

# Quantitative Sovereign Default Models and the European Debt Crisis\*

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December 23, 2018

## Abstract

A large literature has developed quantitative versions of the [Eaton and Gersovitz \(1981\)](#) model to analyze default episodes on external debt. In this paper, we study whether the same framework can be applied to the analysis of debt crises in which domestic public debt plays a prominent role. We consider a model where a government can issue debt to both domestic and foreign investors, and we derive conditions under which their sum is the relevant state variable for default incentives. We then apply our framework to the European debt crisis. We show that matching the cyclicity of public debt—rather than that of external debt—allows the model to better capture the empirical distribution of interest rate spreads and gives rise to more realistic crises dynamics.

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\*Prepared for 2018 International Seminar on Macroeconomics (ISOM) in Dublin, Ireland. We thank Yan Bai and Leonardo Martinez for very insightful discussions. We also thank Charles Engel and Pierre-Olivier Gourinchas for useful comments.

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# 1 Introduction

The model of [Eaton and Gersovitz \(1981\)](#) is the benchmark model to study sovereign debt crises. In its simplest form, this model considers the decision problem of a government that faces income risk and issues non-contingent debt with the option to default. The default option is more valuable when the income of the government is low, when its debt obligations are high, or both. Consequently, income flows and debt repayments are key variables to explain default episodes in this class of models.

Following the work of [Arellano \(2008\)](#) and [Aguiar and Gopinath \(2006\)](#), several researchers have applied this framework to the analysis of debt crises in emerging markets. The canonical approach in the literature consolidates the budget constraint of the government and that of the private sector, implicitly assuming that the government has enough instruments to control the saving behavior of private domestic agents. This literature has shown that quantitative versions of the [Eaton and Gersovitz \(1981\)](#) model can account for the joint behavior of net external debt, income, and interest rate spreads in emerging markets, most notably reproducing the coincidence of debt crises with reversals of current account deficits.

The literature is, however, typically silent on the role of public debt – in particular domestic public debt – because this variable is irrelevant for default incentives under the assumptions emphasized earlier. This is a limitation for the analysis of debt crises in which domestic public debt plays a prominent role.<sup>1</sup> This paper takes a step toward filling this gap in the literature by applying this class of models to the analysis of the recent debt crisis in southern Europe. We first document that in the case of Portugal, Spain, and Italy, total public debt (domestic and foreign) outperforms standard indicators of external debt in accounting for the dynamics of interest rate spreads. We next show that we can reconcile the canonical default model with such evidence. Specifically, we derive theoretical conditions under which total public debt is a sufficient statistic in the decision problem of the government. We then show how to calibrate this model to account for the behavior of income, public debt, and interest rate spreads for southern European economies.

We start by empirically studying the relation between spreads, income, and different indicators of debt for Portugal, Spain, and Italy. We show that different indicators of indebtedness have different dynamics in the data. In particular, total public debt increases substantially after 2008 and mirrors the dynamics of interest rate spreads. This is not true for other measures of indebtedness such as external public debt and net external debt. Simple statistical regressions confirm that total public debt is more tightly associated with

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<sup>1</sup>In our paper, the distinction between domestic and external debt regards the residence of bondholders, as opposed to the jurisdiction under which the bonds were issued or the currency. Most of the outstanding debt securities of European countries were issued under domestic law and in euros.

the behavior of interest rate spreads on government debt securities than more traditional indicators of debt used in the literature for the countries we study.

Motivated by this evidence, we study theoretically the conditions under which total public debt is the only relevant state variable for the default incentives of the government in addition to its income. We consider a framework that allows for a clear distinction between external and public debt. A benevolent government finances public consumption goods using tax revenues and issues government debt to maximize the utility of the representative agent in the economy. Government debt is non-contingent, but the government cannot commit to repaying. Public debt can be held by domestic households and by foreign investors. Importantly, we assume that domestic households can also trade with foreign lenders and that the government has no margin to interfere with private choices.<sup>2</sup>

Our main result is that total government debt (and not its external component) is the only relevant endogenous state variable in the decision problem of the government if the following two conditions hold. First, the government cannot discriminate between domestic and foreign bondholders in default. Second, private financial markets are *sophisticated* in that domestic and foreign private investors can trade securities contingent on the state of the economy and on government policies. Moreover, we show that the government problem in such an environment is isomorphic to the one considered in [Arellano \(2008\)](#), with the exception that the resource constraint of the small open economy is replaced with the budget constraint of the government. While the assumption of sophisticated private financial markets is clearly extreme, it provides a useful theoretical benchmark to contrast with the standard approach in the literature.

Equipped with these results, we explore the effects of fitting the [Arellano \(2008\)](#) model to total public debt rather than to external debt. Our application focuses on Spain. We select model parameters to target the sample mean and the standard deviation of interest rate spreads, the sample mean of the public debt to output ratio, and its correlation with detrended GDP. To contrast our approach with the traditional one in the literature, we consider an alternative calibration that replaces this latter empirical target with the correlation between the trade balance and GDP. The main result from this analysis is that matching the cyclicity of total public debt rather than that of external debt allows the model to better fit interest rate spreads dynamics.

As discussed earlier, external debt and public debt have different cyclicity in our data. External debt decreases after a negative income shock. In the traditional calibration, this association between debt issuances and income is achieved by having an “impatient” government with a discount rate that is substantially above the market discount rate. This feature implies that the government borrows up to the implicit limit induced by default

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<sup>2</sup>This lack of interference is not necessarily optimal. As [Chari et al. \(2016\)](#) and [Perez \(2015\)](#) show, some form of financial repression can be optimal when the government cannot commit to repaying its debt.

risk, and because default incentives are higher in low income states, the government ends up borrowing less when income contracts and more when it expands. Because of that, the traditional calibration implies that interest rate spreads are substantially above zero even in “normal” times, and that they exhibit little volatility: after a bad income shocks, the government cuts its leverage, mitigating the impact of the income shock on equilibrium spreads.

Total public debt, instead, increased in the data during the crisis. Our calibration fits this association by having a substantially less impatient and more “cautious” government relative to the traditional calibration. In normal times, the government is essentially not at risk of default, and interest rate spreads gravitate around zero. However, occasional jumps may occur after a sequence of bad income shocks and increasing debt dynamics. This behavior generates a distribution of interest rate spreads that is remarkably close to the one observed for countries in southern Europe, with mass around zero and rare large spikes.<sup>3</sup>

**Related literature** Our paper contributes to the literature on sovereign debt. Following the work of [Eaton and Gersovitz \(1981\)](#), the papers of [Arellano \(2008\)](#) and [Aguiar and Gopinath \(2006\)](#) were the first to analyze the quantitative performance of this class of models. Subsequent papers have enriched the standard framework with more realistic features. [Hatchondo and Martinez \(2009\)](#), [Chatterjee and Eyigungor \(2012\)](#), and [Hatchondo et al. \(2015\)](#) introduce long-term debt in the canonical framework. [Arellano and Ramanarayanan \(2012\)](#) consider an explicit maturity choice for debt issuances. [Mendoza and Yue \(2012\)](#) propose a model where default costs endogenously arise because of the negative effects of default on financial markets and international trade. [Bianchi et al. \(2012\)](#) apply the canonical default model to explain the coexistence of short-term foreign assets and long-term foreign debt in the government’s balance sheet. All these studies consolidate the budget constraints of the government and the private sector, and they typically fit these models to the behavior of aggregate output, interest rate spreads, and indicators of indebtedness. They show that this class of models can generate countercyclical net external debt for the small open economy—that is, capital flows out of the small open economy in bad times, and flows in during good times.<sup>4</sup> Under this approach, the model-consistent debt variable should be net external debt. Some of these

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<sup>3</sup>Low interest rate spreads between southern European Eurozone members and Germany can also be accounted for by expectations of future bailouts. For instance, see [Dovis and Kirpalani \(2018\)](#).

<sup>4</sup>[Hatchondo and Martinez \(2009\)](#), [Hatchondo et al. \(2015\)](#), and [Bianchi et al. \(2012\)](#) target, instead, the volatility of consumption relative to that of output rather than indicators of net exports. Note, however, that for an emerging market economy the volatility of consumption is typically higher than that of income. In a pure exchange economy model, this moment can be reproduced only by having a countercyclical trade balance. Thus, when fit to emerging markets, this calibration behaves very similarly to the ones that target the cyclicalities of net exports.

papers, however, use public debt to measure the *level* of indebtedness of the country. The implicit assumption is that the private sector does not borrow or save, so that external debt and total public debt are equivalent. While this assumption might be appropriate for emerging market economies, it is not for the European countries in our sample.

In our approach, instead, we focus only on the budget constraint of the government, and we fit the model to the *level and cyclical*ity of public debt. For the Southern European economies under analysis, this variable goes up in bad times and declines in good times, which is the opposite prediction of the standard quantitative sovereign debt model in the literature. Thus, part of our contribution is to study the implications of targeting a different cyclical<sup>ity</sup> of debt in this class of models. We do so by studying the most basic model in the literature, [Arellano \(2008\)](#). While this reduces the realism of our analysis, it helps the reader in understanding how different calibration targets affect the behavior of the benchmark model.<sup>5</sup>

Several recent papers have studied the dynamics and interactions of domestic and foreign debt. [Dovis et al. \(2016\)](#) consider a theoretical model where the distribution of domestic and foreign debt is a key determinant of debt sustainability. [D’Erasmus and Mendoza \(2013\)](#) study a quantitative model in which the government has an incentive to default on domestic debt because of distributional motives. We view our contribution as twofold. First, we find conditions under which only total public debt is relevant for the government problem, not its distribution between domestic and foreign investors. Second, and differently from [D’Erasmus and Mendoza \(2013\)](#), we emphasize how the different cyclical<sup>ity</sup> of public and external debt affects the calibration of the default model.

Finally, several recent papers have applied this class of models to the European debt crisis. See, for example, [Salomao \(2017\)](#), [Bocola and Dovis \(2016\)](#), and [Arellano et al. \(2018\)](#). [Paluszynski \(2016\)](#) emphasizes that the standard default model has a hard time matching the debt crisis in these countries. Specifically, he shows that the model fits poorly because the Portuguese government had more external debt in 2008 than in 2011 while interest rate spreads increased substantially only in 2011. Our analysis confirms this finding, but it also demonstrates that fitting this model to public debt rather than to external debt substantially improves the ability of the model to fit the dynamics of interest rate spreads for Southern European countries.

The paper is structured as follows. In Section 2 we briefly present the dynamics of interest rate spreads and various indicators of debt for Portugal, Italy, and Spain. Section 3 lays out our model and derives the main theoretical results. Section 4 calibrates the model to Spain and presents the main quantitative results. Section 5 concludes.

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<sup>5</sup>The message of the paper is consistent with the richer model considered in [Bocola and Dovis \(2016\)](#) with risk averse lenders, long-term debt, and time-varying rollover risk.

## 2 Empirics

Models of sovereign debt imply that equilibrium interest rate spreads are a function of the sovereign's income and its debt obligations. In this section, we study the relation between interest rate spreads, income, and three different debt indicators for Spain, Italy, and Portugal. We consider these three countries because they were at the center of the European debt crisis. We exclude Greece and Ireland from the analysis because they received support from European institutions earlier in the sample. The three debt indicators we consider are net external debt, public external debt, and total public debt. We find that the total public debt is more tightly associated to interest rate spreads than public external debt or net external debt.

The standard approach when comparing sovereign debt models to the data is to aggregate all agents in the economy into one entity, a benevolent sovereign. As a result, the relevant debt obligations of the sovereign in the model correspond to the sum of debt obligations to foreigners of domestic households, firms, and the government. This is the first debt indicator we consider: net external debt in the economy.<sup>6</sup> In practice, however, researchers often use public external debt when comparing the model's predictions to the data. So the second debt indicator we consider is public external debt. The final debt indicator we consider is total public debt, which includes the obligations of the government to both domestic and foreign lenders.

Figure 1 presents the detrended output, the interest rate spread, and the three debt indicators for Spain, Italy, and Portugal. The time period we consider is 2002Q1 to 2012Q2.<sup>7</sup> The final period of analysis was chosen so that we exclude the periods following the introduction of the Outright Monetary Transactions program by the European Central Bank.

As can be seen in the figure, the three countries went through two recessions during this period, the first in 2008 and the second in 2011. While both recessions were of similar magnitude in their effect on output, a substantial difference can be seen in the behavior of spreads between the two recessions. Following the 2008 recession, spreads went up from about 0% to 1.0-1.5% in the first quarter of 2009. Following the 2011 recession, spreads reached levels of about 5% in Spain and Italy, and about 12% in Portugal.

The last three panels of Figure 1 present the behavior of net external debt, public external debt, and total public debt. As is apparent in the figure, the three debt indicators have different dynamics during the crisis. Following the 2008 recession, total public debt

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<sup>6</sup>We define net external debt to be the negative of the net international investment position.

<sup>7</sup>We obtain real GDP from OECD quarterly national accounts for the period 1960Q1-2012Q2. Detrended output is defined as the difference between log real GDP and a quadratic time trend. Interest rate spreads on bonds with a five-year residual maturity are obtained from Markit. As for the three debt series, we obtain total public debt as a fraction of GDP from the OECD, public external debt from the Bruges data set, and the net international investment position from Eurostat.

Figure 1: Data

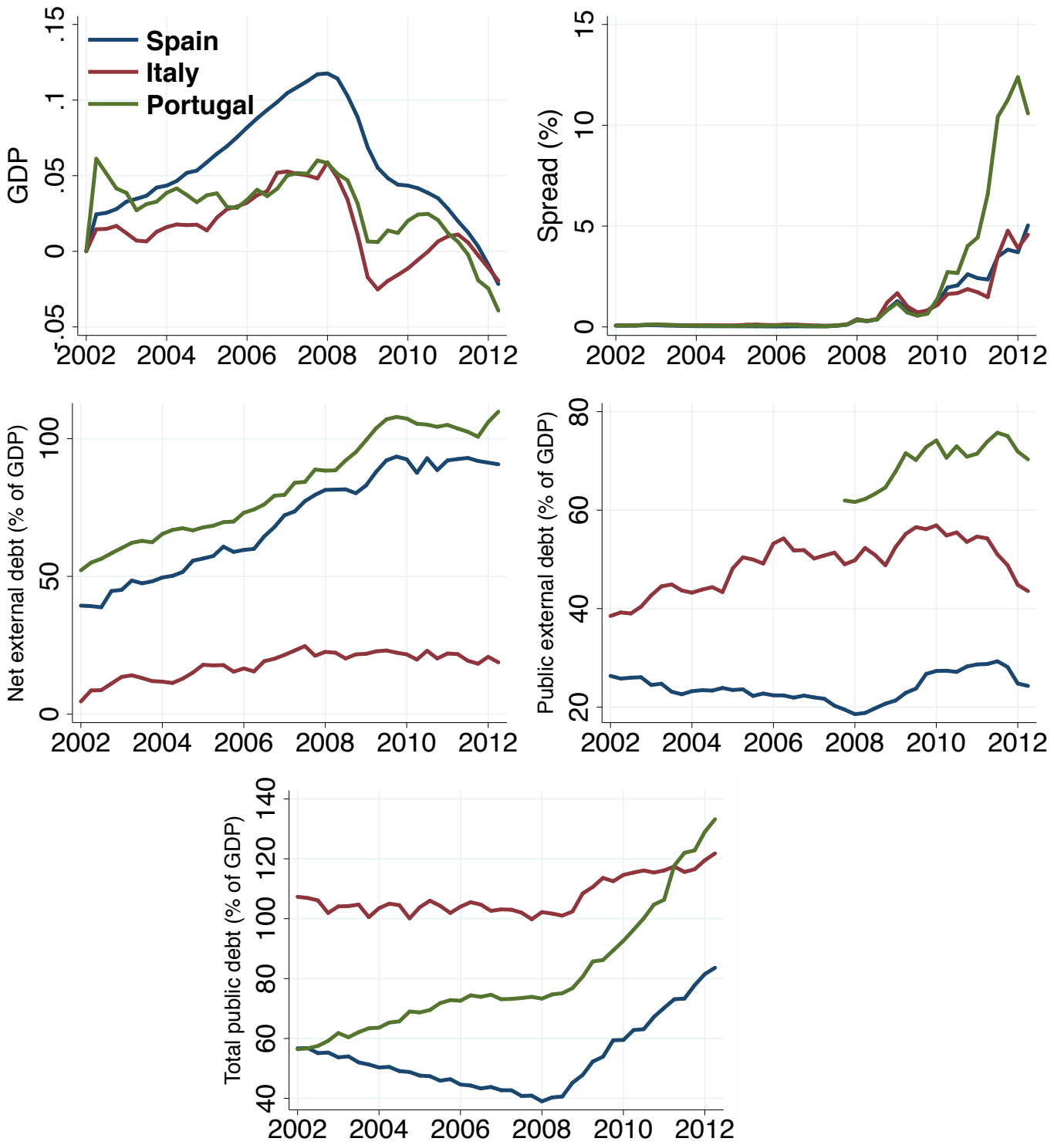


Table 1: **Regression results**

Dependent variable: *spread*

| Country  |                | Output only        | Debt only         |                   |                   | Output and debt    |                    |                   |
|----------|----------------|--------------------|-------------------|-------------------|-------------------|--------------------|--------------------|-------------------|
|          |                |                    | PED               | NED               | TPD               | PED                | NED                | TPD               |
| Spain    | Debt           |                    | 0.25***<br>(0.05) | 0.05***<br>(0.01) | 0.10***<br>(0.01) | 0.06<br>(0.14)     | 0.05***<br>(0.01)  | 0.17***<br>(0.01) |
|          | Output         | -0.23***<br>(0.06) |                   |                   |                   | -0.19<br>(0.13)    | -0.23***<br>(0.03) | 0.28***<br>(0.03) |
|          | R <sup>2</sup> | 0.39               | 0.30              | 0.46              | 0.79              | 0.40               | 0.85               | 0.91              |
| Italy    | Debt           |                    | 0.04<br>(0.03)    | 0.08***<br>(0.02) | 0.17***<br>(0.03) | 0.03<br>(0.03)     | 0.09***<br>(0.02)  | 0.18***<br>(0.03) |
|          | Output         | -0.31***<br>(0.08) |                   |                   |                   | -0.31**<br>(0.09)  | -0.31***<br>(0.08) | 0.03<br>(0.06)    |
|          | R <sup>2</sup> | 0.30               | 0.02              | 0.10              | 0.67              | 0.31               | 0.41               | 0.67              |
| Portugal | Debt           |                    | 0.54***<br>(0.13) | 0.11***<br>(0.03) | 0.15***<br>(0.02) | -0.03<br>(0.17)    | 0.02<br>(0.02)     | 0.13***<br>(0.02) |
|          | Output         | -1.21***<br>(0.19) |                   |                   |                   | -1.33***<br>(0.27) | -1.12***<br>(0.21) | -0.25<br>(0.16)   |
|          | R <sup>2</sup> | 0.67               | 0.34              | 0.34              | 0.83              | 0.66               | 0.67               | 0.84              |

Notes: NED: net external debt; PED: public external debt; TPD: total public debt. \*\*\* - significant at a 1% level, \*\* - significant at a 2% level.

increases substantially in all three countries. The two indicators of external debt, instead, show little sign of change, and in the case of external public debt, they decline toward the end of the period.

To formally assess the relation between each debt indicator and interest rate spreads, we regress spreads on each debt indicator and on detrended output. We assume that spreads are approximated by the following equation:

$$r_t = \alpha + \beta X_t + \varepsilon_t,$$

where  $r_t$  is the spread at time  $t$ , and  $X_t$  is a set of covariates at time  $t$ . We consider specifications in which  $X_t$  includes only the level of debt and ones in which it includes both the level of debt and detrended output. We use the different debt indicators, one at a time, as the level of debt in the regression. We estimate this regression separately for Spain, Italy, and Portugal. The results are reported in Table 1.

The first column presents the regression results when the only covariate is output. We can see that the  $R^2$  varies from 0.3 in the case of Italy to 0.67 in Portugal. So variation in output goes a long way toward explaining variations in the spread, as predicted by a standard default model.



The next three columns present the regression results when the only covariate is the level of debt in the economy. We find that the  $R^2$  when using total public debt is two to seven times larger than when using the other two debt indicators. A similar result is obtained when considering the specification in which both output and the level of debt is used (the three last columns in the table). This result indicates that out of the three debt indicators considered, total public debt is the most tightly linked to interest rate spreads.

### 3 Model

We now theoretically explore under what conditions total public debt is the only relevant endogenous state variable for the government's default incentives. Specifically, we show that if default is non-discriminatory and private financial markets are sophisticated, then only total public debt is relevant for the government problem in addition to exogenous shocks. We say that private financial markets are sophisticated if private agents (domestic and foreign) can trade securities contingent on the state of the economy and government policies, in particular government default. This view contrasts with the standard way to think about sovereign debt (or better, external debt) in emerging market economies. In the traditional approach, private markets are extremely unsophisticated: there are no private markets. In this case, total external debt is the relevant variable for the government problem and interest rate spreads.

#### 3.1 Environment

We follow most of the literature and consider a model of a government that receives a stochastic flow of tax revenues and borrows to smooth public consumption across time and states. Alternatively, we could have considered a tax-smoothing model in the Ramsey tradition in which the government issues debt to smooth the costs of distortionary taxes. As shown in [Aiyagari et al. \(2002\)](#), under certain conditions these two problems admit the same representation.<sup>8</sup>

Time is indexed by  $t = 1, 2, \dots$ . The state of the economy is  $s_t \in S$  where  $S$  is some arbitrary set. Let  $s^t = (s_0, s_1, \dots, s_t)$  and  $\mu(s^t|s_0)$  be the probability of drawing a history  $s^t$  conditional on  $s_0$  in period 0. The economy is populated by a benevolent government, domestic consumers, and foreign lenders. The domestic consumers have preferences over private consumption good,  $c(s^t)$ , and public consumption good,  $g(s^t)$ , given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t|s_0) U(c(s^t), g(s^t)), \quad (1)$$

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<sup>8</sup>See also [Pouzo and Presno \(2014\)](#).

where

$$U(c, g) = u(c) + \omega(g).$$

The functions  $u$  and  $\omega$  are increasing, concave, and differentiable. For brevity, we refer to  $g(s^t)$  as public consumption, but we interpret  $g(s^t)$  as being public consumption plus transfers, capturing all the expenditures made by the government. The consumer receives an income  $Y(s_t)$  in each period.

Foreign lenders have a large endowment in each period and evaluate repayments with a stochastic discount factor  $M(s_{t+1}, s_t)$ .

The government is benevolent and maximizes the utility of the domestic consumer.<sup>9</sup> In each period it receives tax revenues  $T(s_t) = \tau Y(s_t)$ , and it can borrow from domestic and foreign agents by issuing uncontingent one-period debt. If the government defaults on its debt, it is excluded from capital markets for a random period, and it has a probability of reentering capital markets equal to  $\zeta$ . While in default, the government suffers a loss in tax revenues equal to  $\chi_t$ . This is motivated by evidence that sovereign defaults lead to severe financial and output disruptions (Hébert and Schreger, 2017; Bocola, 2016), and they should therefore imply a loss in fiscal revenues for the government.

### 3.2 Recursive equilibrium

We follow most of the literature and consider equilibria with a Markovian structure where equilibrium strategies and prices depend only on the current state. The government acts first and chooses its policy which consists of the decision of repaying inherited debt and the new debt issued (conditional on repayment). We denote the new debt issued by  $B'$  and the repayment decision by  $\delta \in \{0, 1\}$  where  $\delta = 1$  stands for repayment and  $\delta = 0$  stands for default. We denote a policy by  $\pi = (\delta, B')$ . The state the government faces when choosing its policy is given the exogenous state  $s$  and the endogenous state  $S = (B_H, B_F, A(\cdot), d)$  where  $B_H$  denotes the domestic holdings of government debt,  $B_F$  the foreign holdings of government debt,  $A(\cdot)$  the portfolios of Arrow securities held by domestic agents conditional on the policy chosen by the government, and  $d$  is an indicator variable that captures the ability of the government to issue new debt:  $d = 1$  if new debt can be issued and  $d = 0$  if no new debt can be issued. Private agents make their choices after the government and so the aggregate state they face is  $(S, s, \pi)$ .

**Private agents** We start setting up the equilibrium by describing the decisions of private agents given the current policy  $\pi$  and the policy rule  $\mathbf{p}(S, s)$ . The policy rule  $\mathbf{p}(S, s)$  de-

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<sup>9</sup>The results of this section hold even if we allow the government to have a different discount factor than consumers. In particular, we could allow the government discount factor to be lower than the one for domestic consumers.

scribes the policies chosen by future governments. The problem for an individual household with financial wealth  $a$  is to choose how much to consume and how to allocate his savings between defaultable government debt,  $b'$ , and Arrow securities contingent on the aggregate state next period,  $a'$ :

$$v(a, S, s, \pi) = \max_{c, b', a'(\cdot)} u(c) + \beta \sum_{s'} \mu(s'|s) \sum_{\pi'} \Pr(\pi' = \mathbf{p}(S', s') | S', s') v(\tilde{a}(S', s', \pi'), S', s', \pi') \quad (2)$$

subject to

$$c + \sum_{s'} \sum_{\pi'} Q(S, s, \pi | S', s', \pi') a(S', s', \pi') + q(S, s, \pi) b' \leq (1 - \tau) Y(s) + a$$

where  $Q(S, s, \pi | S', s', \pi')$  is the price of an Arrow security that pays if state  $(S', s', \pi')$  is realized next period,  $q(S, s, \pi)$  is the price of government debt,  $\tilde{a}(S', s', \pi')$  is the financial wealth next period in aggregate state  $(S', s', \pi')$  given by the sum of the Arrow security and the return on the defaultable debt holdings,

$$\tilde{a}(S', s', \pi') = a(S', s', \pi') + b' \delta(S', s' | \pi'),$$

and the evolution of the aggregate state  $S'$  is given by

$$S' = \mathbf{S}'(S, s, \pi) = (\mathbf{B}'_H(S, s, \pi), \mathbf{B}'_F(S, s, \pi), \mathbf{A}'(\cdot)(S, s, \pi), \mathbf{d}'(S, s, \pi)),$$

where  $\mathbf{S}' = (\mathbf{B}'_H, \mathbf{B}'_F, \mathbf{A}'(\cdot), \mathbf{d}')$  is the equilibrium law of motion for the aggregate state.

Since we focus on symmetric equilibria where all households are identical, it is convenient to define the allocation rules for the representative agent,

$$C(S, s, \pi) = c(\bar{a}, S, s, \pi), \mathbf{B}'_H(S, \pi, s) = b'(\bar{a}, S, s, \pi), \mathbf{A}'(\cdot)(S, \pi, s) = a'(\cdot)(\bar{a}, S, s, \pi) \quad (3)$$

where  $\bar{a} = A(S, s, \pi) + b\delta$  is the financial wealth for the representative agent. Thus the equilibrium law of motions for domestic asset holdings,  $\mathbf{B}'_H(S, \pi, s)$  and  $\mathbf{A}'(\cdot)(S, \pi, s)$ , are equal to the allocation rules for the domestic household evaluated at the average inherited financial wealth  $\bar{a}$ . The evolution for the foreign holdings of government debt,  $\mathbf{B}'_F(S, s, \pi)$ , is residually pinned down by the market clearing condition for government debt,  $B' = \mathbf{B}'_H(S, s, \pi) + \mathbf{B}'_F(S, s, \pi)$ . Finally, the transition for  $d$  is such that if there is repayment in the current period then  $\mathbf{d}' = 1$ , while if there is a default then  $\mathbf{d}' = 1$  with probability  $\zeta$  and  $\mathbf{d}' = 0$  with complementary probability, capturing the assumption that a government regains access to capital markets after a default with probability  $\zeta$ .

The no-arbitrage conditions from the foreign lenders require that

$$Q(S, s, \pi | S', s', \pi') = M(s', s) \Pr(\pi' = \mathbf{p}(S', s') | S', s') \mathbb{I}_{\{S' = \mathbf{S}'(S, \pi, s)\}} \quad (4)$$

$$q(S, s, \pi) = \sum_{s'} M(s', s) \delta(\mathbf{S}'(S, \pi, s), s'). \quad (5)$$

That is, the price of an Arrow security that pays one unit of the final consumption good next period in the aggregate state  $(S', s', \pi')$  is either  $M(s', s)$  if it is possible to observe  $(S', s', \pi')$  next period or zero otherwise. In particular, the price is zero if the probability of observing a policy  $\pi'$  in state  $(S', s')$  is zero,  $\pi' \neq \mathbf{p}(\mathbf{S}'(S, \pi, s), s')$ . From (5), the price of defaultable government debt is the discounted value of repayments obtained in the next period.

**Government problem** We now examine the problem of the government in state  $(S, s)$ . The government takes as given the choices of future governments, the price rule  $q(S, s, \pi)$ , the aggregate consumption rule  $C(S, s, \pi)$ , and the law of motion for the aggregate state  $\mathbf{S}'(S, s, \pi)$ . The government chooses the current policy  $\pi$  and government spending  $g$  to solve

$$W(S, s) = \max_{\pi=(\delta, \mathbf{B}'), g} u(C(S, s, \pi)) + \omega(g) + \beta \sum_{s'} \mu(s'|s) W(\mathbf{S}'(S, s, \pi), s') \quad (6)$$

subject to the government budget constraint if  $d = \delta = 1$ ,

$$g + B_H + B_F \leq \tau Y(s) + q(S, s, \pi) B'$$

and if  $\delta = 0$  or  $d = 0$  the government has no access to capital markets so

$$g = \tau Y(s) - \chi(s),$$

where  $\chi(s)$  is the default cost.

**Equilibrium** We can then define a recursive equilibrium for this economy:

**Definition.** A *recursive equilibrium* is a value for the government  $W$  and the associated policy rule  $\mathbf{p} = (\delta, \mathbf{B}')$ , the household value  $v$  and associated individual allocation rules, the aggregate allocation rules, the law of motion for aggregate state variables, and prices for Arrow securities  $Q$  and government debt  $q$  such that i) the government value and the associated policy rules solve (6), ii) the household value and the individual allocation rules solve (2), iii) the aggregate allocation rules satisfy the representativeness condition

(3), iv) the prices for Arrow securities and government debt are given by (4) and (5), and v) the market for government debt clears in that

$$B' = \mathbf{B}'_H(S, s, \pi) + \mathbf{B}'_F(S, s, \pi)$$

where recall that  $\pi = (\delta, B')$ .

### 3.3 Primal Markov problem

We next show that, from period 1 onward, debt issuances and default decisions are functions only of the total public debt and exogenous shocks and do not depend on the composition of the holdings of sovereign debt. Total public debt and income are sufficient statistics for the problem of the government and the price of government debt  $q$ . This result is driven by the fact that the domestic household's financial wealth and marginal utility of private consumption do not depend on government policies from period 1 onward. This is because private agents are insured against a government default. Thus the composition of government debt holdings does not affect the government's default decisions.

**Implementability conditions** We first establish that from period 1 onward the domestic household's financial wealth and marginal utility of private consumption are independent from government policies. From the first-order conditions of the household's problem and the no-arbitrage condition (4), we obtain that for all  $S' = \mathbf{S}(S, s, \pi)$ ,

$$\Pr(\pi' = \mathbf{p}(S', s') | S', s') \beta \frac{u'(C(S', s', \pi'))}{u'(C(S, s, \pi))} = \Pr(\pi' = \mathbf{p}(S', s') | S', s') M(s', s),$$

where we used the representativeness condition  $C(S, s, \pi) = c(\bar{a}, S, s, \pi)$ . For policies along the equilibrium path, we have that  $\Pr(\pi' = \mathbf{p}(S', s') | s') > 0$ , and so it must be that

$$\beta \frac{u'(C(S', s', \pi'))}{u'(C(S, s, \pi))} = M(s', s). \quad (7)$$

For policies off the equilibrium path, instead,  $\Pr(\pi' = \mathbf{p}(S', s') | S', s') = 0$ , and so the risk-sharing condition (7) does not necessarily hold. This is because histories that are not reached in equilibrium are irrelevant from the household's perspectives and so the household's financial position is indeterminate in such histories. In what follows, we select the holdings of Arrow securities assuming that the government can tremble and choose all the possible policies with a small probability and take the limit of such probability to zero. With this selection, the risk-sharing condition (7) must hold for any policies  $\pi'$ , on and off

the equilibrium path. This implies that the marginal utility of private consumption for the domestic household is constant across different policies. The domestic consumer and the foreign agent perfectly share both the exogenous risk,  $s$ , and the endogenous policy (default) risk,  $\pi'$ . This implies a restriction on the portfolios of Arrow securities that can be chosen in equilibrium. Take two states with the same output but with different policies. Suppose  $\pi_A$  has repayment and  $\pi_B$  has default. Budget feasibility requires that

$$A(S', \pi_B) = A(S', \pi_A) + B'_H. \quad (8)$$

Thus, for each state  $s'$  and  $S'$ , and all policies  $\pi_A$  and  $\pi_B$ , the portfolio of Arrow securities must satisfy (8) to guarantee that (7) holds under both policies.

**Debt distribution does not matter for government problem** We next use the result above to show that the equilibrium policy from period 1 onward solves a simple problem with total debt as the unique endogenous state variable and the government only considers the utility from public consumption when making its choices. In particular, from period 1 onward, the value for the government from government consumption is

$$\Omega(B, s) = \max \left\{ \Omega^r(B, s), \Omega^d(B, s) \right\}, \quad (9)$$

where  $\Omega^r$  is the value from government consumption if there is repayment,

$$\Omega^r(B, s) = \max_{G, B'} \omega(G) + \beta \sum_{s'} \mu(s'|s) \Omega(B', s') \quad (10)$$

subject to the budget constraint

$$G + B \leq \tau Y(s) + q(s, B') B',$$

and  $\Omega^d$  is the value from government consumption if there is default,

$$\Omega^d = \omega(\tau Y(s) - \chi(s)) + \beta \sum_{s'} \mu(s'|s) \left[ (1 - \zeta) \Omega^d(s') + \zeta \Omega(0, s') \right]. \quad (11)$$

The government then chooses to repay its debt,  $\delta(B', s') = 1$ , if and only if  $\Omega^r(B, s) \geq \Omega^d(B, s)$ . The pricing schedule for government debt in (5) then simplifies to

$$q(s, B') = \sum_{s'} M(s', s) \delta(S'(s')). \quad (12)$$

The problem in period 0 is different because the asset holdings of the domestic house-

holds do not necessarily satisfy (8), and thus a default decision can affect domestic private wealth, so the distribution of government debt between domestic and foreign agent is relevant. To set up the problem for the government in period 0, we define the value for the equilibrium private consumption given initial financial wealth  $A$ ,  $v(A, s)$ , that is the unique solution to the following functional equation:<sup>10</sup>

$$V(A, s) = \max_{c, A(\cdot)} u(c) + \beta \sum_{s'} V(A(s'), s')$$

subject to

$$c + \sum_{s'} M(s', s) A(s') \leq (1 - \tau) Y(s) + A.$$

The value of repaying the inherited debt in period 0 given  $S_0 = (B_{H0}, B_{F0}, A_0(\cdot), 1)$  can be written as

$$\Omega_0^r(S_0, s) = V(A_0(S, s, \delta = 1), s) + \Omega^r(B, s) \quad (13)$$

where  $A_0(S, s, \delta = 1)$  is the value of inherited Arrow securities that pay in case of no default in period 0. The value of defaulting is given by

$$\Omega_0^d(S_0, s) = V(A_0(S, s, \delta = 0), s) + \Omega^d(s) \quad (14)$$

where  $A_0(S, s, \delta = 0)$  is the value of inherited Arrow securities that pay in case of a default in period 0. The government then defaults in period 0 if and only if<sup>11</sup>

$$\Omega_0^r(S_0, s) \geq \Omega_0^d(S_0, s). \quad (15)$$

The next proposition states the main result of this section:

**Proposition.** *Given an initial state  $S_0$ , the equilibrium outcome  $\{B(s^t), \delta(s^t), q(s^t)\}$  can be recovered as the solution to the quasi-recursive problem in (9)–(15).*

The proof is straightforward. The risk-sharing condition (7) implies that the consumption profile of the domestic household does not depend on the government policies from period 1 onward both on and off the equilibrium path. From period 1 onward, the domestic households are insured from government default as a consequence of (7), and we have that (8) holds. Thus, along the equilibrium path, when a government contemplates whether to default, the changes in the private net foreign asset positions are not affected by this choice. Hence, the government can ignore the private sector in its default and debt issuance choices. Thus, from period 1 onward, government policies solve (9).

<sup>10</sup>Note that  $V(A) = v(A, S, s)$ .

<sup>11</sup>Under our construction,  $W(S, s) = \max\{\Omega_0^r(S, s), \Omega_0^d(S, s)\}$ .

### 3.4 Discussion

The representation in (9) looks identical to the setup in canonical models used to study sovereign default for emerging market economies. There is, however, an important difference in the interpretation of the state variable  $B$ . Canonical models that follow [Eaton and Gersovitz \(1981\)](#) consider the polar opposite of the case considered here: private agents cannot borrow from foreign lenders.<sup>12</sup> The canonical sovereign debt model admits the representation in (9), but the relevant statistic is external debt (private and public), and not total public debt as in the case studied in this section.

Clearly, we need strong assumptions to compare these two cases. In particular, by assuming that private financial markets are sophisticated, we obtain that there are no spillovers from government decisions to the private sectors and the distribution of public debt holdings is not relevant. This contrasts with the findings of two large and growing literatures. One literature documents the pass-through of government interest spreads on private interest rates ([Bocola \(2016\)](#) and [Arellano et al. \(2017\)](#)). Another literature emphasizes that the composition of holdings of government debt is an important determinant of interest rate spreads and debt sustainability more broadly ([Perez \(2015\)](#), [Gennaioli et al. \(2014\)](#), [Broner et al. \(2014\)](#), [Chari et al. \(2016\)](#), [Dovis et al. \(2016\)](#), and [D’Erasmus and Mendoza \(2013\)](#)). We are not advocating that these features are not relevant, but we are abstracting from these features to make the comparison between our model and the canonical model in the starkest way possible.

Which of these two polar opposites is more relevant? The answer may depend on the particular application, especially on the sophistication of financial markets and the ability of governments to interfere with private domestic contracts. See [Arellano et al. \(2016\)](#) for a documentation of how different governments have varying ability to interfere with private contracts. In Europe, private external financial markets are fairly advanced and sophisticated, and EU regulations require free mobility of capital within member states. Thus, we think our model is a useful benchmark to study the European debt crisis. Moreover, the insurance and liquidity provided by the ECB to banks may be a proxy for the credit default swaps present in the model.

## 4 Calibration and workings of the model

In this section, we consider two different calibrations of the model in (9) using Spanish data. Our calibration maps the variable  $B$  to total public debt, while the traditional calibration maps it to total external debt. We then compare the performance of these two

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<sup>12</sup>Alternatively, there is no distinction between private foreign liabilities and public foreign liabilities, and all external debt is ultimately a liability of the government.



different approaches.

## 4.1 Calibration

We make the following functional form assumptions. We follow [Bocola and Dovis \(2016\)](#) and assume that the preferences for public good consumption are given by

$$\omega(g) = \frac{(g - \underline{g})^{1-\sigma} - 1}{1 - \sigma}, \quad (16)$$

where  $\sigma \geq 0$  and  $\underline{g} \geq 0$  is a minimal level for the public consumption good. We are not literally interpreting  $\underline{g}$  as a subsistence level for consumption, but our preferred interpretation is that  $\underline{g}$  stands in for the level that captures the components of public spending that are hardly modifiable by the government in the short run, such as wages of public employees and pensions.<sup>13</sup> The presence of  $\underline{g}$  breaks the homotheticity of the government preferences and is critical in accounting for the data in our interpretation of the model, as we will make clear later.

We further assume that foreign lenders are risk neutral so  $M(s', s) = \Pr(s'|s) / (1 + r^*)$  where  $r^*$  is the risk-free real interest rate. The output process,  $Y = \exp\{y\}$ , is an AR(1) process,

$$y' = \rho_y y + \sigma_y \varepsilon', \quad \varepsilon' \sim \mathcal{N}(0, 1). \quad (17)$$

The default costs are parametrized following [Chatterjee and Eyigungor \(2012\)](#),

$$\chi(Y) = \max\{0, d_0 \tau Y + d_1 (\tau Y)^2\}. \quad (18)$$

We select the model parameters to match a set of moments summarizing the behavior of public finances and interest rates. We set  $r^*$  so that the annual risk-free real interest rate is 1.8 percent. We set  $\zeta = 0.282$  following [Arellano \(2008\)](#). We then set  $\sigma = 2$ . The parameters of the output process in equation (17) are estimated on log deviations of real GDP from a quadratic time trend estimated over the 1960Q1-2012Q2 period.

The remaining parameters,  $\beta$ ,  $d_0$ ,  $d_1$ , and  $\underline{g}$ , are chosen to match a set of empirical targets. We include in the set of empirical targets statistics that summarize the behavior of outstanding debt and interest rate spreads. In both calibrations, we consider the sample mean of the debt services to output ratio, and the mean and standard deviation of the yields differential between a Spanish and German Treasury bill with a residual maturity

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<sup>13</sup>One can also think of  $\underline{g}$  as a form of habit: households are used to (or the government is committed to) a certain level of public expenditures and transfers, and it is costly for the government to deviate from this level of expenditures.

Table 2: Model parameters

|                    | Our calibration | Traditional calibration    |                     |                               |
|--------------------|-----------------|----------------------------|---------------------|-------------------------------|
|                    |                 | Calibrated $\underline{g}$ | $\underline{g} = 0$ |                               |
| $r^*$              | 0.0045          | 0.0045                     | 0.0045              | Annual risk free rate of 1.8% |
| $\sigma$           | 2               | 2                          | 2                   | Conventional value            |
| $\zeta$            | 0.28            | 0.28                       | 0.28                | Arellano (2008)               |
| $\rho_y$           | 0.97            | 0.97                       | 0.97                | Estimates of equation (17)    |
| $\sigma_y$         | 0.01            | 0.01                       | 0.01                | Estimates of equation (17)    |
| $\beta$            | 0.98            | 0.86                       | 0.82                | Method of simulated moments   |
| $\tilde{d}$        | 0.09            | 0.02                       | 0.02                | Method of simulated moments   |
| $d_{\text{ratio}}$ | 2.07            | 1.15                       | 1.15                | Method of simulated moments   |
| $\underline{g}$    | 0.85            | 0.36                       | 0                   | Method of simulated moments   |

Notes: We reparametrize the  $d(\cdot)$  function in equation (18) in order to make our results more easily interpretable. The parameter  $\tilde{d}$  stands for the percentage loss in output after a default when output is at its median values. The parameter  $d_{\text{ratio}}$  is the ratio between the default cost over output for top income vs. median income.

of one year.<sup>14</sup> These are standard targets for quantitative sovereign debt models. The two calibrations differ, however, in the measure of debt cyclicity. In our calibration, we target the correlation between the debt services to output ratio and detrended real GDP, while in the traditional calibration, we substitute this moment with the cyclicity of the trade balance over GDP,  $\text{corr}(\text{TB}/Y, \ln Y)$ .<sup>15</sup>

We solve the model on a grid of points for  $(\beta, d_0, d_1, \underline{g})$  and select the parametrization that minimizes a weighted distance between sample moments and their model-implied counterparts.<sup>16</sup> Model-implied moments are computed on a long simulation ( $T = 100,000$ ), and we weight the distance between a sample moment and its model counterpart by the inverse of the standard deviation of the sample moment squared. For the traditional calibration we also parametrize the model by setting  $\underline{g} = 0$  and choosing  $(\beta, d_0, d_1)$  to match the same set of moments. Table 2 reports the value for the model's parameters.

<sup>14</sup>We decided to keep average debt services constant across the two calibrations in order to isolate the effects that the cyclicity of debt has on the the behavior of the model. The empirical target is constructed using public debt. We use the average debt maturity—6.5 years during the sample—and assume that a constant fraction of debt matures in every period. The average debt maturity along with total public debt allows us to compute the debt service payments in every period. The average net foreign debt for Spain is comparable to average public debt (see Section 2). Thus, we would not expect our results to change dramatically if we were to use a more appropriate average external debt for the traditional calibration.

<sup>15</sup>Certain papers in the literature (e.g., Bianchi et al. (2012)) target the variance of aggregate consumption relative to output which is typically larger than one for emerging economies. In a pure exchange economy, however, this moment is tightly associated to the cyclicity of the trade balance. In particular, to have consumption more volatile than output it must be that the trade balance is countercyclical.

<sup>16</sup>The model is solved using a value function iteration procedure similar to the one used in Arellano (2008). We use an equidistant income grid between 3 standard deviations below and above the ergodic

Table 3: **Calibration targets**

| Statistic                 | Data     |        | Our calibration | Traditional calibration    |                     |
|---------------------------|----------|--------|-----------------|----------------------------|---------------------|
|                           | Estimate | (S.E.) |                 | calibrated $\underline{g}$ | $\underline{g} = 0$ |
| Average spread            | 0.32     | (0.28) | 0.09            | 0.44                       | 0.52                |
| Spread volatility         | 0.88     | (0.29) | 0.83            | 0.51                       | 0.57                |
| Average debt service/GDP  | 8.43     | (0.60) | 8.52            | 8.32                       | 8.28                |
| Debt service cyclicality  | -0.87    | (0.30) | -0.29           | -                          | -                   |
| Trade balance cyclicality | -0.72    | (0.28) | -               | -0.08                      | -0.07               |

Notes: Data moments computed using GMM, standard errors using Newey-West with 8 lags. Weighting matrix is a diagonal with inverse of standard errors squared.

The first and second columns of Table 3 show that our calibration of the model has good in sample fit. As in the data, the government debt service to output ratio ( $B/Y$  in the model) is negatively correlated with output. Interest rate spreads are typically close to zero in model simulations, with an annualized average value of 0.09% relative to the 0.32% observed in the sample, and they are as volatile as in the data. Both with a calibrated  $\underline{g}$  and with  $\underline{g} = 0$ , the traditional calibration does not fit as well on the targeted moments, as it produces a coefficient of variation for interest rate spreads of roughly one, substantially below that of the data. [Aguiar et al. \(2016\)](#) have previously pointed out the difficulties of the traditional calibration to produce volatile interest rate spreads. Targeting countercyclical debt issuances allows the model to better fit this moment. It is also worth noting that the two calibrations imply different behavior also for the face value of debt: in our calibration,  $\text{corr}(B', \ln Y) = -0.25$ , while in the traditional calibration  $\text{corr}(B', \ln Y) = 0.98$ .

## 4.2 Workings of the model

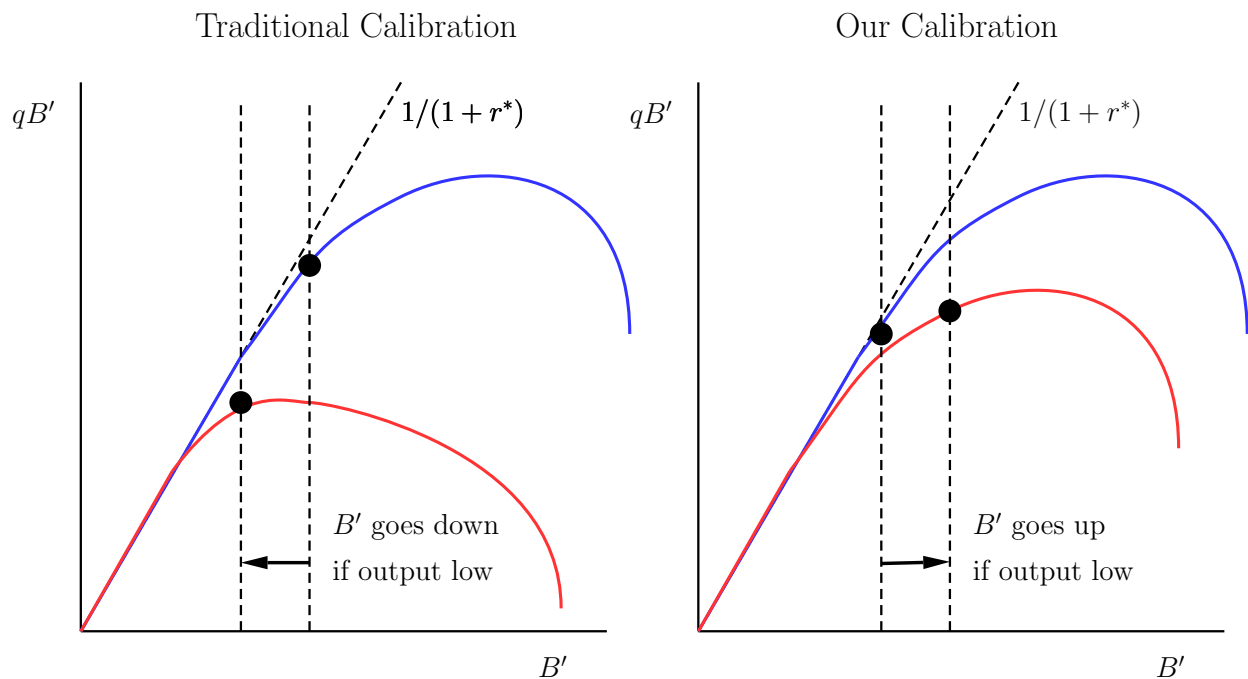
We now contrast the workings of our calibration with the traditional one. We start by considering the traditional calibration. How can the model generate a pro-cyclical debt issuance in a pure exchange economy? [Arellano \(2008\)](#) shows that it takes a combination of i) convex default costs and persistent output, and ii) a high degree of impatience relative to the international risk-free rate and weak precautionary saving motives. The first set of ingredients imply that the pricing schedule that the government faces expands after a positive income shock and contract after a negative one: if the output costs of default are convex, then the government finds it more attractive to repay its debt obligations in good times. If, in addition, the income process is persistent, then the government can borrow more in good times as it can credibly promise to repay more in the following period. The

mean. The income grid consists of 25 points and the debt grid contains 501 points.

second set of ingredients implies that the government behaves myopically, and uses the debt market mostly to front-load consumption rather than smooth it across states of the world.

The combination of the cyclical nature of the pricing schedule and government's impatience, then, implies that debt issuances are procyclical. This mechanism is illustrated in the left panel of Figure 2, where we plot the revenues from debt issuances,  $q(s, B')B'$ , as a function of the face value of debt,  $B'$ , for a high (blue line) and a low (red line) current realization of output. When output is high, the risk of future default is small, and the government is able to raise more resources by issuing debt. Conversely, low-income states are associated with tighter pricing. Thus, the revenues schedule shifts inward restricting the ability of the government to front-load consumption. Because of high impatience and low precautionary motives, the government finds it optimal to borrow more in a high output state (the optimal debt choices are depicted with circles on the schedule  $q(s, B')B'$ ). In a low output state, however, the government is forced to cut its indebtedness because such large debt would not be credibly repaid. Hence, the impatience of the government, coupled with the endogenous borrowing limits implied by default risk, leads to procyclical debt issuances.

Figure 2: The cyclical nature of debt issuances



Notes: The filled circles represent the optimal debt issuances in the high and low output states.

Our baseline calibration works differently because the government is more cautious and has less incentives to front-load consumption (higher  $\underline{g}$  and higher  $\beta$ ). The right

panel of Figure 2 plots the revenue schedules,  $q(s, B')B'$ , for two realizations of current output and the respective optimal debt issuances for our baseline calibration. The revenue schedules in the two calibrations are qualitatively similar: they both define a Laffer curve for debt issuances, and they both shift inward when the economy is hit by negative output shocks. The two calibrations differ in the debt choices made by the government. In our parametrization, the government uses the debt markets mostly for consumption smoothing, so it optimally keeps a low debt in high-output states, while it borrows in the face of bad output shocks. In the next section we explore how this different behavior for debt issuances under the two calibrations impact the distribution of equilibrium spreads in the model.

Before moving there, it is worth emphasizing that the presence of the subsistence level of public consumption good,  $\underline{g}$ , in the government's preferences (16) is critical for jointly replicating the countercyclicality of debt issuances and positive and volatile spreads.<sup>17</sup> We show this in Figure 3. The figure displays the correlation of debt and income, the average debt-to-income ratio, the average spread, and the standard deviation of spreads as functions of the discount factor  $\beta$ , keeping the other parameters at their levels of the traditional calibration with  $\underline{g} = 0$  (fourth column in Table 2).<sup>18</sup> When  $\beta$  is close enough to  $1/(1+r^*)$ ,<sup>19</sup> the model can potentially generate a negative correlation between the debt-to-GDP ratio and output but with lower debt and essentially no spreads in equilibrium. The idea is that for high  $\beta$ , the government does not want to front-load consumption anymore, and even small precautionary saving motives incentivize it to be far away from the region of the state space with positive spreads (the "risky region" in the language of Arellano, 2008). When  $\underline{g}$  is sufficiently large, the government is forced to increase debt when income realizations are sufficiently negative, and it thus visits with positive probability the risky region.

### 4.3 Behavior of spreads

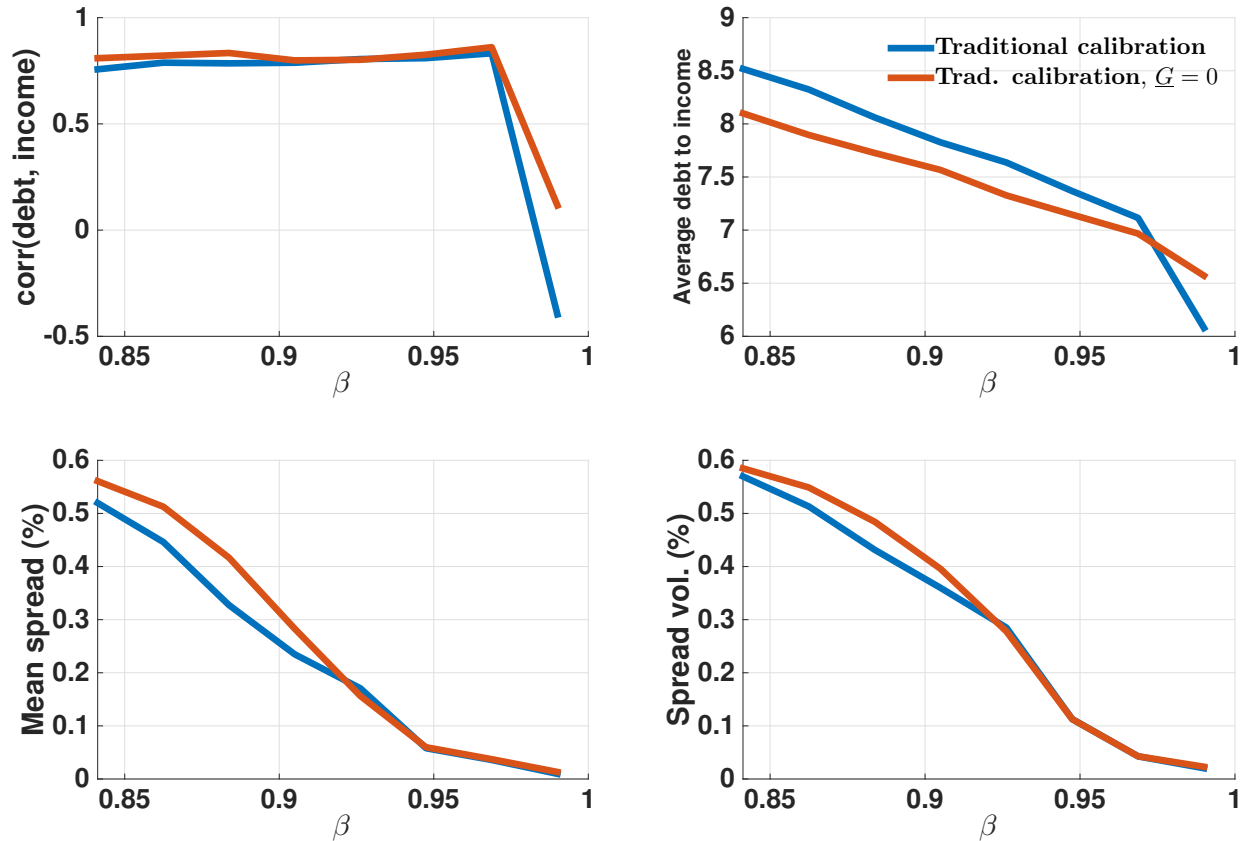
We now discuss how spreads behave in the two calibrations. Our calibration generates interest rate spreads that are on average close to zero, and they jump to substantially positive values only conditional on sufficiently low income realizations or high inherited debt. Figure 4 plots the distribution of spreads generated by our calibration together with the data for Spain. To make the two distributions more comparable to the data, we scale spreads by their mean. Our calibration matches the entire distribution of spreads

<sup>17</sup>This is true also true in the richer model considered in Bocola and Dovis (2016)..

<sup>18</sup>Figure 3 also plots the same four statistics generated by varying  $\beta$  and keeping all the other parameters at the values of the calibration in the third column in Table 2. We can see that the results are similar.

<sup>19</sup>We only consider cases with  $\beta(1+r^*) < 1$  because we know that if  $\beta(1+r^*) = 1$  then the government would have incentives to accumulate assets.

Figure 3: Discount factor and target moments



Notes: To generate this figure, we set all the model parameters except  $\beta$  to their values in the traditional calibration and then solve the model for different values of  $\beta$ . We simulate artificial data under these different parametrizations and report the model-generated statistics.

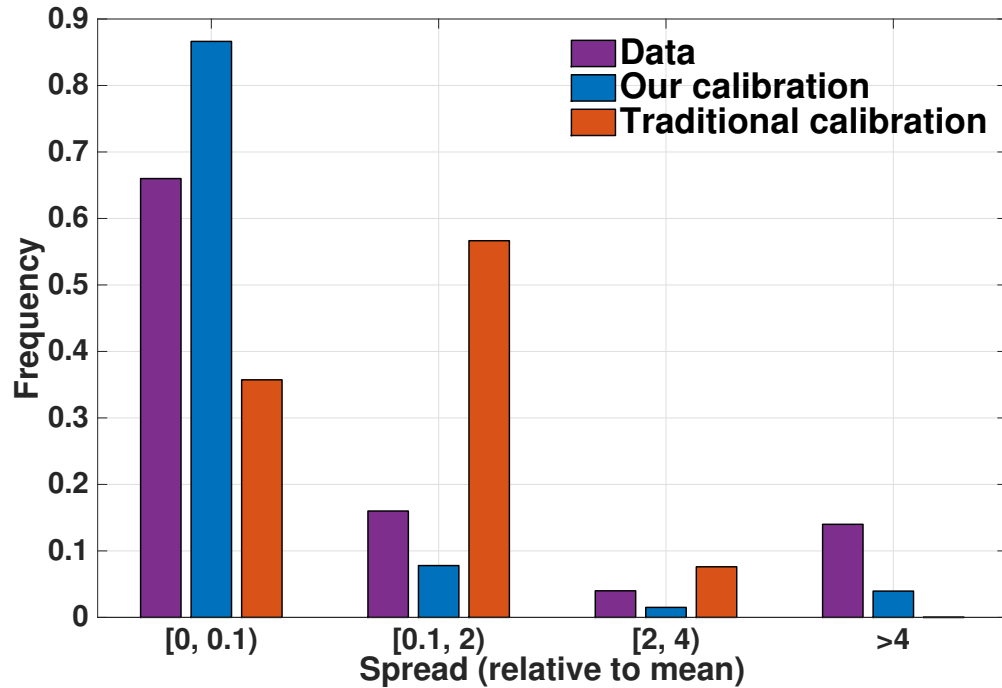
fairly well despite targeting only the first two moments. A salient feature of the data is that spreads are less than 10% their average value most of the time, approximately 65%. Our calibration is consistent with this fact: spreads are below 10% of their sample mean roughly 85% of the times.<sup>20</sup>

Figure 4 also reports the distribution of spreads obtained in the traditional calibration. The distribution is more centered on the mean, unlike in the data. Spreads are below 10% of their mean value only about 35% of the time. As explained in the previous section, in this calibration, the government acts in a myopic way. So the government is at risk of a default most of the time, with interest rate spreads being positive even when output is above average.

Note also that our calibration is able to generate more extreme observations than the traditional calibration (spreads that are four times above their mean value). To understand why spreads do not reach higher levels in the traditional calibration, note that

<sup>20</sup>Our calibration strategy can generate similar distribution for interest rate spreads also with long-term debt. See [Bocola and Dovis \(2016\)](#).

Figure 4: Distribution of interest rate spreads



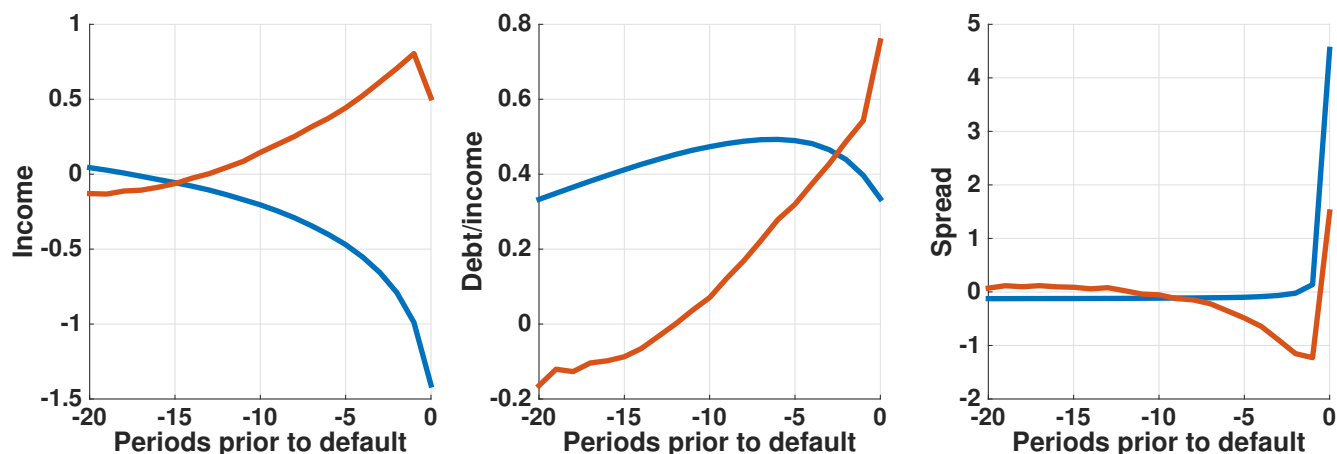
equilibrium spreads in the model are driven by current income and newly issued debt,  $\text{spr}(y, b'(y, b))$ . In the traditional calibration, a negative income shock increases spreads directly, but it decreases them indirectly via the reduction in newly issued debt associated with it,  $\partial b'/\partial y < 0$ . This indirect effect moderates the impact of a negative income shock on equilibrium spreads. In contrast, in our calibration,  $\partial b'/\partial y > 0$ . Thus, both effects increase the spreads, and the model can generate spreads significantly higher than the mean when hit by a negative income shock.

#### 4.4 Dynamics in a typical crisis

We now contrast the typical dynamics leading to a debt crisis under our calibration and the traditional one. We define a crisis as a period in which the interest rate spread is higher than its mean value plus one standard deviation. The typical path for income, the ratio of debt to income,  $B/Y$ , and spreads are reported in Figure 5. All variables are reported in standard deviations relative to their means. Blue lines are for our calibration, while red lines correspond to the traditional one.

In our calibration, the driving force leading to a crisis is a sequence of negative income shocks. Along this path, the government issues more debt to keep its consumption away from the subsistence level  $\underline{g}$ . The path for the debt-to-GDP ratio is initially increasing but eventually, the government must reduce its debt when interest rates start to raise. This can

Figure 5: Typical outcome path leading to a crisis



Notes: The blue lines report crisis events under our calibration while red lines report the same experiment in the traditional calibration. All variables are reported in standard deviations relative to their mean.

account for the fact that fiscal policy turns to be procyclical during crisis even in advanced economies where policy is countercyclical in normal times. Spreads are negligible until one quarter prior to the start of the crisis, even though income is below its mean and debt is above its mean in all preceding periods. The spread then jumps to more than four standard deviations above the mean. This highlights that the sensitivity of spreads to output and debt is highly non-linear in the model.

The sequence of shocks that lead to a crisis in the traditional calibration is very different. A crisis is preceded by a sequence of increasing positive income shocks until the period in which a crisis starts, where the government experiences a negative innovation to its income. Why such pattern? In order to have spreads higher than average, the level of debt needs to be above mean. Because in the traditional calibration there is a positive comovement between debt issuances and GDP, the model needs a series of positive income shocks to increase debt above its mean. Along this path with increasing income, spreads are decreasing and below average until the period in which the crisis starts.

The increase in spreads in the typical crisis is smaller than in our calibration: in our calibration spreads are more than four standard deviations higher than the average while in the traditional calibration they are only one and a half standard deviation higher. This is because under our calibration, both debt and income contributes to higher spreads (debt-to-GDP is above average and income is below average), while in the traditional calibration this is not the case: debt is above average only if income is above average essentially by construction because of the imposed procyclicality of debt issuances.

Note that the dynamics leading to a crisis under the traditional calibration does not need to be preceded by a sequence of positive shocks. This depends on model parameters. It is possible that the typical path leading to a crisis or a default has income shocks



below average. However, along such paths the debt would be decreasing over time. The traditional calibration, essentially by construction, cannot generate contemporaneously patterns of increasing debt and income below trend, a feature that appears important to account for the southern European experience.

## 5 Conclusion

This paper uses a standard quantitative sovereign default model to study the recent debt crisis in southern Europe. We first document that in the case of Portugal, Spain, and Italy, total public debt outperforms standard indicators of external debt in accounting for the dynamics of interest rate spreads. Motivated by this evidence, we derive theoretical conditions under which total public debt is the only relevant state variable in the decision problem of the government other than GDP, while most of the existing literature focuses on external debt. We then show that a calibrated version of the model can account for the behavior of income, public debt, and interest rate spreads for southern European economies. Compared to the traditional calibration in the literature that targets the behavior of external debt and in particular its cyclicity, our approach improves the fit of the model regarding interest rate spreads and produces more realistic crises dynamics.

We purposely analyzed the most basic default model to illustrate the mechanism in a transparent way. Extending the model to account for the joint behavior of external and public debt is a promising avenue for future research. One may also want to extend the model by introducing other realistic features that can increase the fit of the model such as long-term debt, risk-averse lenders, partial recovery of debt, and a production economy with investment. We leave these topics for future research. The main message of the paper extends to these more realistic economies: matching the countercyclicality of public debt issuances allows the model to reproduce more realistic behavior for interest rate spreads with a higher mass at zero and few large spikes.<sup>21</sup>

Finally, studying whether our results are also applicable to emerging market economies is an important area for future research. It is not obvious, however, that the results of our empirical analysis would extend to emerging market economies. First, in most emerging market economies, debt issued under domestic law is denominated in domestic currency while debt issued under foreign law is denominated in foreign currency. Since debt crises are typically associated with currency depreciations, the foreign currency value of domestic debt is likely to fall during crisis. Second, we expect the cyclicity of total public debt in emerging economies to be less countercyclical than in advanced economies and closer to the cyclicity of the trade balance. This is related to the observation that fiscal policy

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<sup>21</sup>See for instance [Bocola and Dovis \(2016\)](#).

in emerging economies (Latin American in particular) is often procyclical in contrast to advanced economies (See [Gavin and Perotti, 1997](#) and [Kaminsky et al., 2004](#)).

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