma)$. Using (A.6) and (A.7), we have that

$$\begin{aligned}
&\left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\frac{q(s_0)}{\pi(s_0)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right) - \sum_{s\in\overline{S}}\left[q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\right]\\
&\leq \left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\min_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right) - \left(\sum_{s\in\overline{S}}q(s)\right)\times\mathcal{C}\left(\max_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}\frac{u'(c_1)}{(1+\bar{\kappa})\beta}\right)\\
&= \left(\sum_{s\in\overline{S}}q(s)\right)\times\frac{1}{\sigma}\log\left(\frac{\max_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}}{\min_{s\in S}\left\{\frac{q(s)}{\pi(s)}\right\}}\right) = \left(\sum_{s\in\overline{S}}q(s)\right)\times\frac{1}{\sigma}\log g_1 \leq \frac{\overline{q}}{\sigma}\log g_1,
\end{aligned}$$

where we also used the functional form of $\mathcal{C}$ for this case, used that $g_1 \geq 1$, and that $\overline{q} \geq \sum_{s\in\overline{S}}q(s)$.

Inequality (A.5) then implies that

$$g_0 + \frac{\overline{q}}{\sigma}g_1 > \overline{w},$$

a violation of the condition in Assumption 2, part (c). ‖

A.2. *Proof of Lemma 2*

Under Assumptions 1 and 2, a consumption allocation $(c_1,\{c_2(s)\})$ *and money holdings* m *are part of an equilibrium given the exchange rate policy* $(e_1,\{e_2(s)\})$ *if and only if there exists an* i *such that:*

$$y_1-c_1+\overline{q}(Y_2-C_2)=\Delta(i)\overline{w}, \tag{3.14}$$

$$\frac{\overline{q}u'(c_1)}{\beta U'(C_2)}=1+\Delta(i)\geq 1, \tag{3.15}$$

$$h'\left(\frac{m}{e_1}\right)=u'(c_1)\frac{i}{1+i}, \tag{3.16}$$

and $\{c_2(s)\}$ *solves the static planning problem (SP) given* C_2*; and where* Y_2 *and* U *are defined in (SP) and (3.4). Household welfare in this equilibrium is*

$$u(c_1)+h(m/e_1)+\beta U(C_2). \tag{3.17}$$

Proof. We will prove the necessary and sufficient parts independently.

The "only if" part. Equation (3.16) follows immediately from the household first-order condition with respect to money balances.

From Lemma 1, we know that equal gaps allocations are the only possible equilibrium under Assumption 2. As a result, $\{c_2(s)\}$ solves problem (SP) with $\bar{q}C_2=\sum_s q(s)c_2(s)$. Note also that (SP) implies $\bar{q}\frac{\pi(s)}{q(s)}u'(c_2(s))=U'(C_2)$.

Let $1+\kappa\equiv\frac{q(s)e_1}{p(s)e_2(s)}=\frac{q(s)}{\beta\pi(s)}\frac{u'(c_1)}{u'(c_2(s))}$, which holds for any s. Note that $\kappa\geq 0$, from (3.6). Under equal gaps, it follows that $\kappa=\Delta(i)$, and thus $1+\Delta(i)\geq 1$. The definition of κ implies that (3.15) holds.

From the resource constraint, (3.9), we have

$$y_1-c_1+\sum_{s\in S}q(s)(y_2(s)-c_2(s))=\Delta(i)\overline{w}.$$

Using $\bar{q}C_2=\sum_s q(s)c_2(s)$, and the definition of Y_2, delivers (3.14).

The "if" part. Consider C_2, c_1, i, and m that solves (3.14)–(3.16). Let $\{c_2(s)\}$ be the associated solution to the (SP) problem.

Let us conjecture an equilibrium with the following properties:

$$p(s)=\frac{\beta\pi(s)u'(c_2(s))e_1}{u'(c_1)e_2(s)};\quad f^*(s)=f(s)=d_1^*=m^*=0;\quad M=m$$

$$F=\frac{1}{\bar{q}}(y_1-c_1+\overline{w});\quad A=\frac{e_1\bar{q}F-M}{\bar{p}};\quad T_2(s)=F-\frac{A+M}{e_2(s)}$$

$$a^*(s)=e_2(s)[y_2(s)+F-c_2(s)];\quad a(s)=A-a^*(s);\quad d_2^*(s)=\frac{a^*(s)}{e_2(s)}$$

The conjectures above guarantee that the budget constraints of intermediaries, households, and the monetary authority are holding, as well as market clearing in both money and domestic securities.

We need to show that $F\geq 0$, and $a^*(s)\geq 0$, and that the households and intermediaries are optimizing.

Toward showing that $F\geq 0$, we note that $c_1\leq c_1^{fb}$ (an implication of the results in Corollary 1). In addition, from Assumption 1,

$$\sum_{s\in S}q(s)\max\{y_2(s)-c_2^{fb}(s),0\}\leq\overline{w}.$$

From the resource constraint of the first-best problem, we can rewrite the above as

$$c_1^{fb}-y_1+\sum_{s\in S}q(s)\max\{c_2^{fb}(s)-y_2(s),0\}\leq\overline{w}.$$

So $c_1^{fb}\leq y_1+\overline{w}$. It follows that $c_1\leq c_1^{fb}\leq y_1+\overline{w}$, and thus $F\geq 0$.

Note that from (3.15), and using that (SP) implies $\bar{q}\frac{\pi(s)}{q(s)}u'(c_2(s))=U'(C_2)$, it follows that

$$c_2(s)=\mathcal{C}\left(\frac{q(s)}{\pi(s)}\frac{u'(c_1)}{(1+\Delta(i))\beta}\right).$$

Toward showing that $a^*(s) \geq 0$, note that, using (3.15),

$$\frac{a^*(s)}{e_2(s)} = y_2(s) + F - c_2(s) = y_2(s) + \frac{1}{\overline{q}}(y_1 - c_1 + \overline{w}) - \mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right).$$

And then from (3.14), we have that

$$y_1 - c_1 + \overline{q}Y_2 - \sum_{s \in S} q(s)\mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right) = \Delta(i)\overline{w}.$$

Using this in the previous equation:

$$\begin{aligned}
\overline{q}\frac{a^*(s)}{e_2(s)} &= \overline{q}y_2(s) - \overline{q}Y_2 + \sum_{s' \in S} q(s')\mathcal{C}\left(\frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right) + (1+\Delta(i))\overline{w} - \overline{q}\mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right) \\
&\geq \overline{w} - \left[\sum_{s \in S} q(s)y_2(s) - \overline{q}y_2(s)\right] - \left[\overline{q}\mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right) - \sum_{s' \in S} q(s')\mathcal{C}\left(\frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)\right] \\
&\geq \overline{w} - g_0 - \left[\overline{q}\mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right) - \sum_{s' \in S} q(s')\mathcal{C}\left(\frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)\right] \equiv H,
\end{aligned}$$

where the first inequality follows from $\Delta(i)\overline{w} \geq 0$, and the second from $g_0 \geq \sum_{s \in S} q(s)y_2(s) - \overline{q}y_2(s)$ (as shown in footnote 33).

To show that $a^*(s) \geq 0$, it suffices to show that, under Assumption 2, $H \geq 0$. We proceed by parts.

Under Assumption 2, part (a). In this case, the terms within square brackets in H cancel, and

$$H = \overline{w} - g_0 \geq 0,$$

where the inequality follows by the condition in Assumption 2, part (a). Hence, $a^*(s) \geq 0$ in this case.

Under Assumption 2, part (b). In this case,

$$\begin{aligned}
H &= \overline{w} - g_0 - \left[\frac{\overline{q}\mathcal{C}\left(\frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)}{\sum_{s' \in S} q(s')\mathcal{C}\left(\frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)} - 1\right]\left[\sum_{s' \in S} q(s')\mathcal{C}\left(\frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)\right] \\
&\geq \overline{w} - g_0 - \left[\frac{\mathcal{C}\left(\min_{s \in S} \frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)}{\mathcal{C}\left(\max_{s \in S} \frac{q(s')}{\pi(s')} \frac{u'(c_1)}{(1+\Delta(i))\beta}\right)} - 1\right](y_1 + \overline{q}Y_2) = \overline{w} - g_0 - (g_1^{1/\sigma} - 1)(y_1 + \overline{q}Y_2),
\end{aligned}$$

where, similar to the Proof of Lemma 1, the first inequality follows from $\sum_{s \in S} q(s)c_2(s) \leq y_1 + \overline{q}Y_2$, and the arguments used to derive (A.6) and (A.7). Hence, the condition in Assumption 2, part (b), guarantees that $H \geq 0$ and thus $a^*(s) \geq 0$ in this case.

Under Assumption 2, part (c). In this case, using the same arguments we used for (A.6) and (A.7), we get

$$H \geq \overline{w} - g_0 - \overline{q}\left[C\left(\min_{s \in S} \frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))}\right) - C\left(\max_{s \in S} \frac{q(s)}{\pi(s)} \frac{u'(c_1)}{(1+\Delta(i))}\right)\right] = \overline{w} - g_0 - \frac{\overline{q}}{\sigma} g_1 \geq 0,$$

where the equality follows from the functional form for $\mathcal{C}$, and that last inequality follows from Assumption 2, part (c). Hence, $a^*(s) \geq 0$ in this case.

The final step is to check the optimality of the households and intermediaries. Given the domestic security prices we conjectured, the households are on their Euler equation for domestic securities, and their money balances are consistent with optimality given (3.16). Given that $q(s) \geq \frac{p(s)e_2(s)}{e_1}$ (which follows from equal gaps and $\Delta(i) \geq 0$), we have that $f(s) = 0$ is optimal for the household.

For the intermediaries, note that given that $q(s) = (1+\Delta(i))\frac{p(s)e_2(s)}{e_1}$, then from (3.8), the intermediaries are indifferent between any of the domestic securities. Note that $i \geq 0$, implying that $m^* = 0$ is also consistent with intermediaries' optimality.

Household utility. In any equal gaps allocation, the utility of the household equals (3.17) by the definition of U in problem (SP). ‖

A.3. *Proof of Corollary 1*

Suppose $\Delta(i)\overline{w} < y_1 + \overline{q}Y_2$. *There is a unique pair* (c_1, C_2) *that solves* (3.14) *and* (3.15). *When* $\Delta(i)=0$, c_1 *coincides with the first-best consumption. In addition,* c_1 *strictly decreases with* $\Delta(i)$ *and strictly decreases in* $\overline{w}$ *for* $\Delta(i)>0$.

Proof. Uniqueness of (c_1, C_2) follows from the strict concavity of $u(\cdot)$ and $U(\cdot)$ (the latter follows from standard arguments). It is also straightforward to verify that equations (3.14) and (3.15) are the solution to the problem that defines the first-best consumption allocation when $\Delta(i)=0$. To demonstrate the comparative static results, we can combine equation (3.14) with (3.15) to obtain

$$\frac{\overline{q}u'(c_1)}{\beta U'\left(\frac{y_1 - c_1 + \overline{q}Y_2 - \Delta(i)\overline{w}}{\overline{q}}\right)} = (1+\Delta(i)).$$

Total differentiation of the above expression leads to the following expression:

$$\frac{\partial c_1}{\partial \Delta(i)} = \frac{1}{\overline{q}}\left\{\frac{1 - \overline{w}U''(C_2)/U'(C_2)}{u''(c_1)/u'(c_1) + (1/\overline{q})U''(C_2)/U'(C_2)}\right\}$$
$$\frac{\partial c_1}{\partial \overline{w}} = -\Delta(i)\frac{1}{\overline{q}}\left\{\frac{U''(C_2)/U'(C_2)}{u''(c_1)/u'(c_1) + (1/\overline{q})U''(C_2)/U'(C_2)}\right\}.$$

The first expression is always negative because of the strict concavity of $u(\cdot)$ and $U(\cdot)$, implying that c_1 strictly decreases with $\Delta(i)$. The second expression tells us that c_1 strictly decreases with $\overline{w}$ when $\Delta(i)>0$. ‖

A.4. *Proof of Proposition 1*

Suppose Assumptions 1 and 2 hold. If $\Delta(0)\leq 0$, *then the best equilibrium features* $(c_1^{fb}, C_2^{fb}, m, i)$ *where*

$$C_2^{fb} = \sum_{s\in S} q(s)c_2^{fb}/\overline{q},$$

$i \geq 0$ *and such that* $\Delta(i)=0$,

m such that $h'(m/e_1) = u'(c_1^{fb})\frac{i}{1+i}$.

Proof. The monetary authority cannot implement an interest rate i such that $\Delta(i)<0$. Thus, we must have $\Delta(i)\geq 0$. Because $\Delta(0)\leq 0$, and $\Delta(.)$ is increasing in i, there exists a level of $i\geq 0$ such that $\Delta(i)=0$. Denote this level by $\bar{i}$. Any equilibrium must feature $i \geq \bar{i}$.

By Lemma 2, we know that if $i=\bar{i}$, the resulting consumption allocation in the monetary equilibrium equals that of the first best, and money demand satisfies

$$h'(m/e_1) = u'(c_1^{fb})\frac{\bar{i}}{1+\bar{i}}.$$

Consider now any $i' > \bar{i}$, and denote by (c_1', C_2', m') the resulting allocation in the monetary equilibrium. By the definition of first best, we know that $u(c_1') + \beta U(C_2')$ cannot be higher than the utility level achieved at (c_1^{fb}, C_2^{fb}). Moreover, by Corollary 1, we know that $c_1' < c_1^{fb}$, and thus the utility generated by the consumption bundle is strictly below the first best one. In addition, $c_1' < c_1^{fb}$ implies, by equation (3.16), that m'/e_1 is smaller than the value achieved at $\bar{i}$. Thus, because $h(.)$ is increasing, the utility from real money balances under $\tilde{i}$ is weakly lower than that achieved at $\bar{i}$. It follows that household welfare is maximized at $\bar{i}$, thus proving the result. ‖

A.5. *Proof of Proposition 2*

Suppose Assumptions 1 and 2 hold. If $\Delta(0)>0$ *and* $\Delta(0)\overline{w} < y_1 + \overline{q}Y_2$, *then the best equilibrium features* (c_1, C_2, m, i) *such that*

$i=0$, $\dfrac{m}{e_1} \geq \bar{x}$, *and*

(c_1, C_2) *are the unique solution to* (3.14) *and* (3.15).

Proof. Because of the zero lower bound constraint, we have that $i \geq 0$. Denote by (c_1, C_2, m) the consumption allocation and money demand that is achieved in a monetary equilibrium with $i = 0$. Following the same steps as in the Proof of Proposition 1, we can verify that the welfare of the representative household is maximized when the monetary authority sets $i = 0$. Consider any $\tilde{i} > 0$, and denote by $(\tilde{c}_1, \tilde{C}_2, \tilde{m})$ the associated allocation in the monetary equilibrium. First, because $\Delta(\tilde{i}) > \Delta(0)$, we have that $u(\tilde{c}_1) + \beta U(\tilde{C}_2)$ is below $u(c_1) + \beta U(C_2)$. Second, money demand is satiated at zero interest rates, which implies that $h(\tilde{m}/e_1)$ is lower than the one achieved at $i = 0$. It follows that the best equilibrium features $i = 0$. ‖

A.6. *Proof of Corollary 2*

Implementation away from the zero lower bound: suppose Assumptions 1 and 2 hold. If $\Delta(0) < 0$*, the monetary authority implements the best equilibrium with any* $F \in [0, (y_1 - c_1^{fb} + \overline{w})/\overline{q}]$.

Proof. The first observation we make is that by Lemma 1, the monetary equilibrium implemented features equal gaps. Hence, for given i, we have that (c_1, C_2, m) solve (3.14)–(3.16). We now argue that if $F \in [0, (y_1 - c_1^{fb} + \overline{w})/\overline{q}]$, we must have $\Delta(i) = 0$. Suppose by contradiction that $\Delta(i) > 0$. From Corollary 1, we have that $c_1 < c_1{}^{fb}$. In addition, from the trade balance equation,

$$c_1^{fb} > c_1 = y_1 + \overline{w} - \left(\sum_{s \in S} q(s) f(s) + \bar{q} F \right) \geq c_1{}^{fb} - \sum_{s \in S} q(s) f(s) = c_1{}^{fb}.$$

The first equality follows from the fact that $\Delta(i) > 0$ implies that $\sum_{s \in S} \frac{p(s) a^*(s)}{e_1} = \overline{w}$ and $m^* = 0$. The first inequality follows from $F \leq (y_1 - c_1^{fb} + \overline{w})/\overline{q}$, and the last equality follows from household optimality which implies $f(s) = 0$ in an equal gaps equilibrium.

Now that we have proved that $\Delta(i) = 0$, the next step is to show that the consumption allocations correspond to the first best. We know from Corollary 1 that there is a unique pair of c_1, C_2 that solves (3.14) and (3.15). Since (3.14) is the resource constraint, the first-best allocation (3.2) and (3.15) together with (SP) imply that (3.3) is satisfied, and it follows that $c_1 = c_1^{fb}, C_2 = C_2^{fb}$, and $c_2(s) = c^{fb}(s)$. Finally, we know that i is such that $\Delta(i) = 0$ and m follows from (3.16). ‖

A.7. *Proof of Corollary 3*

Implementation at the zero lower bound: suppose Assumptions 1 and 2 hold. If $\Delta(0) > 0$*, the monetary authority implements the best equilibrium with* $F = (y_1 - c_1^* + \overline{w})/\overline{q} > 0$*, where* c_1^* *is the best equilibrium first-period consumption.*

Proof. The first observation we make is that by Lemma 1, the monetary equilibrium implemented features equal gaps. Let us first show that F induces $c_1 = c_1^*$. From the trade balance equation, (3.10), we have

$$c_1 = y_1 + \overline{w} - \left(\sum_{s \in S} q(s) f(s) + \bar{q} F \right) = c_1^*, \tag{A.8}$$

where we substituted the value of F and used that $f(s) = 0$ in an equal gaps equilibrium.

Next, we show that $i = 0$. Suppose $i > 0$. From Corollary 1, we would have $c_1 < c_1^*$, contradicting (A.8). Hence, $i = 0$. Using $\Delta(0)$ and $c_1 < c_1^*$, we can obtain C_2 as the unique solution to (3.14). Finally, since $i = 0$, we must have that $m \geq \overline{m}$. ‖

B. DERIVATION OF EQUATION 5.2

Using the budget constraint of the households and the government, market clearing, and solving this time for $a(s)$, we can obtain

$$y_1 = c_1 + \sum_{s \in S} \frac{p(s) e_2(s)}{e_1} (c_2(s) - y_2(s)) + \sum_{s \in S} \left(q(s) - \frac{p(s) e_2(s)}{e_1} \right) (f(s) + F) + \left[\sum_{s \in S} p(s) - 1 \right] \frac{m^*}{e_1}.$$

The last term is zero, as intermediaries do not hold money unless at zero domestic interest rates. In addition, the households do not purchase a foreign security if there is an arbitrage gap in it. As result, the above can be written as

$$y_1 - c_1 + \sum_{s \in S} q_d(s) (y_2(s) - c_2(s)) - \sum_{s \in S} q_d(s) \left(\frac{1}{\bar{p}} \frac{e_1}{e_2(s)} - \frac{1}{\bar{q}} \right) \bar{q} F = 0,$$

where $q_d(s) = p(s) \frac{e_2(s)}{e_1}$, the domestic price. Note that in effect, this way of writing the resource constraint discounts the arbitrage losses using the domestic asset prices, $q_d(s)$.

Finally, using equal gaps, (3.11), the definitions of $\Delta(i)$ in (3.12), and Y_2 and C_2, we get

$$y_1 - c_1 + \frac{\bar{q}}{1+\Delta(i)}(Y_2 - C_2) = \frac{\Delta(i)}{1+\Delta(i)}\bar{q}F,$$

which is equation (5.2).

C. RESOURCE COSTS IN AN INFINITE HORIZON MODEL

In this appendix, we derive the one-period-ahead resource loss (5.3) in an infinite horizon version of the model. Time is indexed by $t=0,1....$ We denote by s^t the history of states up to time t; that is, $s^t = s_0,(s_1,...,s_t)$.

The budget constraint for households in state s^t is

$$y(s^t)+T(s^t)+f(s^t)+\frac{a(s^t)+m(s^{t-1})}{e(s^t)} = c(s^t)+\sum_{s_{t+1}\in S}\frac{p(s^t)}{e(s^t)}a(s^t,s_{t+1}) + \frac{m(s^t)}{e(s^t)}$$
$$+\sum_{s_{t+1}\in S} q(s_{t+1},s^t)f_{t+1}(s_{t+1},s^t)) \, . \tag{C.1}$$

The government budget constraint in state s^t is

$$\frac{A(s^t)+M(s^{t-1})}{e(s^t)} + F(s^t) = T(s^t) + \frac{M(s^t)}{e(s^t)} + \sum_{s_{t+1}\in S}\left[\frac{p(s_{t+1},s^t)}{e(s^t)}\,A(s_{t+1},s^t) + q(s_{t+1},s^t)F_{t+1}(s_{t+1},s^t))\right], \tag{C.2}$$

where we allowed the government to hold Arrow–Debreu securities. This allows us to consider situations in which the government holds different assets (maturity, currency, risk) in its portfolio.

Combining the households' and the government's budget constraints (C.1)–(C.2), we obtain

$$\frac{A(s^t)}{e(s^t)}+\frac{a(s^t)}{e(s^t)}+m(s^{t-1})-M(s^{t-1})+F(s^t)+f(s^t)+y(s^t) = c(s^t)+\sum_{s_{t+1}\in S}\frac{p(s_{t+1},s^t)}{e(s^t)}\left[A(s_{t+1},s^t)\right.$$
$$\left.+a(s_{t+1},s^t)\right] + \frac{m(s^t)-M(s^t)}{e(s^t)} + \sum_{s_{t+1}\in S} q(s_{t+1},s^t)\left[f(s_{t+1},s^t)+F(s_{t+1},s^t)\right]. \tag{C.3}$$

Updating one period forward and rearranging, it follows that

$$\frac{A(s_{t+1},s^t)}{e(s_{t+1},s^t)}+\frac{a(s_{t+1},s^t)}{e(s_{t+1},s^t)} = c(s_{t+1},s^t)-\left[y(s_{t+1},s^t)+F(s_{t+1},s^t)+ \; f(s_{t+1},s^t)+m(s^t)+M(s^t)\right]$$
$$\sum_{s_{t+1}\in S}\frac{p(s_{t+2},s^{t+1})}{e(s_{t+1},s^t)}\left[A(s_{t+2},s^{t+1}) + a(s_{t+2},s^{t+1})\right] + \frac{m(s^{t+1})}{e(s_{t+1},s^t)} - \frac{M(s^{t+1})}{e(s_{t+1},s^t)}$$
$$+\sum_{s_{t+1}\in S}[q(s_{t+2},s^{t+1})\left(f(s_{t+2},s^{t+1})+F(s_{t+2},s^{t+1})\right)\,]. \tag{C.4}$$

Substituting (C.4) into (C.3) and rearranging, it follows that

$$\frac{A(s^t)+a(s^t)}{e(s^t)}+F(s^t)+f(s^t)+y(s^t)+m(s^{t-1})+M(s^{t-1})-c(s^t) =$$
$$\sum_{s_{t+1}\in S}\left[q(s_{t+1},s^t)-\frac{p(s_{t+1},s^t)e(s_{t+1},s^t)}{e(s^t)}\right]\left(f(s^t,s_{t+1})+F(s^t,s_{t+1})\right)\,]$$
$$+\left(1-\sum_{s_{t+1}\in S}p(s_{t+1},s^t)\right)\left(\frac{M(s^t)+m(s^t)}{e(s^t)}\right)$$
$$+\sum_{s_{t+1}\in S}\frac{p(s_{t+1},s^t)}{e(s^t)}\left[c(s_{t+1},s^t)-y(s_{t+1},s^t)+\sum_{s_{t+1}\in S}\frac{p(s_{t+2},s^{t+1})}{e(s_{t+1},s^t)}\left[A(s_{t+2},s^{t+1}) + a(s_{t+2},s^{t+1})\right]\right.$$
$$\left.+\frac{M(s^{t+1})}{e(s_{t+1},s^t)} + \frac{m(s^{t+1})}{e(s_{t+1},s^t)} + \sum_{s_{t+1}\in S}\left[q(s_{t+2},s^{t+1})\left(f(s_{t+2},s^{t+1})+F(s_{t+2},s^{t+1})\right)\right]\right].$$

Using market clearing $A(s^t)+a(s^t)=a^*(s^t)$, $m(s^t)+m^*(s^t)=M(s^t)$, we obtain

$$\frac{a^*(s^t)}{e(s^t)}+F(s^t)+f(s^t)+y(s^t)-\frac{m^*(s^{t-1})}{e(s^t)}-c(s^t)=$$

$$\sum_{s_{t+1}\in S}\left[q(s_{t+1},s^t)-\frac{p(s_{t+1},s^t)e(s_{t+1},s^t)}{e(s^t)}\right]\big(f(s^t,s_{t+1})+F(s^t,s_{t+1})\big)\ ]+i\frac{m^*(s^t)}{e(s^t)}$$

$$\sum_{s_{t+1}\in S}\frac{p(s_{t+1},s^t)e(s_{t+1},s^t)}{e(s^t)}\left[c(s_{t+1},s^t)-y(s_{t+1},s^t)+\sum_{s_{t+1}\in S}\frac{p(s_{t+2},s^{t+1})}{e(s_{t+1},s^t)}\,a^*(s_{t+2},s^{t+1})\right.$$

$$\left.-\frac{m^*(s^{t+1})}{e(s_{t+1},s^t)}+\sum_{s_{t+1}\in S}\left[q(s_{t+2},s^{t+1})\Big(f(s_{t+2},s^{t+1})+F(s_{t+2},s^{t+1})\Big)\right]\right]. \tag{C.5}$$

Let us define

$$\Delta(s^t)\equiv\sum_{s\in S}\frac{q(s_{t+1},s^t)e(s^t)}{e(s_{t+1},s^t)}(1+i(s^t))-1.$$

Notice that under equal gaps,

$$\sum_{s_{t+1}\in S}\left[q(s_{t+1},s^t)-\frac{p(s_{t+1},s^t)e(s^{t+1})}{e(s^t)}\right]\left[F(s_{t+1},s^t)+f(s_{t+1},s^t)\right]$$

$$=\frac{\sum_{s_{t+1}\in S}q(s)\left[F(s_{t+1},s^t)+f(s_{t+1},s^t)\right]\Delta(s^t)}{1+\Delta(s^t)} \tag{C.6}$$

and

$$\sum_{s_{t+1}\in S}\frac{p(s_{t+1},s^t)e(s_{t+1},s^t)}{e(s^t)}\left[y(s_{t+1},s^t)-c(s_{t+1},s^t)\ \right]=\frac{\bar{q}(\tilde{Y}_2(s^t)-C_2(s^t))}{1+\Delta(s^t)}. \tag{C.7}$$

Let us define

$$\tilde{y}(s^t)\equiv a^*(s^t)+F(s^t)+f(s^t)+y(s^t)-\frac{m^*(s^{t-1})}{e(s^t)} \tag{C.8}$$

and

$$\tilde{Y}_2(s^t)\equiv Y_2(s^t)+\sum_{s_{t+1}\in S}\frac{p(s_{t+2},s^{t+1})}{e(s_{t+1},s^t)}\left[a^*(s_{t+2},s^{t+1})\right]$$

$$+\sum_{s_{t+1}\in S}\frac{p(s_{t+1},s^t)}{e(s^t)}\sum_{s_{t+1}\in S}\left[q(s_{t+2},s^{t+1})\Big(f(s_{t+2},s^{t+1})+F(s_{t+2},s^{t+1})\Big)\right], \tag{C.9}$$

where Y_2 and C_2 are defined analogously as in (SP) and (3.4). Using expressions (C.6)–(C.9) and substituting the optimality conditions for $m^*(s^t)$ and $f(s^t)$ into (C.5), we obtain the following resource constraint:

$$\tilde{y}(s^t)-c(s^t)+\frac{\bar{q}(\tilde{Y}_2(s^t)-C_2(s^t))}{1+\Delta(s^t)}=\frac{\Delta(s^t)}{1+\Delta(s^t)}\sum_{s_{t+1}\in S}F(s_{t+1},s^t)q(s_{t+1},s^t). \tag{C.10}$$

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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