The Pass-Through of Sovereign Risk

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This paper examines the macroeconomic implications of sovereign risk in a model in which banks hold domestic government debt. News of a future sovereign default hampers financial intermediation. First, it tightens the funding constraints of banks, reducing their resources to finance firms (liquidity channel). Second, it generates a precautionary motive to deleverage (risk channel). I estimate the model using Italian data, finding that sovereign risk was recessionary and that the risk channel was sizable. I also use the model to measure the effects of subsidized long-term loans to banks. Precautionary motives at the height of the crisis imply that bank lending to firms responds little to these interventions.

I. Introduction

At the end of 2009, domestic government debt holdings by banks in European peripheral countries—Greece, Italy, Portugal, and Spain—were equivalent to 93 percent of banks' total equity. At the same time, domestic financial intermediaries in these economies were providing roughly two-thirds of the external financing of local firms. It is therefore not sur-

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prising that many empirical studies have documented a severe disruption of financial intermediation and a substantial increase in the borrowing costs of firms during the sovereign debt crisis.[1] One proposed explanation of these findings is that the exposure to distressed government bonds hurts the ability of banks to raise funds in financial markets, leading to a pass-through of their increased financing costs into the lending rates paid by firms (Committee on the Global Financial System 2011). This view was at the core of policy discussions in Europe and was a motive for major interventions by the European Central Bank (ECB).

I argue, however, that this view is incomplete. A sovereign default can in fact be the trigger of a severe recession and have adverse effects on the performance of firms. Consequently, as an economy approaches a sovereign default, banks may start perceiving firms as more risky, and they may demand higher returns when lending to them as a compensation for holding this additional risk. If this mechanism is quantitatively important, policies that address the heightened liquidity problems of banks but do not reduce the increased riskiness of firms may prove ineffective in encouraging bank lending.

I analyze this mechanism in a quantitative model with financial intermediation and sovereign default risk. In the model, the news that the government may default in the future has adverse effects on the funding ability of banks (*liquidity channel*), and it raises the risks associated with lending to the productive sector (*risk channel*). I structurally estimate the model on Italian data and find that the risk channel is quantitatively important: it explains up to 45 percent of the impact of the sovereign debt crisis on firms' borrowing costs. I then use the estimated model to assess the consequences of credit market interventions adopted by the ECB in mitigating the implications of increased sovereign default risk.

My framework builds on a business cycle model with financial intermediation, in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In the model, banks collect savings from households and use these funds, along with their own wealth (net worth), to buy long-term government bonds and to lend to firms. This intermediation is important because firms need external financing to buy capital goods. The model has three main ingredients. First, an agency problem between households and banks generates a constraint on the borrowing ability of the latter. This constraint on bank leverage binds only occasionally, typically when bank net worth is low. Second, financial intermediation is risky: bank net worth varies over time mainly because banks finance long-term risky assets with short-term riskless debt. Third, the probability

[1] See, e.g., the evidence in Acharya et al. (2014) using data on syndicated loans originated by European banks and the evidence in Bofondi, Carpinelli, and Sette (2013) and Bottero, Lenzu, and Mezzanotti (2015) using the Italian Credit Register.

that the government defaults on its debt in the future is time varying and follows a reduced-form rule.

To understand the mechanisms of the model, consider a scenario in which agents' expectations of a future sovereign default rise. The prospect of losses on government bonds depresses their market value today. Because banks are exposed to these assets, their net worth declines and their ability to borrow is impaired. As a result, bank lending to the productive sector drops, leading to a contraction of capital accumulation. This is a conventional liquidity channel in the literature.

In addition, the expectation of a future government default has adverse effects on the willingness of intermediaries to hold financial claims of the firms, even when banks are currently not constrained in their borrowing ability. Indeed, the news that the government may default raises the chance of large balance sheet losses and tight funding constraints for the banks in the future. This has two effects. First, in anticipation of hitting their funding constraints, banks act more cautiously and demand higher compensation for holding risky assets. Second, they attach a higher probability of incurring losses to their holdings of firms' claims: these assets, in fact, will lose value if the default occurs next period because they will be subject to fire sales by constrained intermediaries. Effectively, the banks now perceive private-sector claims as more risky. The combination of these two effects generates a precautionary motive for banks to deleverage and to reduce their holdings of firms' claims. I refer to this second mechanism as the risk channel.

I measure the quantitative importance of liquidity and risk by estimating the structural model with Italian data. The major challenge is to separate these two propagation mechanisms because they have qualitatively similar implications for indicators of financial stress commonly used in the literature (e.g., credit spreads). I demonstrate, however, that the Lagrange multiplier on the funding constraints of banks is a function of observable variables. I construct a time series for the Lagrange multiplier and use it in estimation, along with indicators of real economic activity, to measure the cyclical behavior of the financial frictions in the model. In addition, I use credit default swap spreads on Italian government bonds and detailed data on holdings of domestic government debt by Italian banks to measure the time-varying nature of sovereign risk and the exposure of banks to this risk. The estimated model has good in-sample fit, and its asset pricing implications are broadly consistent with indicators of liquidity and risk premia obtained from the cross section of Italian stock returns.

I then use the estimated model for two applications. First, I quantify the effects of the sovereign debt crisis on the financing premia of firms and on output, and I assess the relative importance of the two propagation mechanisms. I estimate that the sovereign debt crisis in Italy was re-

sponsible for a rise in the financing premia of firms that reached 60 basis points in 2011:Q4, with the risk channel explaining up to 45 percent of the overall effects. This increase in the financing costs of firms was associated with a decline in real economic activity: at peak (2011:Q4), the sovereign crisis was responsible for a decline in output of 1.4 percent in annualized terms.

Second, I evaluate the effects of a major credit market intervention adopted by the ECB in the first quarter of 2012, the longer-term refinancing operations (LTROs). I model the policy as a subsidized long-term loan offered to banks and study its effects conditioning on the state of the Italian economy in 2011:Q4. I find that the average effects of LTROs on credit to firms and output are small. The reason is that precautionary motives were sizable when the policy was enacted. Banks thus have little incentive to increase their exposure to firms, and they mainly use LTROs to cheaply substitute liabilities they have with the private sector.

This research is related to several strands of the literature. Many empirical studies have documented a strong link between sovereign and private-sector interest rates, both in emerging economies and more recently in southern European countries. Several authors have recognized the importance of this relationship in different settings: see Neumeyer and Perri (2005) and Uribe and Yue (2006) in the context of business cycles in emerging markets and Corsetti et al. (2013) for the implications of the sovereign risk pass-through for fiscal multipliers. However, in these and related papers, the reasons underlying the connection between sovereign spreads and private-sector interest rates are left unmodeled. Part of the contribution of this paper is to microfound this relation in a fully specified dynamic equilibrium model.

In doing so, my paper relates to recent studies that analyze the links between sovereign defaults and the domestic banking sector. Motivated by robust empirical evidence, Sosa Padilla (2013), Gennaioli, Martin, and Rossi (2014b), and Perez (2015) study the effects of sovereign defaults on domestic banks and the impact that the associated output losses have on the government's incentives to default.[2] My research is complementary to theirs: I take sovereign default risk as given, but I explicitly model the behavior of private credit markets when sovereign risk increases. In contrast to the above papers, the mere anticipation of a future sovereign default is recessionary, a feature that is essential for the present analysis because an actual default has not been observed over the sample. Anticipation effects have been studied in related environments by Cooper and

[2] Kumhof and Tanner (2005) and Gennaioli, Martin, and Rossi (2014a) document that banks are highly exposed to domestic government debt in a large set of countries. Borensztein and Panizza (2009) show that sovereign defaults typically occur in proximity of banking crises.

Nikolov (2013), Acharya, Drechsler, and Schnabl (2014), Broner et al. (2014), and Fahri and Tirole (2014). These papers analyze in various forms the feedback loops arising between sovereigns and banks, a feature that is absent in my analysis. However, they abstract from the effects that sovereign risk has on the perceived riskiness of firms, which is the focus of my research.

This paper is also related to recent work that studies the macroeconomic implications of shocks to the balance sheet of financial intermediaries. The modeling builds on the framework developed in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), where the limited enforcement of debt contracts leads to constraints on intermediaries' leverage. In contrast to these papers, the analysis here emphasizes the precautionary motives associated with the possibility that these constraints will bind in the future, the risk channel. Technically, I am able to capture these effects because I solve the model numerically using global methods.[3] These nonlinearities have been studied in different environments by Bianchi and Mendoza (2012), He and Krishnamurthy (2012), Bianchi (2013), Maggiori (2013), and Brunnermeier and Sannikov (2014), among others. A distinctive feature of my work is the emphasis on the quantification of these effects. By focusing on a specific historical event, the sovereign debt crisis in the euro area, I am able to measure more directly the balance sheet risk faced by intermediaries and their associated precautionary motives. The analysis documents that the risk channel was quantitatively sizable during this episode. Moreover, it shows that the measurement of these effects provides policy makers with useful information to understand the impact of credit market interventions.[4]

Methodologically, I draw from the literature on the Bayesian estimation of dynamic equilibrium economies (Del Negro and Schorfheide 2011), more specifically of models in which nonlinearities feature prominently (Fernández-Villaverde and Rubio-Ramírez 2007). To my knowledge, this is the first paper that estimates a model with occasionally binding financial constraints using global methods and nonlinear filters. However, other papers use related techniques in different applications (see Bi and Traum 2012; Gust, Lopez-Salido, and Smith 2013; Guerrieri and Iacoviello 2015).

Layout.—The paper is organized as follows. Section II presents the model, and Section III describes the economic mechanisms at work. Section IV estimates the structural model and analyzes its fit. Section V studies the propagation of a sovereign risk shock and presents the results of

[3] In contemporaneous work, Prestipino (2013) studies optimal policy in a Gertler and Karadi (2011) economy solved numerically using global methods.

[4] There are a number of papers that study the effects of credit market interventions in related environments. See Bianchi and Bigio (2014) and references therein.

the first quantitative experiment. Section VI discusses the macroeconomic implications of LTROs. Section VII presents conclusions.

II. The Model

I consider a standard growth model enriched with a financial sector as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). I introduce in this setting long-term government bonds to which financial intermediaries are exposed. These securities pay in every state of nature unless the government defaults. The probability of a government default varies over time according to a reduced-form stochastic process.

The economy is populated by households, final good producers, capital good producers, and a government. Each household is composed of two types of members: *workers* and *bankers*. Workers supply labor to final good firms. Bankers borrow from capital markets in order to invest in government bonds and in claims of the firms. Firms rent labor from workers and buy capital from capital good producers in order to produce a homogeneous good. Their capital expenses are financed by the bankers. The government issues bonds and taxes households in order to finance government spending and can default on its debt. The actions of the government are determined via fiscal rules.

In the remainder of this section, I describe the agents' decision problems, define the equilibrium, and sketch the algorithm used for the numerical solution of the model. I denote by $\mathbf{S}$ the vector that collects the current value for the state variables, to be defined explicitly later on. The vector $\mathbf{S}'$ denotes the value of the state vector next period.

A. *Agents and Their Decision Problems*

1. Households

A household is composed of a fraction f of workers and a fraction $1 - f$ of bankers, with perfect consumption insurance among them. I denote by $\Pi(\mathbf{S})$ the net profits that the household receives from holdings of economic activities, $\tau(\mathbf{S})$ the lump-sum taxes from the government, and $W(\mathbf{S})$ the wage that workers receive from supplying labor to final good firms. The household values consumption c and dislikes labor l according to the flow utility $u(c, l)$ and discounts the future at the rate β. He makes contingent plans for consumption, labor supply, and savings in order to maximize lifetime utility. Savings are managed by financial intermediaries that are run by bankers that belong to other households, and they earn a risk-free return $R(\mathbf{S})$. Taking prices as given, a household solves the problem

$$v^h(b;\mathbf{S}) = \max_{b',c\geq 0,l\in[0,1]} \{u(c,l) + \beta \mathbb{E}_{\mathbf{S}}[v^h(b';\mathbf{S}')]\}$$

subject to

$$c + \frac{1}{R(\mathbf{S})}b' \leq W(\mathbf{S})l + \Pi(\mathbf{S}) + b - \tau(\mathbf{S}),$$
$$\mathbf{S}' = \Gamma(\mathbf{S}),$$

where $\mathbf{\Gamma}(\cdot)$ describes the law of motion for the aggregate state variables. For future reference, I denote by $\Lambda(\mathbf{S}',\mathbf{S}) = \beta[u_c(c',l')/u_c(c,l)]$ the marginal rate of substitution for the household. In the empirical analysis, the flow utility will be $u(c,l) = \log(c) - \chi l^{1+\nu}/(1+\nu)$.

2. Bankers

A banker uses his accumulated net worth, n, and households' savings, b', to buy government bonds and claims on the firms. I denote by a_B and Q_B the quantity and price of government bonds acquired by the banker and by $R_B(\mathbf{S}',\mathbf{S})$ the realized returns on these assets next period. The banker buys equity of the firms a_K at price Q_K, and he obtains the return $R_K(\mathbf{S}',\mathbf{S})$ next period.

The timing of events within a period is as follows. After observing the realization of $\mathbf{S}$, the banker collects the returns from government bonds, firms' equity, and he pays back his creditors. Next, a banker learns whether he exits or not. If he exits, he gives the accumulated net worth as a dividend to his household. If he does not exit, he issues new debt and buys new assets. Exit is stochastic, occurring with fixed probability $1-\psi$.[5]

Taking prices as given, a banker chooses $\{a_B, a_K, b'\}$ to maximize the present discounted value of dividends paid to his household. I follow Gertler and Karadi (2011) and introduce an agency problem between bankers and their creditors. After making the portfolio choice, the banker can divert a fraction λ of the total assets and transfer these resources to his household. The costs of this action are that the creditors can force the banker into bankruptcy and recover the remaining fraction $(1-\lambda)$ of the assets. The decision problem is then

$$v^b(n;\mathbf{S}) = \max_{a_B,a_K,b'} \mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}',\mathbf{S})[(1-\psi)n' + \psi v^b(n';\mathbf{S}')]\}$$

[5] A banker who exits is replaced by a worker from his household. This new banker receives from the household an endowment of wealth to start the business. This endowment is equal to a fraction ω of the assets that bankers have intermediated in the previous period evaluated at current prices. The banker who exits becomes a worker, so that the relative proportion of types within the household is constant over time.

subject to

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \leq n + \frac{b'}{R(\mathbf{S})},$$

$$\lambda\left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j\right] \leq v^b(n;\mathbf{S}),$$

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S}',\mathbf{S})Q_j(\mathbf{S})a_j - b',$$

$$\mathbf{S}' = \Gamma(\mathbf{S}).$$

The first line in the feasible set states that the total assets acquired by a banker cannot exceed its liabilities. The second line is the incentive constraint: the value for the banker of defaulting on his creditors cannot exceed the value of running the business. Finally, net worth next period equals the difference between the returns on assets acquired today and the payments promised to households. Proposition 1 further characterizes this decision problem.

PROPOSITION 1. A solution to the banker's dynamic program is

$$v^b(n;\mathbf{S}) = \alpha(\mathbf{S})n.$$

The marginal value of wealth, $\alpha(\mathbf{S})$, solves

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}',\mathbf{S})[(1-\psi)+\psi\alpha(\mathbf{S}')]R(\mathbf{S})\}}{1-\mu(\mathbf{S})}, \tag{1}$$

and the Lagrange multiplier on the incentive constraint satisfies

$$\mu(\mathbf{S}) = \max\left\{1 - \left(\frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}',\mathbf{S})[(1-\psi)+\psi\alpha(\mathbf{S}')]R(\mathbf{S})\}N}{\lambda[Q_K(\mathbf{S})A_K + Q_B(\mathbf{S})A_B]}\right), 0\right\}, \tag{2}$$

where A_B and A_K are, respectively, aggregate bankers' holdings of government bonds and firms' assets, and N is the aggregate bankers' net worth at the beginning of the period (after the realization of $\mathbf{S}$).

Proof. See Appendix A.

This proposition helps us to understand the role of bank net worth in the model. Because of the linearity of the value function, we can write the incentive constraint as

$$\frac{\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda}, \tag{3}$$

implying that the leverage of a banker cannot exceed the threshold $\alpha(\mathbf{S})/\lambda$. Bank net worth is thus a key variable regulating financial intermediation: when net worth is low, the constraint in equation (3) is more

likely to bind. When this happens, the banker obtains fewer resources from households, and, as a result, he reduces his demand for government and firms' claims.

Proposition 1 also implies that heterogeneity in bankers' net worth and in their asset holdings does not affect aggregate dynamics. Hence, aggregate net worth will be a sufficient statistic for the state of the financial sector, a feature that makes the numerical analysis of the model tractable.

3. Producers

There are two types of firms: capital good producers and final good producers. The capital good producers build new capital goods using the technology $\Phi(i/K)K$, where K is the aggregate capital stock and i the input used in production. Taking prices as given, their decision problem is

$$\max_{i \geq 0} \left[Q_i(\mathbf{S}) \Phi\left(\frac{i}{K}\right) K - i \right].$$

In anticipation of the market-clearing condition, the price of new capital goods is $Q_i(\mathbf{S}) = [\Phi'(I(\mathbf{S})/K)]^{-1}$, where $I(\mathbf{S})$ is equilibrium aggregate investment. In the empirical analysis, I will set $\Phi(x) = a_1 x^{1-\xi} + a_2$, where $\xi \in [0, 1]$ parameterizes the elasticity of Tobin's q with respect to the investment-capital ratio. If $\xi > 0$, the price of capital moves over time, and it will be low in periods of low aggregate investment.

Final good firms produce a final output using a constant returns to scale technology, $y = k^{\alpha}(e^{z}l)^{1-\alpha}$, where k is the stock of capital goods, l stands for labor services, and z is a neutral technology shock that follows an AR(1) process in growth rates:

$$\Delta z' = (1 - \rho_z)\gamma + \rho_z \Delta z + \sigma_z \varepsilon_z', \quad \varepsilon_z' \sim \mathcal{N}(0, 1). \tag{4}$$

Labor is rented in competitive markets at $W(\mathbf{S})$. Profit maximization implies that equilibrium wages and profits per unit of capital are

$$W(\mathbf{S}) = (1 - \alpha)\frac{Y(\mathbf{S})}{L(\mathbf{S})}, \quad Z(\mathbf{S}) = \alpha \frac{Y(\mathbf{S})}{K}, \tag{5}$$

where $Y(\mathbf{S})$ and $L(\mathbf{S})$ are equilibrium aggregate output and labor.

Capital goods depreciate every period at the rate δ. Firms need external financing to purchase new capital goods. At the beginning of the period, they issue claims a_K to bankers at a price of $Q_K(\mathbf{S})$.[6] In exchange, the firms pledge the realized return on a unit of the capital stock next period to the banker:

[6] No arbitrage implies that the price of a unit of new capital equals in equilibrium the price of a claim issued by firms, $Q_i(\mathbf{S}) = Q_K(\mathbf{S})$.

$$R_K(\mathbf{S}', \mathbf{S}) = \frac{(1-\delta)Q_K(\mathbf{S}') + Z(\mathbf{S}')}{Q_K(\mathbf{S})}. \tag{6}$$

The realized returns $R_K(\mathbf{S}', \mathbf{S})$ move over time because of two factors: variation in firms' profits and variation in the market value of firms' claims.

4. The Government

In every period, the government engages in public spending. Public spending as a fraction of output follows

$$\log(g)' = (1-\rho_g)\log(g^*) + \rho_g \log(g) + \sigma_g \varepsilon_g', \quad \varepsilon_g' \sim \mathcal{N}(0,1). \tag{7}$$

The government finances its expenditures by issuing long-term bonds to bankers and by levying lump-sum taxes on households. Long-term debt is introduced as in Chatterjee and Eyigungor (2012). In every period, a fraction π of bonds matures, and the government pays back the principal to investors. For the fraction $(1-\pi)$ that does not mature, the government pays the coupon ι, and investors retain the right to the principal in the future. The government can default on its debt in every period by writing off a fraction $D \in [0, 1]$ of its outstanding obligations. Denoting by $Q_B(\mathbf{S})$ the pricing function for government securities, tomorrow's realized returns on a dollar invested in government bonds are

$$R_B(\mathbf{S}', \mathbf{S}) = (1 - d'D)\left\{\frac{\pi + (1-\pi)[\iota + Q_B(\mathbf{S}')]}{Q_B(\mathbf{S})}\right\}, \tag{8}$$

where d' is an indicator variable equal to one if the government defaults next period. The realized returns $R_B(\mathbf{S}', \mathbf{S})$ vary over time because of two sources. First, when the government defaults, it imposes the "haircut" D on bondholders. Second, $R_B(\mathbf{S}', \mathbf{S})$ is sensitive to variation in the price of government securities: for instance, a decline in $Q_B(\mathbf{S}')$ lowers the resale value of government bonds and generates capital losses for bondholders. This second effect is present even when the government does not default ($d' = 0$) and to the extent that government debt is not entirely short-term ($\pi < 1$).

Denoting by B' the stock of public debt, the budget constraint of the government is

$$Q_B(\mathbf{S})\Big[\underbrace{B' - (1-\pi)B(1-dD)}_{\text{Newly issued bonds}}\Big] = \underbrace{[\pi + (1-\pi)\iota]B(1-dD)}_{\text{Payments of principals and coupons}} + \underbrace{gY(\mathbf{S}) - \tau(\mathbf{S})}_{\text{Primary deficit}}, \tag{9}$$

with taxes following the fiscal rule

$$\tau(\mathbf{S}) = t^* \exp\{z_t\} + \gamma_\tau B.$$

In order to close the model, we need to specify how sovereign risk evolves over time. I assume that in every period the economy is hit by a shock ε_d that follows a standard logistic distribution. The default process d' follows

$$d' = \begin{cases} 1 & \text{if } \varepsilon_d' - \Psi(\mathbf{S};\theta_2) \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

where θ_2 is a vector of parameters. We can thus write the conditional probability of a sovereign default tomorrow as a logistic function of $\Psi(\mathbf{S};\theta_2)$,

$$p^d(\mathbf{S}) \equiv \text{Prob}(d' = 1|\mathbf{S}) = \frac{\exp\{\Psi(\mathbf{S};\theta_2)\}}{1 + \exp\{\Psi(\mathbf{S};\theta_2)\}}. \tag{11}$$

By appropriately choosing the function $\Psi(\mathbf{S};\theta_2)$, one can flexibly incorporate key drivers of sovereign credit risk into the analysis. For example, $\Psi(\mathbf{S};\theta_2)$ could depend on B or on Δz, typical determinants of sovereign risk identified in the literature (Arellano 2008).

In the empirical application I will consider a simple specification $\Psi(\mathbf{S};\theta_2) = s$, with s being an AR(1) process

$$s' = (1 - \rho_s)\log(s^*) + \rho_s s + \sigma_s \varepsilon_s', \quad \varepsilon_s' \sim \mathcal{N}(0,1). \tag{12}$$

This choice is motivated by two main considerations. First, there is substantial empirical evidence that a large share of the variation in Italian sovereign spreads during the European crisis was driven by factors orthogonal to domestic fundamentals (Bahaj 2013). These findings are consistent with the view, shared by many economists and policy makers, that self-fulfilling beliefs and contagion through common creditors were key drivers of sovereign risk during the European crisis. The s-shock is intended to capture these considerations. Second, this formulation allows us to clearly isolate the economic mechanisms underlying the propagation of sovereign risk without relying on reduced-form channels built in the function $\Psi(\cdot)$.

Importantly, as I will discuss in Section IV, the exogeneity of sovereign credit risk will not be used as a restriction when estimating the structural model. Thus, the identification of key model parameters will not rely on this assumption.

B. Equilibrium

Let $\mathbf{S} = [K, B, P, \Delta z, g, s, d]$ be the state vector, with P being the gross payments that bankers return to households at the beginning of the period.[7] A *recursive competitive equilibrium* for this economy is given by value functions for households and bankers $\{v^h, v^b\}$, policy functions for households $\{c, l, b'\}$, and policy functions for bankers $\{a_K, a_B, b'\}$ such that, given prices $\{W, Q_K, Q_B, R\}$, (i) bankers' and households' policies and value functions solve their decision problems; (ii) the government budget constraint is satisfied; (iii) the market for firms' claims clears, $\int a_K(n_i; \mathbf{S})di = K'(\mathbf{S})$; (iv) the market for government bonds clears, $\int a_B(n_i; \mathbf{S})di = B'(\mathbf{S})$; (v) the market for households' savings clears, $\int b'(n_i; \mathbf{S})di = b'(\mathbf{S})$; (vi) the goods market clears, $Y(\mathbf{S})(1 - g) = C(\mathbf{S}) + I(\mathbf{S})$; and (vii) $\mathbf{\Gamma}(\cdot)$ is consistent with agents' optimization and the government fiscal rules.

C. Numerical Solution

The solution to the competitive equilibrium is obtained using global numerical methods. Here I briefly sketch the main steps of the algorithm, relegating to the online appendix the details of the implementation and a discussion of its accuracy. Because the nonstationary technology process induces a stochastic trend in several endogenous variables, it is convenient to express the model in terms of detrended variables. For a given variable y, I define its detrended counterpart as $\tilde{y} = y/z$.[8] Let $X(\mathbf{S}) = \{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ be the control variables of the model, with x being an element of X. I approximate each decision rule $x(\mathbf{S})$ using piece wise smooth functions that are summarized by two sets of coefficients, $\{\gamma^x_{d=0}, \gamma^x_{d=1}\}$,

$$x(d, \tilde{\mathbf{S}}) = (1 - d)\gamma^x_{d=0}\mathbf{T}(\tilde{\mathbf{S}}) + d\gamma^x_{d=1}\mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s]$ is the vector of state variables that excludes d, and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. Thus, the numerical solution is indexed by the coefficients $\{\gamma^x_{d=0}, \gamma^x_{d=1}\}_x$. These coefficients are chosen so that the equilibrium conditions of the model, defined in the online appendix, are satisfied at a set of collocation points $(d^i, \tilde{\mathbf{S}}^i) \in \{0, 1\} \times \tilde{\mathcal{S}}$. The collocation points $\tilde{\mathcal{S}}$ and the set of polynomials $\mathbf{T}(\cdot)$ are chosen using the approach of Smolyak (Krueger and Kubler 2004). The use of a sparse grid allows one to soften the curse of dimensionality that arises when solving the model globally.

[7] As explained in the online appendix, along with the other state and control variables, P will be a sufficient statistic for beginning-of-period aggregate net worth N.

[8] The endogenous state variables of the model are detrended using the level of technology lagged one period.

III. The Bankers' Euler Equations

Before turning to the empirical analysis, a useful step is to understand how sovereign risk affects the funding costs of firms and real economic activity in the model. It is convenient to organize the discussion around the Euler equation that governs capital accumulation in the economy.

Given the linearity of the value function of bankers, we can write their optimization problem as

$$\max_{\{a_B,a_K\}} \mathbb{E}_{\mathbf{S}}\left\{\hat{\Lambda}(\mathbf{S}',\mathbf{S})\left\{\sum_{j=\{B,K\}}[R_j(\mathbf{S}',\mathbf{S})-R(\mathbf{S})]Q_j(\mathbf{S})a_j+R(\mathbf{S})n\right\}\right\}$$

subject to the constraint that their demand for assets does not exceed a multiple of their net worth $\lambda[Q_B(\mathbf{S})a_B+Q_K(\mathbf{S})a_K]\leq\alpha(\mathbf{S})n$. In the above notation, $\hat{\Lambda}(\mathbf{S}',\mathbf{S})$ is the effective discount factor of the bankers, and it is related to the marginal rate of substitution of the household as follows:

$$\hat{\Lambda}(\mathbf{S}',\mathbf{S})=\Lambda(\mathbf{S}',\mathbf{S})[(1-\psi)+\psi\alpha(\mathbf{S}')]. \tag{13}$$

Taking first-order conditions for the above program, we can write the Euler equation governing the demand for firms' claims as follows:

$$\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R_K(\mathbf{S}',\mathbf{S})]=\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R(\mathbf{S})]+\lambda\mu(\mathbf{S}). \tag{14}$$

At optimum, the demand for firms' assets is such that their risk-adjusted return equals the shadow value of funds for the bankers. As a benchmark, when $\mu(\mathbf{S})=0$ for all $\mathbf{S}$, equation (14) collapses to the Euler equation of the corresponding frictionless model. Indeed, the bankers will always have enough funds to arbitrage away differences between $\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R_K(\mathbf{S}',\mathbf{S})]$ and $\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R(\mathbf{S})]$ when their funding constraints are not binding. Moreover, they will discount payoffs using the marginal rate of substitution of the household because $\alpha(\mathbf{S})=1$ for all $\mathbf{S}$ in this scenario.

In general, though, variation in the net worth of bankers implies that their funding constraints will bind in some states of the world, and this will affect the behavior of excess returns. In order to understand that, we can rearrange equation (14) as follows:

$$\begin{aligned}\mathbb{E}_{\mathbf{S}}[R_K(\mathbf{S}',\mathbf{S})-R(\mathbf{S})]=&\underbrace{\frac{\lambda\mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})]}}_{\text{Liquidity Premium}}\\&\underbrace{+\operatorname{cov}_{\mathbf{S}}\left[-\frac{\hat{\Lambda}(\mathbf{S}',\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})]},R_K(\mathbf{S}',\mathbf{S})\right]}_{\text{Risk Premium}}.\end{aligned} \tag{15}$$

Expected excess returns arise for two distinct reasons. First, high excess returns on private-sector claims may indicate the inability of bankers to exploit profitable arbitrage opportunities because of binding funding constraints. I refer to this component, represented by the first term on the right-hand side of equation (15), as a *liquidity premium*. Second, high excess returns may simply reflect compensation that bankers demand for holding risk, that is, a premium for assets whose payouts covary negatively with the stochastic discount factor. This is a canonical *risk premium*. Importantly, the stochastic discount factor used by the bankers differs from that of the corresponding frictionless model: it is not only a function of the marginal rate of substitution of households but also a function of the marginal value of wealth for the banker. This aspect allows the model to generate quantitatively meaningful variation in risk premia.[9]

When the funding constraints bind, bankers shed some of their assets in order to meet their leverage requirements: this implies that the demand for assets falls, leading to an increase in the liquidity premium. Even when the constraints are currently not binding, the expectations that they may bind in the future can affect excess returns by changing the risk premium component (Aiyagari and Gertler 1999). This happens mainly because $\alpha(\mathbf{S}')$ and $R_K(\mathbf{S}', \mathbf{S})$ tend to move in opposite directions when the funding constraints bind. In order to understand this property, consider a state $\overline{\mathbf{S}}'$ in which the leverage constraint binds. By equation (1), $\alpha(\overline{\mathbf{S}}')$ is high in this state: intuitively, a dollar is more valuable to a banker when he is facing a tight funding constraint. In addition, $R_K(\overline{\mathbf{S}}', \mathbf{S})$ is likely to be low because the weak demand for firms' assets by constrained bankers puts downward pressure on the price of capital, leading to low ex post returns (see eq. [6]). Taken together, these two facts imply that $\overline{\mathbf{S}}'$ is a state in which claims on the productive sector pay out little precisely when bankers are in most need of wealth. Hence, any shock that raises the likelihood of $\overline{\mathbf{S}}'$ tends to increase the risk premium demanded by the bankers.

In this environment, news about a future sovereign default influences private-sector spreads by affecting both the liquidity and the risk premium of equation (15). Starting with the former, an increase in $p(d' = 1|\mathbf{S})$ triggers a fall in the price of government bonds and a drop in the net worth of the banks.[10] When these losses are sufficiently large, the leverage con-

[9] Importantly, $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ is an example of a leverage-based pricing kernel. Indeed, we can see from eq. (3) that the leverage of bankers is proportional to $\alpha(\mathbf{S})$ when $\mu(\mathbf{S}) > 0$. Adrian, Etula, and Muir (2014) provide empirical evidence in support of leverage-based pricing kernels for the US economy, and He and Krishnamurthy (2013) discuss their asset-pricing implications in endowment economies.

[10] This effect is present only when the government debt held by bankers is long term ($\pi < 1$). When $\pi = 1$, the bankers do not make capital losses on bondholdings unless the government defaults (see eq. [8]).

straints may bind, leading to higher liquidity premia. I refer to this mechanism as the *liquidity channel*. In addition, an increase in $p(d' = 1|\mathbf{S})$ will tend to raise risk premia. When the likelihood of a future government default increases, bankers assign a higher probability of experiencing losses in their holdings of government debt next period. This is a signal that their funding constraints may bind in the future. Therefore, they act more cautiously and demand higher risk premia for holding firms' claims in their balance sheet. This is referred to as the *risk channel*.

This variation in liquidity and risk premia induced by sovereign risk is associated with adjustments in macroeconomic quantities that are well understood in the literature. Specifically, both the liquidity and the risk channel will discourage capital accumulation in the economy, and their aggregate implications will be qualitatively similar to a shock to the marginal efficiency of investment studied in the business cycle literature. Section V discusses these issues in more detail.

It is important to stress that, despite being highly intertwined in the model, the two mechanisms are conceptually different. The liquidity channel indicates that financial intermediaries are not lending to firms because they are currently facing funding constraints. The risk channel, instead, indicates that financial intermediaries are not demanding firms' claims because they are afraid of hitting their funding constraints next period, and they know that these will be bad assets to hold in case this event occurs. Section VI shows that this distinction has important implications for policy evaluation.

IV. Empirical Analysis

The model is estimated using Italian quarterly data. This section proceeds in three steps. Section IV.A describes the data used in estimation. Section IV.B illustrates the estimation strategy. Section IV.C presents diagnostics regarding model fit.

A. Data

Although the previous section has described the economic mechanisms of interest, their quantitative relevance rests on the numerical value of the model parameters, especially those governing the stochastic properties of sovereign risk, the exposure of bankers to this risk, and the macroeconomic implications of the financial frictions studied in this paper. This section describes the key time series used in estimation to inform these parameters.

I use credit default swap (CDS) spreads on Italian government securities with a 1-year maturity to ensure that the time-varying nature of sovereign risk in the model is realistic. The exposure of Italian banks to this

risk is measured using the 2011 stress test of the European Banking Authority (EBA). Through this source, I construct net holdings of domestic government debt, classified by their maturity, for the five largest Italian banks.[11] This information is matched with consolidated balance sheet data at the end of 2010, obtained from Bankscope. Thus, for each of these banks, we have a measure of their holdings of Italian government debt as a fraction of the bank's total assets.

A major challenge in the literature is to select observables that are informative for the parameters governing the financial frictions in this class of models. The approach here consists in deriving a model-consistent indicator of agency costs. Specifically, Appendix B, Section C, shows that the Lagrange multiplier on the incentive constraints of bankers is, in equilibrium, a function of their leverage and of the spread between the interbank rate and the risk-free rate,[12]

$$\mu_t = \frac{\left(\frac{R_{\text{interbank},t} - R_t}{R_t}\right)\text{lev}_t}{1 + \left(\frac{R_{\text{interbank},t} - R_t}{R_t}\right)\text{lev}_t}. \tag{16}$$

Therefore, equation (16) can be used to generate a time series for μ_t. I measure $R_{\text{interbank},t}$ with the prime rate on interbank unsecured loans with a weeklong duration for Italian banks belonging to the Euro Interbank Offered Rate (Euribor) panel. The risk-free rate R_t is matched with yields on contracts of similar duration from the interbank secured market, the Euro G.C. Repo Market (Eurepo). The leverage of financial intermediaries is calculated using the Italian flow of funds.[13] Figure 1 reports the time series for μ_t along with real GDP growth in Italy. Two main facts stand out. First, the Lagrange multiplier is countercyclical, rising substantially in periods in which GDP growth is markedly below average. Second, it is close to zero for most time periods. Thus, the constraint seems to bind only occasionally in the sample.

In addition to the liquidity needs of banks, spikes in the Euribor-Eurepo spread might also reflect heightened credit risk in interbank

[11] The five institutions are Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena, Banco Popolare, and Unione di Banche Italiane.

[12] An interbank market can be introduced in the model by allowing the bankers to trade a risk-free asset in zero net supply. The presence of this asset does not alter allocations. See App. B, Sec. C, for further details.

[13] When constructing the flow of funds, the Bank of Italy follows the conventions of the European System of Accounts under which financial assets and liabilities are reported at market value. This principle, however, is not applied to activities for which there is no secondary market, with the exception of stocks and bonds not listed on official exchanges whose market value is imputed. Therefore, the empirical indicator of financial leverage is not fully consistent with the one in the model, where all assets and liabilities are mark to market.

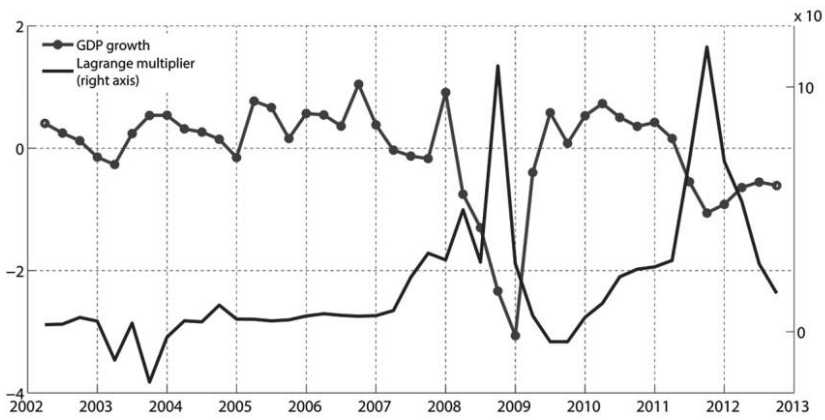

FIG. 1.—GDP growth and the Lagrange multiplier: 2002:Q2–2012:Q4. Color version available as an online enhancement.

transactions. This second component could, in principle, contaminate the proposed measurement and lead to an overstatement of the importance of funding constraints of banks over the sample. However, there are several reasons why the credit risk component may indeed be quantitatively small. First, the interbank rate used here relates to transactions among prime and large credit institutions. Second, the choice of using loans with a short duration (1 week) is intended to minimize the impact of credit risk on the measured Lagrange multiplier. Consistent with these arguments, we can verify that the measured Lagrange multiplier is significantly associated to indicators of bank funding constraints measured through surveys. The Bank of Italy administers a quarterly survey to senior loan officers at Italian banks. The survey addresses issues such as credit standards for approving loans as well as credit terms and conditions applied to enterprises. The first question in the survey asks whether a bank's credit standards to enterprises have changed in a given quarter. Question 2 asks loan officers to rate several factors that contribute to a change in the credit standards. Among those factors, the survey considers three entries related to the "costs of funds and balance sheet constraints" for the bank: (i) cost related to a bank's capital position, (ii) the ability of a bank to access market financing (e.g., money or bond market financing), and (iii) a bank's liquidity position. Section D in Appendix B documents that the Lagrange multiplier tracks closely the net percentage of respondents reporting a tightening of credit (R^2 of 41 percent in a univariate regression). Moreover, it is strongly associated to the net percentage of respondents who answered that one of the three factors described above had somewhat or considerably contributed to the change in credit standards (R^2 of, respectively, 19 percent, 59 percent,

and 45 percent). Appendix table B2 further documents that the remaining "factors" that respondents can select from question 2 are almost uncorrelated with the measured Lagrange multiplier.

In addition to the data discussed, the estimation uses additional indicators from OECD Quarterly National Accounts and other sources. See Appendix B for a detailed description of the data.

B. Estimation

I denote by $\theta \in \Theta$ the vector of model parameters. It is convenient to organize the discussion around the following partition, $\theta = [\theta_1, \theta_2]$:

$$\theta_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z, \exp^{bg}, \pi, g^*, \rho_g, \sigma_g, i^{bg}/y^{bg}, l^{bg}, R^{bg}, \alpha, \nu, \gamma_\tau, \text{lev}^{bg}, q_b^{bg}, \text{adj}^{bg}],$$

$$\theta_2 = [D, s^*, \rho_s, \sigma_s].$$

Conceptually, we can think of θ_1 as indexing a restricted version of the model without sovereign risk, whereas θ_2 collects the parameters determining the sovereign default process.[14] Although a nonlinear analysis of the model is necessary to address the questions asked in this paper, it complicates inference substantially because repeated numerical solutions of the model are computationally costly. I therefore estimate θ using a two-step procedure. In the first step, I infer θ_1 by estimating the model without sovereign risk on the pre–sovereign debt crisis subsample (2002:Q2–2009:Q4). This restricted version of the model has fewer state variables and is easier to analyze numerically. In the second step, and conditional on the first-step parameters, I estimate θ_2 using a retrieved time series of sovereign default probabilities.

An attractive feature of this two-step procedure is that it is more robust to misspecifications of the sovereign default process relative to a full information approach. For instance, the two-step approach does not use information from the sample correlation between GDP growth and CDS spreads in the estimation of θ. On the contrary, a full information procedure would use this moment to inform the values for these parameters. This could be problematic, because the model interprets this moment as purely reflecting the causal effect of sovereign risk on real economic activity.

[14] I have reparameterized $[\beta, \lambda, \omega, \delta, \chi, \iota, t^*, a_1, a_2]$ with balanced growth values for, respectively, the risk-free rate (R^{bg}), the Lagrange multiplier (μ^{bg}), the leverage ratio (lev^{bg}), the investment-output ratio (i^{bg}/y^{bg}), worked hours (l^{bg}), the price of government securities (q_b^{bg}), the ratio of government securities held by bankers to their total assets ($\exp^{bg}$), and the size of capital adjustment costs (adj^{bg}).

1. Estimating the Model without Sovereign Risk

The model without sovereign risk has five state variables, $\mathbf{S}_t = [\hat{K}_t, \hat{B}_t, \hat{P}_t, \Delta z_t, g_t]$. The parameters are

$$\theta_1 = \Bigg[\underbrace{\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z}_{\tilde{\theta}_1}, \underbrace{\exp^{bg}, \pi, g^*, \rho_g, \sigma_g, i^{bg}/y^{bg}, l^{bg}, R^{bg}, \alpha, \nu, \gamma_\tau, \text{lev}^{bg}, q_b^{bg}, \text{adj}^{bg}}_{\theta_1^*}\Bigg].$$

I construct the likelihood function of the model using time series for GDP growth and the Lagrange multiplier described earlier. The cyclical behavior of the model's financial frictions is important for assessing the impact of sovereign risk on the real economy: a likelihood-based approach guarantees a high degree of consistency between the model-implied behavior for these variables and their data counterparts. However, certain parameters are likely to be only weakly affected by the information in the likelihood function, and their identification is problematic. For this reason, I determine a subset of θ_1, θ_1^*, prior to the estimation. Table 1 reports the numerical values for these parameters.

I use the EBA and Bankscope data to make sure that holdings of government securities are equivalent to 7.6 percent of banks' total assets in a balanced growth path of the model and that the average maturity of those bonds equals 18 months (see App. table B1). These numbers pin

TABLE 1
PARAMETERS DETERMINED WITH EXTERNAL INFORMATION

Parameter	Description	Value
$\exp^{bg}$	Bondholdings over total assets	.076
π	Fraction of bond maturing	.056
$\exp\{g^*\}$	Average spending-output ratio	.198
ρ_g	Persistence, spending-output ratio	.894
σ_g	Volatility, spending-output ratio	.012
i^{bg}/y^{bg}	Investment-output ratio	.213
l^{bg}	Hours worked	.318
R^{bg}	Interest rate	1.003
α	Capital share	.300
ν	Inverse Frisch elasticity	.500
lev^{bg}	Leverage	5.000
γ_t	Fiscal rule	1.000
adj^{bg}	Capital adjustment costs	.000
q_b^{bg}	Bond prices	1.000

NOTE.—See App. B for information on data sources.

down $[\exp^{bg}, \pi]$. I select $[g^*, \rho_g, \sigma_g]$ from the estimation of an AR(1) on the spending-output ratio over the 1999:Q1–2009:Q4 period. The parameters $[i^{bg}/y^{bg}, l^{bg}, R^{bg}]$ are set to the sample average of their empirical counterparts, and α is determined using the sample average of the labor income share. The remaining parameters in θ_1^* are determined through normalizations or previous research. The parameter ν is chosen to obtain a Frisch elasticity of 2. The parameter ω is chosen so that the model produces a leverage ratio of 5 in a balanced growth path, in the range of values used in the literature. I set γ_τ to one: this choice has limited effects on aggregate dynamics, because taxes are lump-sum in the model. Following common practice in the profession, I set capital adjustment costs to zero in a balanced growth path and normalize q_b^{bg} to one.

The remaining parameters $\tilde{\theta}_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z]$ are estimated using Bayesian methods. Letting $\mathbf{Y}_t = [\text{GDP growth}_t, \mu_t]'$ and denoting the entire sample by $\mathbf{Y}^T$, I first evaluate the likelihood function of the model $\mathcal{L}(\tilde{\theta}_1|\mathbf{Y}^T)$ using the auxiliary particle filter of Pitt and Shephard (1999). I next characterize the posterior density of $\tilde{\theta}_1$ using the Random Walk Metropolis Hastings developed in Schorfheide (2000). The online appendix provides a description of the likelihood evaluation and the estimation algorithm. The top panel of table 2 reports the prior along with posterior statistics for $\tilde{\theta}_1$.

The prior on the technological process is centered using presample evidence, and I center ξ to 0.5, a conventional value in the literature. I center the prior on μ^{bg} and ψ to their calibrated values in Gertler and Karadi (2011). Regarding posterior estimates, we can observe that agency costs in the model are small: the Lagrange multiplier is estimated to be 0.001 on average, a value that is roughly one-tenth of that in the Gertler

TABLE 2
PRIOR AND POSTERIOR DISTRIBUTION OF ESTIMATED PARAMETERS

Parameter	Prior	Parameter 1	Parameter 2	Posterior Mean	90% Credible Set
$\mu^{bg} \times 100$	Normal	.96	25.00	.10	[.02, .21]
ψ	Normal	.97	.15	.96	[.94, .98]
ξ	Beta	.5	.25	.42	[.31, .52]
$\gamma \times 400$	Normal	1.25	1.00	.43	[.17, .72]
ρ_z	Beta	.35	.25	.51	[.45, .57]
$\sigma_z \times 100$	Inverse gamma	.75	2.00	.59	[.47, .74]
s^*	Normal	−7	5	−7.06	[−9.73, −4.31]
ρ_s	Beta	.5	.3	.95	[.88, .99]
σ_s	Inverse gamma	.75	4	.63	[.52, .76]

NOTE.—Parameter 1 and parameter 2 list the mean and standard deviation for the beta and normal distributions and s and ν for the inverse gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. Posterior statistics are computed using 10,000 draws from the posterior distribution of the model's parameters. The table reports equal tail probability 90 percent credible sets.

and Karadi (2011) calibration. This is not surprising given that the time series for μ_t is close to zero for most of the sample. Capital adjustment costs and the technological process are in the range of what is typically obtained in the literature when using US data.

2. Estimating Sovereign Risk

I now turn to the estimation of $\theta_2 = [D, s^*, \rho_s, \sigma_s]$. The empirical strategy consists of (i) constructing a time series for the probabilities of a sovereign default and (ii) using this time series, along with equations (11) and (12), to estimate θ_2.

The first task is accomplished by exploiting the pricing equation of the model. To do so, it is convenient to introduce the following variable:

$$\hat{p}(\mathbf{S}'|\mathbf{S}) = \frac{R_{\text{interbank}}(\mathbf{S})p(\mathbf{S}'|\mathbf{S})\hat{\Lambda}(\mathbf{S}', \mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})}, \tag{17}$$

with $p(\cdot)$ being the conditional density function of the state vector. The variable $\hat{p}(\cdot)$ is a probability measure, because it is nonnegative and integrates to one.[15] I then price a CDS on a short-term zero coupon bond issued by the government.[16] Using the banker's Euler equation and the definition of $\hat{p}(\cdot)$, we can express the spread on this contract as

$$\text{CDS}_t = \frac{\hat{p}_t^d D}{R_{\text{interbank},t}}, \tag{18}$$

where $\hat{p}_t^d$ stands for the conditional probability of a sovereign default under the $\hat{p}(\cdot)$ measure. Given this relation and the definition of $\hat{p}(\cdot)$ in equation (17), we can write the actual probability of a sovereign default p_t^d as follows:

$$p_t^d = \frac{\text{CDS}_t}{D} \times \frac{\lambda[\text{lev}_t(1 - \mu_t) + \mu_t]}{\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]}. \tag{19}$$

The right-hand side of the above equation is known up to D and $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. I set the haircut D to 0.55, a value that is consistent with the Greek debt restructuring of 2012 (Zettelmeyer, Trebesch, and Gulati 2013). For the second term, I indirectly use the model's restrictions to approximate this object using an empirical counterpart to the model's stochastic discount factor. The online appendix describes this step in detail.

[15] In order to derive this last property, we can write the return on a risk-free security traded by bankers as $R_{\text{interbank}}(\mathbf{S}) = \{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})\}/\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]$ using eq. (14) and eq. (1).

[16] The contract specifies that, in the event of a government default at $t + 1$, the seller of the CDS pays the principal to the buyer in exchange of $(1 - D)$.

After obtaining a time series for p_t^d, we can estimate the logistic model defined by equations (11) and (12). The bottom panel of table 2 reports prior and posterior statistics for $[s^*, \rho_s, \sigma_s]$.

C. Model Fit

I now present an analysis of model fit. First, I verify whether the model performs well in the sample by checking whether it reproduces key moments of the time series used in estimation. The second experiment checks whether the model's predictions for liquidity and risk premia are consistent with comparable measures obtained from the cross section of Italian stock returns.

1. Lagrange Multiplier, Output Growth, and Sovereign Default Probabilities

The analysis of the in-sample fit is conducted through posterior predictive checks.[17] I summarize the behavior of the data by computing a set of sample statistics for the Lagrange multiplier, output growth, and sovereign default probabilities. Then I compute the model-implied posterior distributions for the same statistics via simulation techniques. The check consists of comparing the statistics computed from actual data with their model-implied distribution.

We can start by examining the performance of the model regarding GDP growth and the Lagrange multiplier. Their joint behavior in the data is summarized using mean, standard deviation, first-order autocorrelation, skewness, kurtosis, and their correlation. These sample statistics are collected in $\mathcal{S}$. The model-implied densities for these sample statistics, $p(\mathcal{S}|\mathbf{Y}^T)$, are computed via simulations.[18] Figure 2 reports the 5th and 95th percentiles of the model-implied density (the box) along with its median (the bar) and the sample counterpart (the dot).

The model generates trajectories for the Lagrange multiplier and GDP growth whose moments are in line with those observed in the data. The main discrepancy in the data lies in the excess kurtosis for the GDP growth trajectory: the model has difficulty replicating the depth of the 2009:Q1 recession. The model captures part of the left skewness of GDP growth observed in the data. This nonlinearity is the result of two prop-

[17] See Aruoba, Bocola, and Schorfheide (2013) for a discussion of the use of posterior predictive checks for the evaluation of dynamic stochastic general equilibrium models.

[18] Let $\{\theta_n\}_{n\in N}$ be draws from $p(\theta|\mathbf{Y}^T)$. For each θ_n, I solve the model, simulate trajectories of length T for the endogenous variables, and compute the sample statistics on the simulated path, $\mathcal{S}(\theta_n)$. The density $p(\mathcal{S}|\mathbf{Y}^T)$ is then approximated from $\{\mathcal{S}(\theta_n)\}$. In this experiment, I set $N = 100$ and $T = 100$.

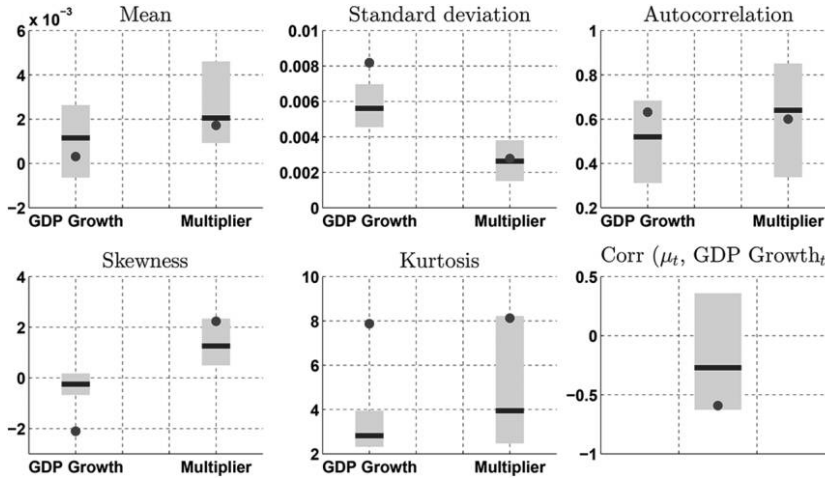

FIG. 2.—Posterior predictive checks: Lagrange multiplier and GDP growth. Dots correspond to the value of the statistic computed from actual data. Solid horizontal lines indicate medians of the sample statistic under the posterior predictive distribution, and the boxes indicate the 90 percent equal tail probability credible set. Color version available as an online enhancement.

erties of the estimated model: the amplification of the leverage constraint and the fact that it binds in recessions.

Table 3 reports the posterior predictive checks for p_t^d. As we can see, the logistic model used in this paper captures key features of the empirical distribution of sovereign default probabilities.

Overall, the results in this section suggest that (i) the cyclical behavior of the leverage constraint in the estimated model is empirically reasonable, and (ii) agents in the model have beliefs about the time-varying na-

TABLE 3
POSTERIOR PREDICTIVE CHECKS: SOVEREIGN DEFAULT PROBABILITIES

Statistic	Data	Posterior Median	90% Credible Set
Median	.02	.08	[.00, 2.42]
Mean	.19	.21	[.01, 7.68]
Standard deviation	.36	.30	[.01, 11.90]
Autocorrelation	.82	.78	[.59, .92]
Skewness	3.02	2.68	[1.44, 5.19]
Kurtosis	12.74	11.05	[4.48, 35.16]

NOTE.—Results are based on 1,000 draws from the posterior distribution of $[s^*, \rho_s, \sigma_s]$. For each draw, I simulate the $\{s_t\}$ process for 100 periods. Statistics are computed on each of these 1,000 samples. The table reports the posterior median and the 90 percent equal tail probability credible set.

ture of sovereign credit risk that closely track what was observed in the data.

2. Evidence from the Cross Section of Stock Returns

I now evaluate the asset-pricing implications of the model. As discussed in Appendix C, the Euler equations of bankers define the following asset-pricing model:

$$\mathbb{E}[R^e_{j,t+1}] = b_0 + b_1\beta_j,$$

$$\beta_j = \frac{\operatorname{cov}\left(-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{j,t+1}\right)}{\operatorname{var}\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)}, \tag{20}$$

where $\hat{\Lambda}_{t,t+1}$ is defined as

$$\hat{\Lambda}_{t,t+1} = \beta \exp(-\Delta c_{t+1})[(1-\psi) + \psi\lambda \text{lev}_{t+1}], \tag{21}$$

and $R_{j,t}$ and $R^e_{j,t}$ are, respectively, realized returns and realized excess returns on asset j at time t. In the above equations, $b_0 = \mathbb{E}[\lambda\mu_t/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]]$ is the mean of the liquidity premium while $b_1 = \operatorname{var}(\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ can be thought of as the price for holding risk. These moments are taken with respect to the ergodic distribution of the model state variables.

The logic of the exercise is as follows. Assuming that equation (20) holds for stocks traded in the Italian exchange, we can estimate b_0 and b_1 using a *two-pass* procedure. We can then ask whether the asset-pricing implications of the model are consistent, qualitatively and quantitatively, with these indicators of liquidity and risk premia. The estimation of equation (20) uses quarterly returns data from Compustat Global along with a time series of the stochastic discount factor in equation (21). Appendix C describes in detail the data and the estimation procedure. Moreover, it confronts this specification with three benchmarks: the capital asset pricing model (CAPM), the three-factor model of Fama and French (1993), and a consumption-based version of $\hat{\Lambda}_{t,t+1}$ obtained by setting ψ to zero in equation (21). In this section, I briefly report the main results.

The first result is that $\hat{\Lambda}_{t,t+1}$ outperforms these benchmark models in pricing the cross section of Italian stock returns. A successful model predicts that the pricing errors, the residuals of the cross-sectional regression, are close to zero. Appendix table C1 shows that the benchmark specification achieves a better fit with respect to that of the CAPM and of the three-factor model of Fama and French (1993). Moreover, the

analysis shows that the leverage component of $\hat{\Lambda}_{t,t+1}$ is key for pricing the cross section of returns, and this mirrors the mechanism in the structural model. These results are consistent with the work of Adrian et al. (2014), who find that a single factor based on the leverage of brokers and dealers prices the cross section of stock returns in the US economy well.

Second, the estimates of equation (20) are consistent with small average liquidity premia. Although imprecise, the point estimate for b_0 implies an average liquidity premium of 36 basis points in annualized terms, remarkably close to the sample average of the interbank spread (8 basis points) used in the calculation of μ_t. This provides additional support for the measurement of the Lagrange multiplier proposed in this paper, and it stands in contrast to calibrations of the Gertler and Karadi (2011) model encountered in the literature, which typically imply substantially larger liquidity premia.

Third, and consistent with the structural model, stocks that pay out little when $\hat{\Lambda}_{t,t+1}$ is high earn higher excess returns relative to stocks that are less correlated to $\hat{\Lambda}_{t,t+1}$. In terms of magnitudes, the estimated b_1 implies an average annual excess return of 14 percent for an asset with $\beta_j = 1$. Section C in Appendix C shows that the structural model can generate values for $\text{var}_t(\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ that are consistent with these estimates.

V. The Propagation of Sovereign Risk

We now turn to the key experiment of this paper and study the macroeconomic effects of an increase in sovereign risk. As a preliminary step, Section V.A uses impulse response analysis to explain the propagation mechanisms of an *s*-shock. Section V.B proposes a counterfactual to measure the effect of the Italian sovereign debt crisis of 2010–11 on the financing premia of firms and real economic activity. Section V.C discusses in more detail the behavior of consumption conditional on an *s*-shock.

A. Impulse Response Functions

We can start by considering the reaction of liquidity and risk premia to a sovereign risk shock. Figure 3 plots the response of key financial variables to an increase in s of 3.5 standard deviations. The model parameters are fixed at their posterior mean $\bar{\theta}$ in these experiments.

The shock increases the probability of a government default next quarter from 0.09 percent to 2.5 percent. Following this impulse, the price of government bonds declines because of the risk of experiencing losses on these assets in the future. This decline in the price of government securities feeds back into the balance sheet of bankers, generating a fall in their net worth. In this example, these losses are large enough to

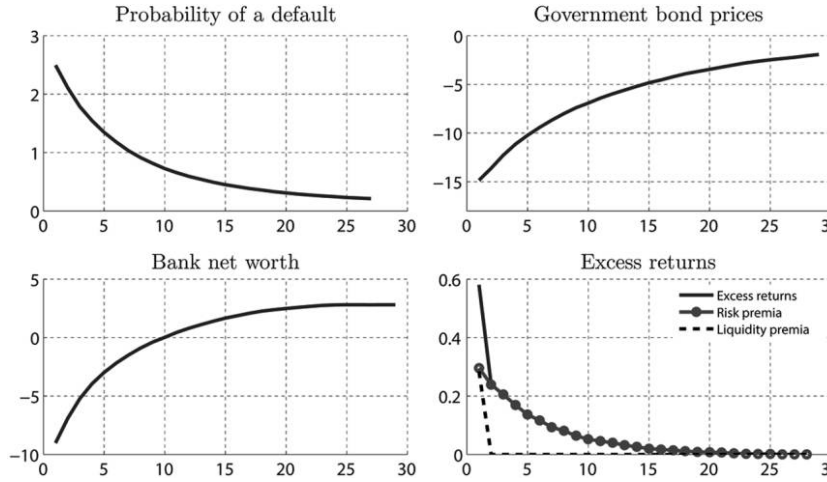

FIG. 3.—Impulse response functions to an *s*-shock: financial variables. Impulse response functions are computed via simulations initialized at the ergodic mean of the state vector. The conditional probability of a default is reported in percentages. Government bond prices and net worth (detrended) are expressed as percentage deviations from their ergodic mean value. Returns are reported in annualized basis points. Color version available as an online enhancement.

trigger the funding constraints of bankers. As a result, the shadow cost of funds for intermediaries increases, and these higher financing costs are passed on to the nonfinancial firms. In figure 3 we can observe that this increase in sovereign risk implies an increase of 30 basis points in the liquidity premium of equation (15).

From the figure, we can also see that the risk premium component increases by 30 basis points following the impulse to *s*. As explained in Section III, this increase in risk premia emerges because bankers now attach a higher likelihood to a state of the world (a sovereign default) in which claims of the productive sector pay out little precisely when bankers are in most need of wealth.

In order to understand the mechanisms that make the *s*-shock a priced factor for firms' claims, we can further decompose the risk premium on these assets as follows:

$$\operatorname{cov}_t\left(-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{K,t+1}\right) \approx \sigma_t\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right) \times \sigma_t(R_{K,t+1}), \tag{22}$$

where the first component, the conditional standard deviation of the (normalized) pricing kernel, can be interpreted as the *price* of risk and the second component, the standard deviation of firms' payouts, can be seen as an indicator of the *quantity* of risk.

TABLE 4
DECOMPOSING RISK PREMIA

	Risk Premia	$\sigma_t\left[\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right]$	$\sigma_t[R_{K,t+1}]$	$\rho_t\left[-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{K,t+1}\right]$
Low s_t	.032	.028	.003	.856
High s_t	.329	.097	.009	.973

NOTE.—The conditional moments are computed by simulations. For the first row, simulate $M = 10{,}000$ realizations for $\{\hat{\Lambda}\}$ and $\{R_K\}$. Each simulation is initialized at the ergodic mean of the state vector and has a length of two periods. The moments are computed using $\{\hat{\Lambda}_2^m\}_{m=1}^M$ and $\{R_{K,2}^m\}_{m=1}^M$. The second row repeats the same procedure, but simulations are initialized as follows: (i) the s-shock is set so that $p_t^d = 0.025$, and (ii) the other state variables are set at their ergodic mean. Risk premia are in annualized basis points.

Table 4 reports these components under two different conditioning sets: one in which the probability of a sovereign default next quarter is 0.09 percent (low s_t) and one in which this probability is 2.5 percent (high s_t). As documented in figure 3, risk premia on firms' claims increase by roughly 30 basis points when default risk changes from 0.09 percent to 2.5 percent. This change in risk premia is due to an increase both in the price and in the quantity of risk: the former increases by a factor of 3.5, and the conditional volatility of firms' payouts triples.

Figure 4 helps us understand this result. There, I plot the density of $\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]$ and $R_{K,t+1}$ under these two conditioning sets. We can observe two main facts from the figure. First, the increase in the price of risk occurs mainly because the right tail of the $\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]$ distribution becomes fatter. After the s-shock, agents place more probability on a default of the government next period. In this state of the world, the funding constraints of bankers are tight because of the large losses on government bond holdings. The anticipation of this event makes bankers effectively act as if they were more averse to risk, and it leads them to charge higher prices for holding risky assets. Second, the increase in the conditional volatility of firms' payouts occurs because the left tail of the $R_{K,t+1}$ distribution becomes fatter. A government default in the model leads to tight funding constraints for bankers. Fire sales of firms' assets by constrained intermediaries put downward pressure on $Q_{K,t+1}$ because of capital adjustment costs. This leads to low ex post returns. When s increases, bankers attach a higher probability to this event, and they therefore perceive future returns on firms' claims as more volatile. In other words, the sovereign default acts as a major economic disaster, and an increase in its likelihood increases the perceived riskiness of the productive sector.[19]

[19] The sources of disaster risk in the model are shocks to the valuation of unproductive assets (government securities) that propagate to the productive ones via the balance sheet of financial intermediaries. In Gourio (2012) and related papers, the sources of disaster risk are shocks to production efficiency.

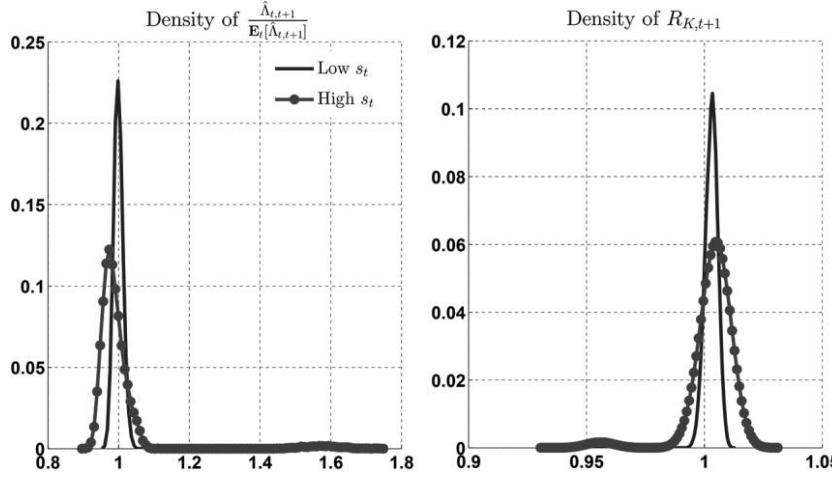

FIG. 4.—Price and quantity of risk. The left panel reports the density of the normalized pricing kernel for different conditioning sets. The solid line is constructed by applying a kernel density smoother to $\{\hat{\Lambda}_2^m\}_{m=1}^M$ simulated conditioning on the ergodic mean as described in the note to table 4 and scaled by their average. The line with circles repeats the same procedure, this time generating the simulations as in the "high s_t" scenario of table 4. The right panel reports the same information for $R_{K,t+1}$. Color version available as an online enhancement.

Figure 5 displays the behavior of macroeconomic aggregates. We can observe that aggregate investment drops by 2 percent on impact. The fall in aggregate investment is related to the behavior of liquidity and risk premia discussed above. On the one hand, the tightening of the funding constraints leads bankers to shed firms' assets. On the other hand, the more cautious attitude of intermediaries and the increase in the perceived riskiness of the firms generate a precautionary motive for the bankers to deleverage and to reduce their exposure to the productive sector. Both of these factors contribute to a reduction in the demand for capital goods. Hours worked and output also decline, respectively, by 0.4 percent and 0.3 percent. Therefore, a mere increase in the probability of a sovereign default depresses real economic activity. The decline in output on impact is entirely driven by the behavior of hours, because capital is predetermined and technology is fixed in this experiment. This drop in hours is motivated by intertemporal labor supply motives: allocating savings to banks is less attractive for households because of agency costs, and this depresses their incentives to work. For this same reason, consumption increases on impact. Therefore, while being recessionary, the *s*-shock generates imperfect comovement between consumption on the one side and investment, output, and hours on the other side. As I explain in Section V.C, the behavior of consumption is not a robust feature

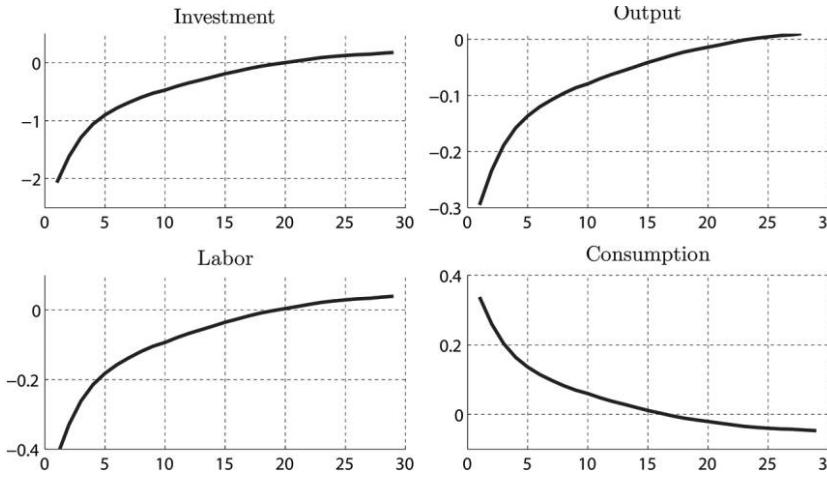

FIG. 5.—Impulse response functions to an s-shock: quantities. See the note to figure 3. Variables are detrended and expressed as percentage deviations from their ergodic mean value. Color version available as an online enhancement.

of the environment, and it can be modified by considering sensible extensions of the model.

Another interesting feature is that the speed of recovery of macroeconomic variables is time varying. In the first few periods after the sovereign risk shock, the leverage constraint binds. The recovery in bank net worth loosens these funding constraints and it stimulates capital accumulation. From period 2 onward, the constraints are not binding anymore and the recovery in net worth has limited positive spillovers: in period 11, net worth is already above trend (see fig. 3), while real economic activity is still depressed partly because of the precautionary motives associated to the risk channel.

B. Sovereign Risk, Firms' Financing Premia, and Real Economic Activity

We can now use the model to measure the effects of the sovereign debt crisis on the financing premia of Italian firms and on real economic activity and to quantify the relative importance of liquidity and risk premia in driving this propagation. This can be done through a counterfactual experiment. Specifically, we can proceed in three steps:

1. Estimate the historical path for the structural shocks. Technically, this is done by applying the particle filter to the observable variables $\mathbf{Y}_t = [\text{GDP growth}_t, \mu_t, p_t^d]$. From this first step, we obtain the

density $p(\varepsilon_t|\mathbf{Y}^t, \theta^i)$ for the structural shocks $\varepsilon_t = [\varepsilon_{z,t}, \varepsilon_{g,t}, \varepsilon_{s,t}]'$ over the period 2010:Q1–2011:Q4.

2. Generate a counterfactual "no sovereign crisis" scenario for the endogenous variables of the model by setting the probability of an Italian default at its ergodic mean throughout the sample. That is, we can feed into the model the historical realizations of the shocks estimated in step 1, with the exception that $\varepsilon_{s,t}$ is set to zero for all t.
3. We can then isolate the effect of heightened sovereign risk on a variable x by taking the difference between its *actual* and *counterfactual* trajectory.[20]

The top panel of figure 6 reports the estimated impact of the sovereign debt crisis of the 2010:Q1–2011:Q4 period on excess returns and on its decomposition into liquidity and risk premia. The model predicts that the sharp increase in the probability of an Italian default in the last two quarters of 2011 led to an increase in excess returns of 60 basis points at peak. The figure also shows that the risk channel was quantitatively an important force in the propagation of sovereign credit risk in Italy: at the end of 2011:Q4, risk premia accounted for 45 percent of the pass-through of sovereign risk on the financing rates of firms. The bottom panel of the figure decomposes further the behavior of risk premia into price and quantity of risk, defined in equation (22). We can verify that both indicators have been substantially affected by the sovereign debt crisis: the price of risk increases by roughly three times, and the conditional volatility of firms' payouts doubles. This last finding suggests that the transmission of risk from the sovereign to the private sector is quantitatively an important force in the model.

We can also explore the effect of the sovereign debt crisis on output. Table 5 reports the difference between actual and counterfactual trajectories in the last two quarters of 2011. The model predicts that output would have been, respectively, 1.05 percent and 1.44 percent higher in annualized terms in the absence of the observed tensions in sovereign markets. Table 5 further computes the predicted effect of the sovereign crisis for 2012. That is, I generate forecasts for linearly detrended output conditioning on two different scenarios in 2011:Q4: the posterior mean of the estimated state vector and its counterfactual mean generated under the "no sovereign crisis" scenario. We can verify that the model estimates output to be persistently below trend following the sharp rise in the probability of an Italian default in 2011.

[20] The actual trajectory for variable x is obtained by fitting the model with the time path for the structural shocks estimated in step 1.

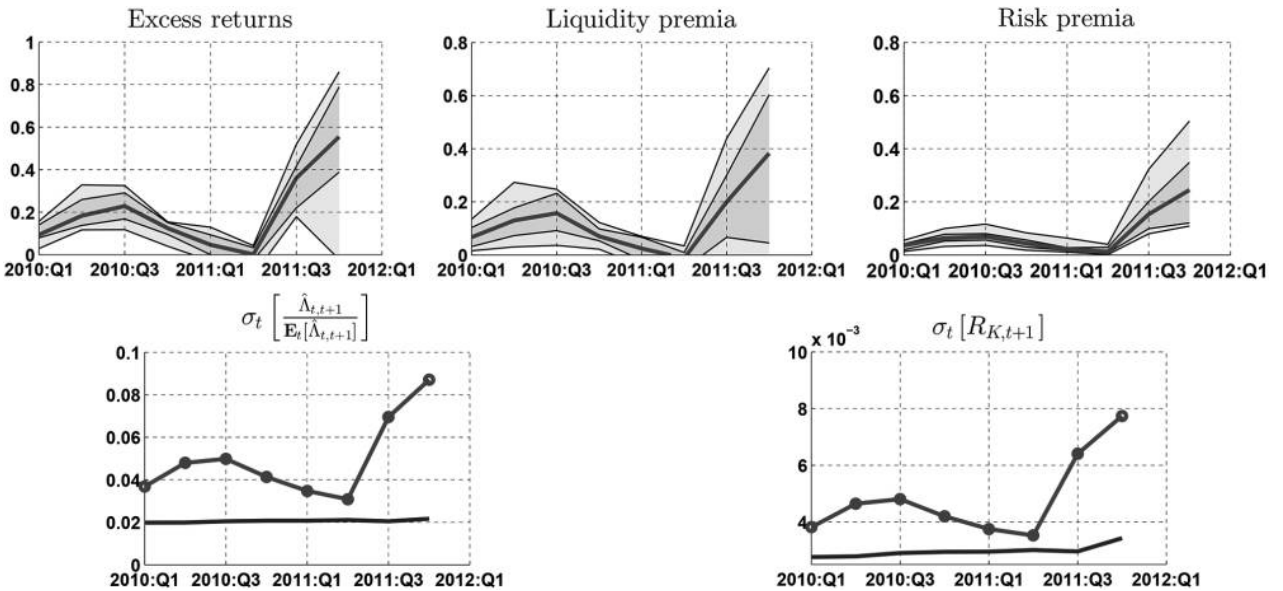

FIG. 6.—Sovereign risk and excess returns. The statistics in the figure are generated as follows. Draw $N = 10$ values of θ from its posterior distribution. For each θ^i, draw $M = 10$ trajectories from $p(\mathbf{S}_t|\mathbf{Y}^t, \overline{\theta})$ for the time period 2010:Q1–2011:Q4. For each trajectory, compute the path for the variables of interest. Next, generate a counterfactual path for the state vector and for the variables of interest by setting $\varepsilon_{s,t} = 0$ for all t. Compute pointwise medians of the difference between these two paths over the M trajectories. The solid lines in the figures are the pointwise 50th percentile of these statistics computed across the N posterior draws. The dark and light shaded areas in the top panels of the figure represent, respectively, a 60 percent and 90 percent equal tail probability credible sets. Color version available as an online enhancement.

TABLE 5
OUTPUT LOSSES OF THE SOVEREIGN CRISIS

	2011:Q3	2011:Q4	2012:Q1	2012:Q2	2012:Q3	2012:Q4
Mean	−1.05	−1.44	−1.53	−1.37	−1.32	−1.23
	[−1.68,	[−2.10,	[−2.21,	[−2.08,	[−2.09,	[−2.04,
	−.32]	−.47]	−.59]	−.38]	−.22]	−.16]

NOTE.—The first two columns report the posterior mean for the differences between the actual and counterfactual levels of output, as described in the notes to fig. 6. The remaining columns report the differences in the point forecasts for output conditioning on the posterior mean of the state vector in 2011:Q4 under the actual and counterfactual scenarios. Equal tail probability 60 percent credible sets are in brackets.

C. *The Response of Consumption to an s-Shock*

As shown in figure 5, when sovereign risk increases, output, investment, and hours fall on impact while consumption rises. This "comovement problem" arises frequently in neoclassical settings when considering the response to nontechnology shocks (Barro and King 1984). Holding technology fixed, the only way that the model can generate a contemporaneous drop in output is through a reduction in hours worked. For this to happen, labor supply needs to decline because K is predetermined and z is held fixed. The comovement problem, then, arises because an increase in consumption is necessary to generate a decline in labor supply under the households' preferences used in this paper.

This feature of the model can be changed with slight modifications to the baseline environment. Specifically, assume that bankers can now borrow from the international capital market at the fixed interest rate R^*. Because of a no-arbitrage condition, the returns on households' savings will also earn the risk-free return R^* in equilibrium. In addition, consider two ingredients that are routinely used in the analysis of small open-economy models: (i) preferences that mute the wealth effect on labor supply and (ii) working capital requirements for the firms. As in Neumeyer and Perri (2005), firms need to borrow a fraction ϕ of the wage bill before production takes place. These loans are obtained from the bankers at the beginning of the period, and they pay the gross return $R_{W,t} = R^* + \lambda(\mu_t/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ at the end of the period.

Figure 7 overlays the responses to a sovereign risk shock for the small open-economy version of the model (lines with circles) and for the benchmark specification (solid lines).[21] First, in the open-economy model, consumption declines conditional on an increase in s. This happens be-

[21] The parameterization of the open-economy model is the same as the benchmark. However, because the preferences described above are not consistent with balanced growth, technology is assumed to follow an AR(1) process. The parameter ϕ is set to one for illustrative purposes.

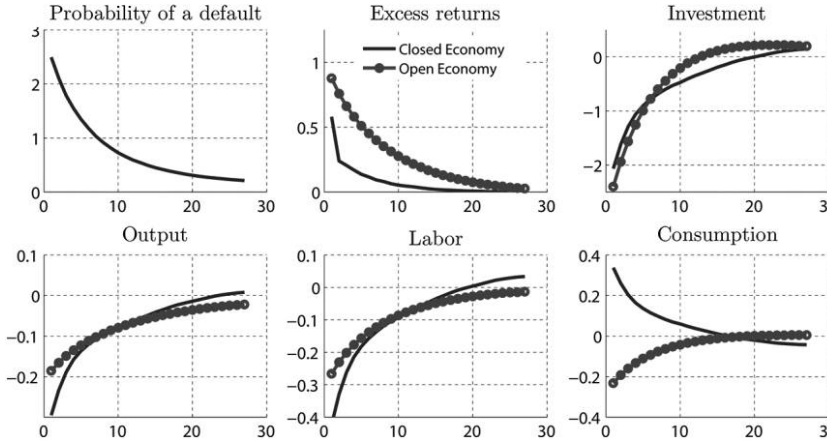

FIG. 7.—Impulse response functions to an *s*-shock: open versus closed economy. See the note to figure 3. Color version available as an online enhancement.

cause the negative wealth effect induced by a tightening of bankers' leverage constraint is not counteracted by the reduction in the risk-free rate, since R^* is now fixed. Second, the implications for investment, output, hours, and excess returns are qualitatively and quantitatively comparable to those obtained in the closed-economy model. However, the underlying forces that generate the decline in hours and output on impact differ. In the open-economy model, the decline in hours worked occurs because the tightening of bankers' funding constraints leads to an increase in $R_{W,t}$, and these higher financing costs for working capital translate into a lower demand for labor by the firm.

VI. Refinancing Operations

In the previous section, we used the structural model to measure the effects of the sovereign debt crisis on the Italian economy and to quantify the relative importance of liquidity and risk in driving this propagation. This section shows that this decomposition provides useful insights into understanding the effects of credit market interventions in this class of models.

This point is illustrated by simulating the effects of the LTROs implemented by the ECB in December 2011 and February 2012. At the height of the euro area sovereign debt crisis, the ECB launched two unconventional LTROs with the aim of loosening funding pressures on the banking sectors of distressed economies. Relative to canonical open market operations in the euro area, these interventions featured a long maturity (36 months), a fixed interest rate, and special rules for the collateral that

could be used by banks.[22] Moreover, the two LTROs were the largest refinancing operations in the history of the ECB, as more than €1 trillion were lent to banks through these interventions. I model LTROs as a nonstationary version of the discount window lending considered in Gertler and Kiyotaki (2010). The government gives bankers the option at $t = 1$ to borrow resources up to a threshold $\overline{m}$. The loans have a fixed interest rate R_m, and bankers repay the principal and all accrued interest at a future date T.[23] The parameters of this policy are calibrated to the actual intervention of the ECB by setting $R_m = 1.00$, T to 12 quarters, and $\overline{m}$ equal to 10 percent of annualized $\hat{Y}^{bg}$.

We can evaluate the effects of the policy on output and the private-sector spreads by simulating the model forward with and without the intervention and by subsequently taking the difference between these two paths.[24] Because the model is nonlinear, the effects of the policy depend on the state variables at which these simulations are initialized. Therefore, an integral part of the evaluation of LTROs is to specify initial conditions. In order to make the exercise more realistic, I choose initial conditions so that the economy is in a state of financial recession. I do so by simulating trajectories from the model and by selecting state variables $\{\mathbf{S}_i^*\}$ at which output growth is 2.5 standard deviations below average and excess returns are 2.5 standard deviations above average. For each $\mathbf{S}_i^*$, I calculate the impact effect of the LTROs on output, excess returns, and their decomposition into liquidity and risk premia. Moreover, I repeat these simulations by drawing initial conditions from $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \bar{\theta})$, the empirical distribution of the state variables for the Italian economy in 2011:Q4.

In order to interpret the results, it is convenient to define the following indicator:

$$\delta_i = \frac{\text{cov}_{\mathbf{S}'_i}\left[-\dfrac{\hat{\Lambda}(\mathbf{S}', \mathbf{S}_i^*)}{\mathbb{E}_{\mathbf{S}'_i}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}_i^*)]}, R_K(\mathbf{S}', \mathbf{S}_i^*)\right]}{\mathbb{E}_{\mathbf{S}'_i}[R_K(\mathbf{S}', \mathbf{S}_i^*) - R(\mathbf{S}_i^*)]}.$$

This variable $\delta_i \in [0, 1]$ can be seen as an indicator of the importance of the precautionary motives described in this paper at a given state $\mathbf{S}_i^*$. When $\delta_i = 0$, expected excess returns purely reflect binding funding constraints for the banks, whereas $\delta_i = 1$ means that the private-sector

[22] Open market operations in the euro area are conducted through refinancing operations. These are similar to repurchase agreements: banks put up acceptable collateral with the ECB and receive cash loans. Prior to 2008, there were two major types of refinancing operations: main refinancing operations (loans with a weekly maturity) and LTROs, with a 3-month maturity.

[23] The government finances this intervention via lump-sum taxes and has perfect monitoring of banks. This last assumption implies that these liabilities do not count for the leverage constraints of bankers and that these government loans do not perfectly crowd out households' deposits. See Gertler and Kiyotaki (2010) for a discussion of this point.

[24] See the online appendix for additional details.

spreads reflect compensation for holding risk. Figure 8 reports the results. Each point represents the impact effect of LTROs on an outcome variable at a given state $\mathbf{S}_i$ and the associated δ_i. The circles report these indicators when the states are drawn from the financial recession simulations while the filled squares report the results when the simulations are initialized from $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \bar{\theta})$.

On average, LTROs lead to an increase in output and a decline in the excess returns that bankers demand for holding firms' assets. However, the figure shows a significant amount of state dependence in the macroeconomic implications of the policy. LTROs are particularly effective in stimulating real economic activity in regions of the state space in which liquidity premia are sizable ($\delta_i \approx 0$). Liquidity constraints, by definition, prevent bankers from undertaking profitable investments: LTROs relax these constraints and stimulate lending to the productive sector. The same intervention, however, has substantially weaker effects when implemented in regions of the space in which risk premia are sizable ($\delta_i \approx 1$). High risk premia are an indication that bankers are not particularly willing to hold firms' claims in their balance sheets. Therefore, refinancing operations tend to have limited effects on real economic activity because bankers use these funds to substitute part of their liabilities with households rather than lending to the productive sector.

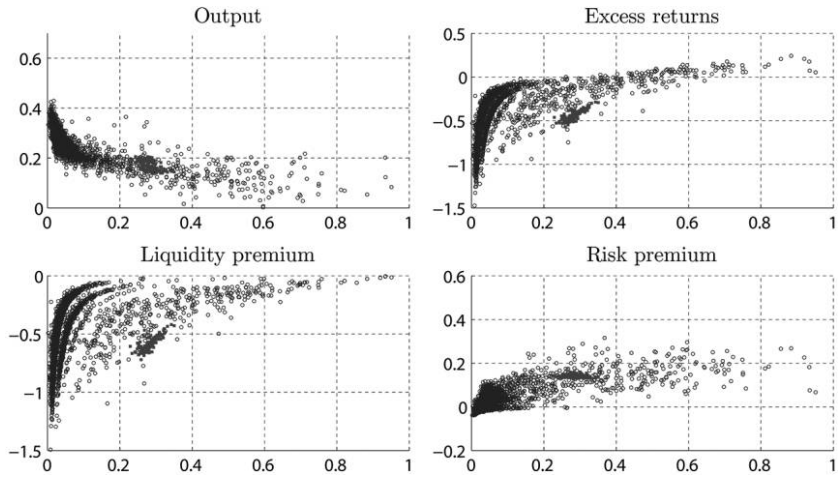

FIG. 8.—The macroeconomic effects of refinancing operations. For each $\mathbf{S}_i$, compute by simulations the impact effect of LTROs on variable x, defined as $\mathbb{E}_{\mathbf{S}_i}[x_1^{\text{ltro}} - x_1^{\text{noltro}}]$ and compute variable δ. Each circle in the figure represents a combination of these two indicators at a particular $\mathbf{S}_i$. The filled circles report the results when $\mathbf{S}_i$ is drawn from the financial recession set, while empty circles report results when simulations are initialized from $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \bar{\theta})$. Returns are in annualized percentages. Color version available as an online enhancement.

The structural model allows one to measure the relative importance of these mechanisms, a decomposition that is important to evaluate these interventions. This can be seen by comparing the effects of LTROs under the two conditioning sets. In a typical financial recession, one would predict refinancing operations to increase output by 0.35 percent on impact and to reduce interest rate spreads by 0.75 percent. When conditioning on the state of the Italian economy in 2011:Q4 instead, these effects are substantially smaller, 0.15 percent and 0.41 percent, respectively. This depends on the fact that precautionary motives, measured by the relative importance of risk premia, were sizable at the height of the debt crisis.

VII. Conclusion

This paper proposed a quantitative model for studying the transmission of sovereign risk on the borrowing costs of firms and on real economic activity. The mere prospect of a future default of the government hampers financial intermediation. On the one hand, banks that are exposed to risky government bonds are less able to borrow from capital markets: this hampers their ability to finance firms (*liquidity channel*). On the other hand, the default of the government may itself be a source of risk for the firms: this reduces the willingness of financial intermediaries to hold claims of the private sector (*risk channel*). The estimation of the model shows that both mechanisms were important during the sovereign debt crisis in Italy. Moreover, the analysis suggests that the provision of liquidity to the banking sector by the ECB had limited effects on bank lending to firms. Indeed, the model detects sizable precautionary motives for financial intermediaries at the height of the debt crisis, which indicates that intermediaries were reluctant to hold claims of the productive sector.

Abstracting from the current application, the analysis underscores the importance of measuring the sources that drive the movements in asset prices. Understanding whether firms' financing premia during crises arise because of lack of liquidity in financial markets or because of compensation for increased risk is key information for policy makers. Incorporating the nonlinearities emphasized in this paper in larger-scale models used for policy evaluation is technically challenging. Moreover, given their relevance for policy, there is a need for developing tools for their empirical validation in the data. These areas represent exciting opportunities for future work.

Appendix A

Proof of Proposition 1

Rewrite the decision problem of a banker by eliminating the demand for households' savings:

$$v^b(n;\mathbf{S}) = \max_{a_B,a_K} \mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}',\mathbf{S})[(1-\psi)n' + \psi v^b(n';\mathbf{S}')]\},$$

subject to

$$n' = \sum_{j=\{B,K\}} [R_j(\mathbf{S}',\mathbf{S}) - R(\mathbf{S})]Q_j(\mathbf{S})a_j + R(\mathbf{S})n,$$

$$\lambda\left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j\right] \leq v^b(n;\mathbf{S}),$$

$$\mathbf{S}' = \Gamma(\mathbf{S}).$$

Guess that the value function is $v^b(n,\mathbf{S}) = \alpha(\mathbf{S})n$, and define $\hat{\Lambda}(\mathbf{S}',\mathbf{S})$ as

$$\hat{\Lambda}(\mathbf{S}',\mathbf{S}) = \Lambda(\mathbf{S}',\mathbf{S})[(1-\psi) + \psi\alpha(\mathbf{S}')]. \tag{A1}$$

Necessary and sufficient conditions for an optimum are

$$\mathbb{E}_{\mathbf{S}}\{\hat{\Lambda}(\mathbf{S}',\mathbf{S})[R_j(\mathbf{S}',\mathbf{S}) - R(\mathbf{S})]\} = \lambda\mu(\mathbf{S}), \tag{A2}$$

$$\mu(\mathbf{S})\left[\alpha(\mathbf{S})n - \lambda\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j\right] = 0. \tag{A3}$$

Substituting the guess in the dynamic program and using the law of motion for n', we can rewrite the decision problem as

$$v^b(n,\mathbf{S}) = \max_{a_B,a_K}\left\{\sum_{j=\{B,K\}} \mathbb{E}_{\mathbf{S}}\{\hat{\Lambda}(\mathbf{S}',\mathbf{S})[R_j(\mathbf{S}',\mathbf{S}) - R(\mathbf{S})]\}Q_j(\mathbf{S})a_j\right\}$$
$$+ \mathbb{E}_{\mathbf{S}}\{\hat{\Lambda}(\mathbf{S}',\mathbf{S})\}R(\mathbf{S})n,$$

subject to the constraint

$$\lambda\left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j\right] \leq \alpha(\mathbf{S})n.$$

The first term on the right-hand side of the objective function equals $\mu(\mathbf{S})\alpha(\mathbf{S})n$.[25] Thus, the value function under the guess takes the following form:

[25] Indeed, when $\mu(\mathbf{S}) = 0$, this term equals zero because risk-adjusted returns on assets demanded by the banker equal $\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R(\mathbf{S})]$ by eq. (A2). When $\mu(\mathbf{S}) > 0$, this term can be written as

$$\mu(\mathbf{S})\lambda\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j.$$

Using the condition in (A3), we can then express $\mu(\mathbf{S})\lambda\Sigma_{j=\{B,K\}}Q_j(\mathbf{S})a_j$ as $\mu(\mathbf{S})\alpha(\mathbf{S})n$.

$$\alpha(\mathbf{S})n = \mu(\mathbf{S})\alpha(\mathbf{S})n + \mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]R(\mathbf{S})n.$$

Solving for $\alpha(\mathbf{S})$ and using the definition of $\hat{\Lambda}(\mathbf{S}', \mathbf{S}')$ in equation (A1), we obtain

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\}}{1 - \mu(\mathbf{S})},$$

which is the expression for the marginal value of wealth in equation (1). The guess is verified if $\mu(\mathbf{S}) < 1$. From equation (A3) we obtain

$$\mu(\mathbf{S}) = \max\left\{1 - \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S})]R(\mathbf{S})\}n}{\lambda[Q_K(\mathbf{S})a_K + Q_B(\mathbf{S})a_B]}, 0\right\} < 1.$$

Finally, financial leverage is the same across bankers when $\mu(\mathbf{S}) > 0$. Therefore, $n/\lambda(\sum_{j=\{B,K\}} Q_j a_j)$ equals $N/\lambda(\sum_{j=\{B,K\}} Q_j A_j)$ when the constraint binds.

Appendix B

Data

A. Credit Default Swap Spread

Daily CDS spreads are on 1-year Italian government securities (RED code: 4AB951), and they are obtained from Markit. The restructuring clause of the contract is CR (complete restructuring). The spread is denominated in basis points and paid quarterly. The data are accessed from Wharton Research Data Services.

B. Banks' Exposure to the Italian Government

The EBA published information on holdings of government debt by European banks participating in the 2011 stress test. Five Italian banks were in this pool: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena, Banco Popolare, and Unione di Banche Italiane. Results of the stress test for each of these five banks are available at http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results. The exposure of each bank to the Italian central and local government is measured as net direct positions (bonds, loans, and advances) plus direct sovereign exposures in derivative contracts. The data are available by the maturity of the financial instrument and reflect positions as of December 31, 2010. This information is reported in table B1. The table reports also total financial assets for these institutions as of December 31, 2010. This latter information is obtained using consolidated banking data from Bankscope, accessed from Wharton Research Data Services.

C. Lagrange Multiplier

As a preliminary step, I introduce an interbank market in the model. Bankers can trade between themselves after observing the realization of **S**, a claim that

TABLE B1
EXPOSURE TO DOMESTIC SOVEREIGN BY MAJOR ITALIAN BANKS: END OF 2010

	3 Months	1 Year	2 Years	3 Years	5 Years	10 Years	15 Years	Total	Total Assets
Intesa	17.18	9.31	2.46	4.87	7.71	6.42	10.12	58.08	658.76
Unicredit	17.78	9.85	2.78	6.12	4.24	5.90	1.44	48.11	929.49
MPS	5.61	4.96	3.92	3.57	1.35	3.71	8.91	32.03	240.70
BPI	3.90	1.65	1.15	3.64	.78	.39	.25	11.76	134.17
UBI	1.27	3.56	.22	.30	.54	2.47	1.76	10.11	129.80
Total	45.75	29.32	10.53	18.5	14.61	18.89	22.48	160.01	2,092.99

NOTE.—Data are reported in billions of euros. MPS = Monte dei Paschi di Siena, BPI = Banco Popolare, and UBI = Unione di Banche Italiane.

offers one unit of the numeraire next period in exchange for $Q_{\text{interbank}}(\mathbf{S})$ units of the numeraire today. I let $a_{\text{interbank}}(n_i, \mathbf{S})$ be the position of a banker with net worth n_i in this market. Market clearing implies $\int a_{\text{interbank}}(n_i, \mathbf{S})di = 0$. Letting $R_{\text{interbank}}(\mathbf{S})$ be the yield on this asset, we can write the bankers' Euler equation as[26]

$$R_{\text{interbank}}(\mathbf{S}) = R(\mathbf{S}) + \frac{\lambda\mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]}. \tag{B1}$$

I now show that the Lagrange multiplier can be expressed as a function of the interbank spread $[R_{\text{interbank}}(\mathbf{S}) - R(\mathbf{S})]$ and of the leverage of bankers.

PROPOSITION 2. In equilibrium, the Lagrange multiplier on the incentive constraint of bankers is a function of financial leverage and of the spread between the interbank rate and the risk-free rate:

$$\mu_t = \frac{\left(\frac{\mathrm{R}_{\text{interbank},t} - R_t}{R_t}\right)\text{lev}_t}{1 + \left(\frac{\mathrm{R}_{\text{interbank},t} - R_t}{R_t}\right)\text{lev}_t}. \tag{B2}$$

Proof. From equation (1), we can write $\mathbb{E}_t[\hat{\Lambda}_{t,t+1}] = \alpha_t(1 - \mu_t)/R_t$. Hence, we can write equation (B1) as

$$\frac{R_{\text{interbank},t} - R_t}{R_t} = \frac{\lambda}{\alpha_t}\frac{\mu_t}{1 - \mu_t}.$$

Equation (B2) follows from the fact that α_t/λ equals financial leverage when $\mu_t > 0$. QED

The data used to construct the time series for the Lagrange multiplier are the following.

Interbank spread.—The prime interbank rate is the Euribor rate on loans with a duration of 1 week. From 2004 onward, this is computed as the average rate for Italian banks on the Euribor panel. Prior to 2004, there is no information on individual banks, and $R_{\text{interbank},t}$ is measured as the average rate across all the banks

[26] Because bankers are indifferent between buying and selling these claims, the introduction of this market does not alter the dynamics of the model.

in the panel. For the computation of the Lagrange multiplier, R_t is the prime rate for interbank secured transactions (Eurepo) on loans with a duration of 1 week. The rates are monthly (2002:M4–2012:M12), and they can be downloaded at http://www.euribor-ebf.eu/euribor-org/euribor-rates.html. Quarterly rates are obtained by compounding the monthly information.

Financial leverage.—I use the Italian quarterly flow of funds (*Conti Finanziari*) to obtain a time series for total financial assets and total equity of the financial sector. First, I match banks in the model with monetary and financial institutions.[27] Second, I construct a time series for bank equity as the difference between total financial assets and total debt liabilities. This latter is defined as total liabilities minus shares and other equities (liabilities) and mutual fund shares (liabilities). Financial leverage is the ratio between total assets and equity. Data can be downloaded at https://www.bancaditalia.it/pubblicazioni/conti-finanziari/.

D. Bank Lending Survey

The indicators of credit tightening and bank funding constraints are constructed from responses to the Bank Lending Survey of the euro area by loan officers at Italian banks. Specifically, I use the responses to question 1 and question 2 in the survey. Data are quarterly (2003:Q1–2012:Q4), and they can be downloaded at https://www.ecb.europa.eu/stats/money/surveys/lend/html/index.en.html.

Question 1: "Over the past three months, how have your bank's credit standards as applied to the approval of loans or credit lines to enterprises changed?" Respondents have five options: "Tightened considerably" (1), "Tightened somewhat" (2), "Remained basically unchanged" (3), "Eased somewhat" (4), and "Eased considerably" (5).

The indicator of credit tightening is the percentage of respondents that indicated either 1 or 2 less the percentage of respondents that indicated either 4 or 5. A value of 100 percent means that all the respondents in the survey indicated that credit standards have tightened considerably or somewhat in the preceding quarter. The indicator is demeaned and standardized.

Question 2: "Over the past three months, how have the following factors affected your bank's credit standards as applied to the approval of loans or credit lines to enterprises changed (as described in question 1)? Please rate the contribution of the following factors to the tightening or easing of credit standards." Respondents have nine options, divided into three categories. Category A ("cost of funds and balance sheet constraints"): "Cost related to your bank's capital position" (1), "Your bank's ability to access market financing (e.g., money or bond market financing, incl. true-sale securitization)" (2), "Your bank's liquidity position" (3). Category B ("Pressure from competition"): "competition from other banks" (4), "competition from non-banks" (5), "competition from market financing" (6). Category C ("Perception of risk"): "expectations regarding general economic activity" (7), "industry or firm-specific outlook" (8), "risk on the collateral demanded" (9).

[27] This category includes commercial banks, money market funds, and the domestic central bank. I use balance sheet information for the Bank of Italy to exclude the latter from this pool.

The indicators of banks' funding constraints are the percentage of respondents who replied that factor 1, 2, or 3 contributed "considerably" or "somewhat" to a tightening of credit less the percentage of respondents who indicated that these same factors contributed "considerably" or "somewhat" to an easing of credit. In the same vein, I obtain indicators for the remaining six factors. These variables are demeaned and standardized.

Table B2 reports the results of a regression of each of these indicators on the measured Lagrange multiplier.

E. Other Time Series

Real gross domestic product: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1999:Q1–2012:Q4.

Real private final consumption expenditures: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1999:Q1–2012:Q4.

Real general government final consumption expenditures: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1999:Q1–2012:Q4.

Real gross capital formation: OECD Quarterly National Accounts, millions of euros, volume estimates (reference year 2010), 1999:Q1–2012:Q4.

TABLE B2
BANK LENDING SURVEY AND THE LAGRANGE MULTIPLIER

Dependent Variable	$\frac{\mu_t-\bar{\mu}_t}{\sigma(\mu_t)}$	R^2
Credit standards	.62	.41
	(5.48)	
Capital position	.42	.19
	(3.02)	
Ability to access market finance	.74	.59
	(2.94)	
Liquidity position	.65	.45
	(2.29)	
Competition from banks	.22	.05
	(2.38)	
Competition from nonbanks	−.16	.03
	(−.71)	
Competition from market	−.28	.08
	(−.98)	
Expectations of economic activity	.31	.10
	(1.76)	
Industry or firm-specific outlook	.10	.01
	(.71)	
Risk on collateral demanded	.16	.03
	(.96)	

NOTE.—Each row reports the results of a univariate regression of the indicators constructed from the Bank Lending Survey on the (standardized) Lagrange multiplier. Intercepts are not reported. Robust *t*-statistics are in parentheses.

Labor income share: EU KLEMS, ratio of total labor compensation and value added, 1970–2007 (annual). Data can be downloaded at http://www.euklems.net/.[28]

Worked hours: EU KLEMS, average numbers of hours worked per year by person engaged, 1970–2007 (annual). The series is scaled by $(24 - 8) \times 7 \times 52$. Data can be downloaded at http://www.euklems.net/.

Appendix C

Evidence from the Cross Section of Italian Stock Returns

From equation (15), we can write expected excess returns on any asset j held by bankers as

$$\mathbb{E}[R^e_{j,t+1}] = b_0 + b_1\beta_j, \tag{C1}$$

where $b_0 = \mathbb{E}[\lambda\mu_t/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]]$ is the mean of the liquidity premium, common to all assets;

$$\beta_j = \frac{\operatorname{cov}\left(-\dfrac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{j,t+1}\right)}{\operatorname{var}\left(\dfrac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)} \tag{C2}$$

is the beta of security j, an indicator of the risk of the asset; and $b_1 = \operatorname{var}(\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ can be thought as the price for holding risk. These moments are taken with respect to the ergodic distribution of the model state variables.

This section explains how (b_0, b_1) are estimated using information from the cross section of stock returns and compares these estimates with corresponding indicators of liquidity and risk premia obtained from the structural model. Section A discusses the data, Section B describes the procedure, and Section C presents the results. The online appendix reports some sensitivity analysis.

A. Data

Data on stocks: End-of-month returns for stocks traded on the Italian exchange are obtained from Compustat Global Security Daily. I select stocks whose country of incorporation is Italy. For each of these stocks, I obtain the following information: global company key, company legal name, Standard Industrial Classification code (SIC), shares outstanding (CSHOC), trading volume (CSHTRD), closing price (PRCCD), daily total return factor (TFRD), adjustment factor (AJEXDI), and end-of-month indicator. The returns for security j in month t are defined as

[28] EU KLEMS is an industry-level and productivity research project funded by the European Commission. The acronym stands for EU-level analysis of capital (K), labor (L), energy (E), materials (M), and service inputs (S).

$$R_{j,t} = \log\left(\frac{\frac{\text{PRCCD}_{j,t}}{\text{AJEXDI}_{j,t}}\text{TFRD}_{j,t}}{\frac{\text{PRCCD}_{j,t-1}}{\text{AJEXDI}_{j,t-1}}\text{TFRD}_{j,t-1}}\right).$$

I construct $R_{j,t}$ for each stock in the sample for the period 1999:M1–2011:M12, and I drop securities that have missing observations over the sample period. This leaves 116 stocks. I further restrict the sample by discarding securities whose average trading volume falls in the bottom 10th percentile. As is customary in the literature, I aggregate these stocks into a smaller number of portfolios. I consider two procedures:

- *Industry and size.* Stocks are partitioned by industry into manufacturing (SIC $\leq$ 30), services (SIC $\in$ [40, 60) and $>$ 70), and financial (SIC $\in$ [60, 70)). Within each group, stocks are sorted by quintiles of the size distribution, obtaining 15 portfolios. Returns on the portfolios are value weighted.
- *Beta.* First, the beta of each stock is estimated via a time-series regression (see below for details). Second, stocks are partitioned by the deciles of the estimated distribution of the betas, obtaining 10 portfolios. Returns on the portfolios are value weighted.

Quarterly returns are obtained by compounding the monthly series.

Risk-free rate: The risk-free rate is matched with the yields on zero-coupon German government securities with a residual maturity of 6 months (Nelson-Siegel-Svensson method). The data are obtained from the Deutsche Bundesbank at http://www.bundesbank.de/Navigation/EN/Statistics/statistics.html.

Stochastic discount factor: I construct a quarterly time series (1999:Q1–2011:Q4) for $\hat{\Lambda}_{t,t+1}$ using equation (21) and the growth rate of real private final consumption expenditures (Δc_{t+1}), the financial leverage indicator (lev_{t+1}), and the posterior mean of (β, ψ, λ).

B. Two-Pass Estimation

The estimation of (b_0, b_1) is accomplished using a two-pass procedure. First, the beta of each portfolio is estimated via time-series regressions. Specifically, I consider a first-order log-linear approximation to the stochastic discount factor

$$\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \approx 1 + \log(\hat{\Lambda}_{t,t+1}) - \mathbb{E}_t[\log(\hat{\Lambda}_{t,t+1})] = 1 + e_{\hat{\Lambda},t+1},$$

and I obtain the innovations $\{e_{\hat{\Lambda},t+1}\}$ by fitting an AR(1) process to $\log(\hat{\Lambda}_{t,t+1})$. The beta defined in equation (C1) is estimated as the slope coefficient of a linear projection of $R_{j,t+1}$ on $e_{\hat{\Lambda},t+1}$ for portfolio j.

Second, after averaging $\{R^e_{j,t}\}$ over time, I estimate (b_0, b_1) via a cross-sectional regression of average excess returns on their respective betas. In the benchmark

specification, I pool the two sets of portfolios to obtain 25 observations in this cross-sectional regression. Moreover, I restrict the estimation to the 1999:Q1–2007:Q4 period in order to minimize the impact of the extremely low realized excess returns observed during the crisis on the sample averages. The online appendix checks the sensitivity of the results to these choices.

C. Results

Column 1 of table C1 reports ordinary least squares estimates for the cross-sectional regression (C1). The remaining columns report estimates for alternative specifications of equation (C1) that differ in the "factors" used to calculate the betas of the portfolios. Columns 2 and 3 estimate two benchmarks in the empirical asset-pricing literature, the CAPM and the three-factor model of Fama and French (1993).[29] In column 4, the betas are calculated with respect to a consumption-based version of $\hat{\Lambda}_{t,t+1}$ obtained by setting ψ to zero in equation (21).[30] The bottom panel of the table reports diagnostics for the cross-sectional regressions: the adjusted R^2, confidence interval for this statistic, the mean absolute pricing error (in annualized percentages), and a test statistic for the null hypothesis that the pricing errors are jointly zero with its associated p-value. Test statistics and p-values are computed using the method of Shanken (1992) that corrects for the uncertainty associated with the estimation of the betas in the first step of the procedure. Confidence intervals on the adjusted R^2 are computed by bootstrap, following Lewellen, Nagel, and Shanken (2010).

Regarding the estimates, the fit of the benchmark specification outperforms that of the CAPM and of the three-factor model of Fama and French (1993). For example, the first specification achieves an R^2 of 49 percent, while these two benchmark asset-pricing models achieve, respectively, 19 percent and 26 percent. Moreover, a comparison of columns 1 and 4 reveals that the leverage component in $\hat{\Lambda}_{t,t+1}$ is important for pricing the cross section of stock returns: the consumption-based version of $\hat{\Lambda}_{t,t+1}$ achieves an R^2 of 1 percent.

The point estimate of b_0 suggests small liquidity premia on average, 36 basis points in annualized terms. However, this coefficient is not significantly different from zero. The point estimate of b_1 implies an annualized premium of roughly 14 percent for an asset with a $\beta_j = 1$. It is interesting to compare the estimated b_1 with comparable indicators in the structural model. Table C2 reports $\text{var}_t(\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ under different conditioning sets, along with other statistics that are relevant to understanding its behavior. Specifically, the table reports the expected value of bankers' marginal value of wealth conditional on the constraints binding next period and the conditional standard deviation of the two components

[29] The "factors" for the CAPM and the three-factor model of Fama and French are downloaded from the Stefano Marmi data library at http://homepage.sns.it/marmi/Data_Library.html.

[30] For these three specifications, the second set of portfolios is generated by sorting stocks according to their betas with the factors considered. For the three-factor Fama and French specification, the stocks are sorted with respect to their beta with the market return.

TABLE C1
ORDINARY LEAST SQUARES ESTIMATES OF EQUATION (C1)

	Benchmark (1)	CAPM (2)	Three-Factor Fama-French (3)	No Leverage (4)
Intercept	.36	10.68	6.38	1.43
	(.23)	(.76)	(.88)	(.26)
$\beta^{\hat{\Lambda}}$	14.94			
	(3.87)			
β^{MKT}		−9.30	−5.92	
		(5.70)	(5.71)	
β^{SMB}			1.68	
			(3.61)	
β^{HML}			7.06	
			(3.86)	
$\beta^{\hat{\Lambda},\psi=0}$				−.00
				(.25)
R^2	.49	.19	.26	.01
Lower-bound R^2	.40	.02	.02	.01
MAPE	4.60	4.56	4.55	5.29
$T^2(\chi^2)$	16.84	22.33	21.75	24.00
p-value	.86	.56	.47	.46

NOTE.—The dependent variable is the sample mean of annualized excess returns for the 25 portfolios. Column 1 estimates eq. (C1). The remaining columns estimate, respectively, the CAPM, the three-factor model of Fama and French, and a specification of eq. (C1) in which ψ is set to zero in eq. (21). The bottom panel reports test diagnostics, including R^2, the lower bound on an 80 percent confidence interval for the sample R^2, the mean absolute pricing errors (MAPE), and a χ^2 statistic that tests whether the pricing errors are jointly zero. Standard errors and p-values are computed using the method of Shanken (1992).

TABLE C2
PRICE OF RISK IN THE MODEL

	Ergodic Mean (1)	$p_t^d = .05$ (2)	$p_t^d = .05$ and Low N_t (3)	Filtered States (4)
$400 \times \text{var}_t\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)$	.24	6.28	19.84	3.75
$\mathbb{E}_t[\alpha_{t+1} \mid \mu_{t+1} = 1]$	1.04	1.21	1.45	1.11
$100 \times \sigma_t(\alpha_{t+1})$	2.30	14.13	32.09	10.57
$100 \times \sigma_t(\Delta c_{t+1})$	.45	.60	.61	.54

NOTE.—The conditional moments are computed by simulation. For col. 1, simulate $M = 10{,}000$ realizations for $\{\hat{\Lambda}\}$, $\{\alpha\}$, $\{\mu\}$, and $\{\Delta c\}$. Each simulation is initialized at the ergodic mean of the state vector and has a length of two periods. The moments are computed using the second period of these simulations. The remaining columns repeat this procedure, but simulations are initialized at a different point of the state space. Column 2 initializes the simulations as follows: (i) the s-shock is set so that $p_t^d = .05$, and (ii) the other state variables are set at their ergodic mean. Column 3 initializes the simulations as in col. 2, with the exception that P is set so that net worth is three standard deviations below average. Column 4 initializes the simulations at the mean of the state vector under $p(\mathbf{S}_t|\mathbf{Y}^{2011:Q4}, \bar{\theta})$, computed with the particle filter.

of the stochastic discount factor: the marginal value of wealth of the banker and aggregate consumption growth.

Column 1 of table C2 reports these statistics when the state vector is at the ergodic mean. We can verify that the price of risk is small in this region of the state space. This happens for two reasons. First, at the ergodic mean, bankers have a buffer stock of wealth that insures them against regular fluctuations in the price of capital. Second, the probability of a government default next period is small ($p^d = .001$). Hence, balance sheet risk for financial intermediaries is negligible, and the model does not generate a particularly volatile stochastic discount factor.

The model, however, can generate substantially larger values for $\text{var}_t(\hat{\Lambda}_{t,t+1}/\mathbb{E}_t[\hat{\Lambda}_{t,t+1}])$ in regions of the state space where the risk faced by intermediaries is not trivial. Column 2 of table C2 reports the same statistics when the likelihood of a sovereign default next period is 5 percent, holding fixed the other state variables at their ergodic mean values. Now bankers anticipate that they may face severe losses next period on their government bond holdings, and these losses may tighten their funding constraints. Moreover, the net worth losses for the bankers are large in those states of the world, implying a substantial increase in their marginal value of wealth: from the second row of table C2, we can verify that the mean of α_{t+1} conditional on the constraints binding next period increases from 1.04 to 1.21. This explains why the conditional volatility of α_{t+1} is high in this region of the state space: bankers now place more mass on the right tail of the α_{t+1} distribution, and this implies that its conditional distribution spreads out. The higher volatility of α_{t+1} translates into a higher conditional volatility for the stochastic discount factor: the price of risk is now 6.28 percent in annualized terms. Interestingly, the last two rows of the table clarify that the asset-pricing properties of the model are mostly driven by α_{t+1} rather than by the behavior of consumption growth. This feature of the model is consistent with the result, discussed earlier, that the leverage component of $\hat{\Lambda}_{t,t+1}$ is essential for pricing the cross section of Italian stock returns. Also, the balance sheet risk implied by the prospect of a sovereign default depends on the other state variables in the model. For instance, column 3 shows that the price of risk reaches 19.84 percent once we condition on $p_t^d = .05$ and on low initial values of bankers' net worth (three standard deviations below average). These examples indicate that the model is capable of generating values for the price of risk in the range of those estimated using cross-sectional information on stock returns.

Column 4 of table C2 reports these calculations for a region of the state space that is empirically relevant for the Italian economy during our sample. Specifically, I use the particle filter to estimate the state of the Italian economy in 2011: Q4, and I compute the above statistics conditioning on these states.[31] In this region of the state space, the model produces a price of risk of 3.75 percent, roughly one-third of the estimated value in table C1.

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[31] See Sec. V.B for details on the filtering procedure.

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